# Conceptualising (mathematics teacher educator) identity work: an enactivist-informed approach

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Research suggests that engaging in identity work is a vital part of a process of becoming a mathematics teacher and that mathematics teacher educators (MTEs) are key to supporting this process, yet we currently know very little about the identity work of MTEs themselves. In this paper, we contribute to this dearth of literature whilst exploring what the perspective of enactivism has to offer identity research given its emphasis on relationships and embodied action. We formulate our conceptualisation of MTE identity work as a set of seven methodological principles informed by the enactivist theory of cognition.

Identity research has contributed significantly to our understanding of mathematics teaching and learning, as well as to our understanding of mathematics teacher development. For the last twenty years or so, there has been a growing interest in teacher identity both within mathematics education and beyond (Darragh, 2016). For some time, scholars have emphasised the importance of understanding how teachers' knowledge and beliefs relate to their classroom practices (e.g. Hill et al., 2007; Philipp, 2007; Sowder, 2007), however, there has been little consensus regarding the nature of this relationship (Skott, 2015) and seldom a strive towards understanding and documenting professional growth and development from teachers' perspectives (Battey & Franke, 2008). The study of mathematics teacher identity emerged partly from the recognition of this gap in understanding and has been further driven by concerns about the challenges of implementing those teaching and learning practices promoted during mathematics teacher education or professional development programmes (Skott, 2019).

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Findings from identity studies provide important insights that translate into implications for the education and professional development of teachers at all stages of their careers (e.g. Lutovac & Kaasila, 2014). Teacher education programmes for prospective mathematics teachers commonly consist of university-based components and school-based components (also referred to as "school placements" or "practicum"). According to Walshaw (2010), both practicum schools and universities are sites for the "production and regulation of teachers' subjectivities" (p. 119). In relation to university-based components of teacher education, Walshaw claims "the university course [is] a powerful factor in the construction of an identity as a teacher [...] through producing mechanisms to shape [teachers'] knowledges, modes of operating and positionings" (p. 119). Since the process of constructing teaching identities is an unavoidable consequence of teacher education programmes, there is potential for mathematics teacher educators (MTEs) to make the most of this opportunity. It is now acknowledged, for example, that *identity work*, a "teacher's active and intentional work on their identities through various reflective activities" (Lutovac & Kaasila, 2018a, p. 254), can be deliberately designed by MTEs and incorporated into teacher education programmes for prospective teachers as a way of shaping identities in relation to, say, mathematics or mathematics teaching. Neumaver DePiper (2013), for example, explored how to support prospective teachers as they engaged in identity work, that is, "their struggles with the internal and external forces that shape their understandings of being a mathematics teacher and of mathematics teaching" (p.9), through a course designed to encourage the prospective teachers to question "prevailing discourses of students, mathematics, and teaching" (p. 10) in relation to their own sense of self. De Freitas (2008), though not using the phrase identity work, facilitated the identity development of a group of prospective mathematics teachers through their participation in a secondary mathematics methods course that included writing a mathematics auto-ethnography designed to "increase pre-service mathematics teachers' awareness of their own privilege and power in teaching such a high-stakes discipline" (p. 50). Lutovac and Kaasila (2011) researched the use of the narrative tools, specifically, narrative rehabilitation and bibliotherapy, as a form of identity work that was found to enhance prospective teachers' views of mathematics. Such examples demonstrate the potential for identity work to be incorporated into teacher education programmes as a counterpart to what seems to be a more common focus on developing pedagogical content knowledge.

Though rarely the focus of research on identity work, MTEs, who shape the learning and development of prospective teachers, are undoubtedly integral participants in the identity work of those they teach. Since MTEs must "contend with multiple and potentially conflicting responsibilities when balancing a focus on [prospective teachers'] personal identity and a focus on mathematics teacher preparation" (Neumayer DePiper, 2013, p.9), researchers could be more concerned with those who *facilitate* the identity work of teachers. For example, if there is an assumption (by an MTE) that prospective teachers need to reposition themselves in relation to mathematics and mathematics teaching in order to become effective teachers, this assumption could be problematised (as opposed to it remaining unquestioned). One way to uncover and challenge these kinds of assumptions is for MTEs to themselves engage in a process of identity work. It is surprising, therefore, that we still know very little about the identity formation of MTEs, including those who transition from first-order teaching (i.e. teaching mathematics to students in schools) to second-order teaching (teaching mathematics teachers in universities), a topic that has been taken up more widely outside of mathematics education (see Williams et al., 2012). The authors of this paper were themselves experienced mathematics teachers before moving to their respective universities as MTEs.

Tracy, as a new MTE in 2016, embarked on her own process of identity work (Helliwell, 2021) when she realised how unequipped she was in her new role having transitioned to becoming a university-based MTE.

The sense of loss I was feeling led me at times to question my move to mathematics teacher education from what had become a comfortable position in school where I felt both confident and respected. I was determined to learn though, to develop myself as a mathematics teacher educator, to feel that sense of belonging that I had felt in school. I recognised that I needed to work differently from the way I would have worked with the mathematics teachers from my own department when I was in school but knowing this was only the initial step to becoming a mathematics teacher educator. With an awareness of the need to change, I found myself facing what felt like a necessary process of letting go of certain ways of being, that I had previously valued so strongly, to become something other. At times, I wondered if it might have been less problematic to have been starting from scratch (Helliwell, 2021, p. 14)

Although not an uncommon route into teacher education (from teaching in schools), research suggests there is still a lack of formal education available for beginning MTEs to support this transition as well as their ongoing work in teaching mathematics teachers. Since identity research within mathematics education has typically focussed on those teaching mathematics and those learning mathematics (Darragh, 2016) and is "seldom [focussed on] the identities of those educating future teachers" (Graven & Heyd-Metzuyanim, 2019, p. 371), there is an opportunity for identity research to start contributing to our understanding of MTE growth and development. With this paper, we contribute to this dearth of literature on the identity formation of MTEs.

In their reviews of identity research within the field of mathematics education, both Darragh (2016) and Lutovac and Kaasila (2018b) report that the majority of research consists of small-scale studies involving a small number of participants (e.g. case studies) that use data consisting of interviews, autobiographies, and narratives. Given the proliferation of identity research based on student and teacher self-reports, Lutovac and Kaasila suggest that "greater emphasis be placed on observations of practice" as a way of providing "new directions and deeper insights into teaching identity" (p. 770). Based on a similar observation, Graven and Hevd-Metzuvanim (2019) urge the field to consider "new and more elaborate ways of looking at identity from action, rather than (only) from selfreports" (p. 374). We take up these challenges by considering what the perspective of *enactivism* has to offer identity research given its emphasis on relationships and embodied action. Specifically we present our conceptualisation of (MTE) identity work that is informed by an enactivist perspective. The inclusion of parentheses around "MTE" is to express the more inclusive nature of our conceptualisation of identity work which we suggest could be applicable beyond the domain of MTEs to other practitioners concerned with their professional and personal growth and development.

Before presenting our conceptualisation of (MTE) identity work, we explore existing research on MTE identity before discussing the theoretical bases of identity research within mathematics education.

## MTE identity research

Although it has become well-acknowledged that the process of identity development is an intrinsic part of learning to teach mathematics, research relating to MTE identity has not yet received substantial attention (Lloyd et al., 2021). The majority of existing studies on MTE identity are conducted by MTE researchers utilising self-based methodologies as a way of researching their own identity formation. According to Chapman (2020), self-based methodologies primarily comprise of *narrative inquiry, self-study*, and *autoethnography* with each one privileging self in the research design in distinct ways. Hamilton et al. (2008, p. 17) describe these distinctions as: "a look at a story of self" (narrative inquiry); "a look at self within a larger context" (auto-ethnography); and "a look at self in action" (self-study) and although each self-based methodology usually incorporates narrative tools (e.g. life writing, collecting personal stories, keeping reflective journals), these tools are employed for different purposes. Self-study scholars, for instance, seek to enhance their teaching practices by examining their practical and personal knowledge (Chapman, 2020) in relation to their values, beliefs, and contexts. Examples of MTE identity research where self-based methodologies are utilised include Lloyd et al. (2021) who use duoethnography as a methodology to investigate their MTE identities in relation to prospective teachers' emergent mathematics instruction; Osborn et al. (2021) who use narrative inquiry to examine their collective identity as MTEs; and Kastberg and Grant (2020) who explore the characteristics of their critical friendship in relation to their MTE identities as part of a self-study (see also Allen et al., 2016; Cross Francis et al., 2022; Knapp, 2017; Weinberg et al., 2021). Beyond those studies of self, research on MTE identity can be found in the work of Karsenty et al. (2021) who analyse the identities of three facilitators of professional development for mathematics teachers (a sub-group of MTEs) as one component of their decision-making and in the work of Goos and Bennison (2019) who use Valsiner's zone theory to understand identity formation of mathematics teachers and MTEs. creating a theory of "goal-directed change" (p. 405) that the authors use to understand identity development.

In relation to the identity formation of individuals who have transitioned from teaching students in schools to teaching teachers, more research exists outside of mathematics education. Dinkelman et al. (2006), for example, report on the struggles that two beginning teacher educators experienced in becoming teacher educators, a process they describe as *recasting* their teacher identities. Similarly, Amott (2018) discusses the problematic nature of the identity transformation of school teachers who become teacher educators, a transition that has been described as "*expert become novice*" (p. 477, emphasis original). Amott advocates for narrative practices as a way of supporting beginning teacher educators in developing their identities.

In their survey of research on teacher educators' identities, Kaasila et al. (2023) distinguish between research on teacher educators' holistic identities (understood as a nexus of teaching and researching identities) and on their teacher identities suggesting it is the latter that is mainly absent from the literature base. According to these authors, the ways in which "teacher educators make sense of their role of being a teacher of teachers has not yet received substantial attention in the literature" (Kaasila et al., 2023, p. 3). To understand who teacher educators

are, Swennen et al., (2010) conducted a review of research literature about teacher educators and found four teacher educator sub-identities (described explicitly or implicitly) that constitute (to differing extents) the main identity of teacher educators: school teacher; teacher in higher education; teacher of teachers; and researcher. These authors also report on the need for teacher educators to transform their identity as teachers in becoming a teacher of teachers. We see in both of these examples, the beginnings of a category-based conception of teacher educator identity, a phenomenon we also observe in the work of Llovd et al. (2021) who establish several sub-identities such as methods course instructor; supervisor; teacher; and researcher, that shape their classroom observations. As the body of research concerning MTE identity grows, one possibility is that it follows a similar direction to research on MTE knowledge that commonly builds on category-based models of mathematics teacher knowledge (e.g. Ball et al., 2008). We see our own research on MTE identity as diverging from a category-based conceptualisation (i.e. the *what* of MTE identity) towards a more relational, process-oriented view of MTE identity (i.e. the *how* of MTE identity).

# The theoretical bases of identity research

Darragh (2016) describes what she views as the two main perspectives on identity development within mathematics education. The first perspective comes from a psychological paradigm where identity is viewed as something internal to an individual, something an individual acquires or possesses. The second perspective comes from a sociological paradigm where identity is considered as an action within specific social situations. From within this second paradigm, Darragh identifies three broad themes of identity research: *performative identity*, which examines how language influences social interactions and shapes individuals' identities in social practices; *narrative identity*, which focuses on a person's evolving life story as they do identity work; and *participatory identity*, which is related to social practice theory and particularly the concepts of figured worlds (Holland et al., 1998) and communities of practice (Wenger, 1998).

How identity is conceptualised (e.g. as an entity/state, or as a process/ action) will determine the methodological decisions that researchers make. A commonly adopted conceptualisation within mathematics education is the one formulated by Sfard and Prusak (2005) who describe identity as a discursive construct and define identities as "collections of stories about persons" (p. 16) thus proposing that multiple identities exist for any one individual (depending on who is doing the telling and to whom). By adopting a narrative-defined concept of identity such as this one, identity can become operationalised through collecting stories from and about research participants such as mathematics teachers. From this perspective, the stories collected do not give researchers access to the participating teachers' identities (as in the psychological sense of the word), rather, these stories are their identities which evolve as new and different stories are told. Here, the process of story-telling is one of identity-building. In defining mathematics teacher identity as a discursive construct, a distinction is being made between who mathematics teachers (say they) are and what mathematics teachers do. According to Sfard and Prusak, teachers' identities *originate* from their daily practices and in turn *shape* their future practices, but the practices themselves (i.e. what teachers do) are not a necessary focus of research.

Within mathematics education research, a misalignment is often reported between mathematics teachers' self-reports of their teaching practices and their observed teaching practices. This distinction has been framed in different ways. Ernest (1989), for example, refers to mathematics teachers' espoused model of teaching and learning mathematics and their enacted model. Some mathematics education research reports on consistencies between teachers' espoused and enacted models (e.g. Stipek et al., 2001), but more commonly, research reports on inconsistencies (e.g. Beswick, 2004; Cooney, 1985; Voss et al., 2013). Sfard and Prusak (2005) are careful to acknowledge that "[w]hat a person endorses as true about herself may be not what others see enacted" (p. 17). They deal with this potential contradiction by distinguishing between the various identities authored by a range of people about a particular individual, maintaining that narratives remain useful in research given their role in *shaping* an individual's actions, "even if they communicate one's experiences only as well as human words can tell" (p. 17).

As MTEs committed to enhancing our teaching practices, we are less interested in the content of the stories we tell (i.e. who we are), and more interested in how we are *shaping* our individual actions (i.e. what we do), since it is our actions that make a difference to the mathematics teachers that we teach (and will in turn shape the stories others tell about our teaching). Sfard and Prusak (2005) suggest the focus of research should not be on the identities (stories) themselves but on the "dialectic between identity-building and other human activities" (p. 17) but in doing so suggest a separation between the two. As an alternative, we turn our attention to *the relationship between* the narratives we tell ourselves (and others) and our enacted practices as MTEs. For this reason, the notion of *identity work* is a particularly useful one which we view, in this study, to be part of a process of becoming MTEs. One way our research diverges from those studies based purely on narrative forms of data, is our focus on those *other human activities*, which for us consist of our actions and interactions with teachers of mathematics whose learning we shape.

In *a conversation with Etienne Wenger* (Farnsworth et al., 2016), Wenger suggests that although reifications (i.e. narrative markers of our identity) are usually easier to collect as visible evidence, they are only "half of the story" (p. 147) of identity where participation is the other half. In their survey of the state of the art in mathematics identity research, Graven and Heyd-Metzuyanim (2019) urge those in the field to consider participation and push forward with "methodologies that extract identity from classroom activity" (p. 374) offering examples where researchers (e.g. Andersson & Wagner, 2019) have studied the discursive actions of individuals as a way of examining their identities, going beyond individuals' self-reports.

Whether identity is conceptualised as a collection of stories about a person or as something that can be "extracted" from the discursive actions of individuals or groups, research often emphasises the rational nature of identity as opposed to, say, its tacit, intuitive and embodied nature. In our own formulation of identity work, we look at what *situated cognitive theories* have to offer identity research, and specifically the enactive approach to cognition with its emphasis on non-rational forms of *knowing as doing* and the *relationship* between that and more rational forms of knowing. In the next section we present those concepts from enactivism that are necessary as the basis of our conceptualisation of identity work before articulating our conceptualisation in methodological terms.

#### Enactivist theory of cognition

Cognitive theories seek to describe the organisation and behaviour of cognitive systems. In mathematics education, for example, cognitive theories are used to explain certain phenomena such as what learning (mathematics or mathematics teaching) is, how it happens, how to observe it, and potentially how to enhance it. When researching particular phenomena, cognitive theories can form the basis of underlying ontological and epistemological assumptions as well as associated methodological implications which include decisions relating to the appropriate units of analysis (e.g. the cognitive *system*).

Whereas some cognitive theories (e.g. cognitivism) conceptualise cognition as the mental processes that occur purely within the brain of an individual, other cognitive theories extend the cognitive system to incorporate the body, where the system is considered to be bounded by the skin of the individual cognising agent. Enactivism as a cognitive theory has its roots in biology, is inspired by phenomenological philosophy (e.g. Merleau-Ponty, 1962), and has been likened to complexity theory (see Davis & Sumara, 2007). The "enactive" (Varela et al., 1991) approach to cognition which breaks from the psychological paradigm, has also been linked to Bateson's (2000) ecological theory of mind (where ecology here refers to the study of relationships). It follows that with Bateson, enactivist scholars conceptualise organisms (e.g. people) and their environments as complex systems. With enactivism, the individual is understood to be part of (i.e. a subsystem to) a series of increasingly complex systems (such as a classroom, a culture, a biosphere, ...) (Sumara & Davis, 1997), an idea that is contrary to those theories that position cognising subjects as distinct from the world. Thus, although an individual is considered a complex system biologically bounded by their skin, the cognitive system is distributed beyond that boundary. Distributed cognitive theories, which view cognitive systems that extend beyond the limits of the individual, are necessarily situated assuming the now familiar brainbody-world formulation, where, in the case of cognitive systems involving humans, the world consists of culturally constructed social and material settings (Hutchins, 2014). One consequence of adopting such a distributed model of cognition during any research project is that the unit of analysis must extend beyond the individual components to consider the relations between them (e.g. the relations between individuals and their environments). In such a framing, the individual is not *contained* within the environment, rather, they are an *integral part* of that environment or context (Sumara & Davis, 1997) with individual and environment being simultaneously co-defined through their ongoing interactions. For the enactivist, cognition is therefore an active process that is neither located within the individual cognising agent (for example, as a *product* of an interaction), or within the environment, but emerging with and existing in the ongoing interactions between all elements of the cognitive system.

Enactivism departs from cognitivist framings of cognition as "the representation of a pregiven world by a pregiven mind" (Varela et al., 1991, p. 9) in which the cognising system (the brain) acts on the basis of internal representations. Rather, for Varela et al., cognition is viewed as "the enactment of a world and a mind on the basis of a history of the variety of actions that a being in the world performs" (p. 9). The consequences of the enactivist view of cognition is the rejection of a representational view of knowledge as some sort of mirror of reality.

For enactivists, "every act is an act of cognition" (Davis, 1996, p. 201). According to Varela, our spontaneous action stems from an "immediate coping with what is confronting us" (Varela, 1999, p. 5) and he urges us to closely consider what it is that we do spontaneously when presented with any given situation.

[W]e are *always* operating in some kind of immediacy of a given situation. Our lived world is so ready-at-hand that we have no deliberateness about what it is and how we inhabit it.

(Varela, 1999, p.9, emphasis original)

Varela distinguishes our immediate coping as the most predominant cognitive mode, at the level of perception, from rational deliberation that is associated with the use of language. From an enactivist perspective, all actions, including unreflective ones, are acts of cognition. The key enactivist aphorism "all doing is knowing, and all knowing is doing" (Maturana & Varela, 1998, p. 26, emphasis original) highlights the emphasis that enactivists place on non-rational, embodied forms of knowing and a view of cognition as "effective action" (Maturana & Varela, 1998, p. 29).

## Enactivism and identity

Enactivism rejects the notion of "a truly existing self or ego" (Varela et al., 1991, p. 79). Personal identity is instead seen to "arise in the complex mix of biological disposition, physical affect, social circumstance, and cultural context as the agent copes with the contingencies of existence" (Davis & Sumara, 2007, p. 468). For the enactivist, perception and action connect directly, there is no core, inner self that controls the actions of the individual, only "perceptually guided action" (Varela et al., 1991, p. 173). Not only is identity conceived of as a process that is in constant flux (as is acknowledged from other perspectives), from an enactivist perspective, identities happen, they are enactments that are "embodied in the nested interactivities of dynamic forms" (p. 468), and as such can be considered as *relational* constructs.

The implications of this relational view of identity are significant. Rather than conceptualising identity as a property of an individual, identity is seen as "an emergent property of a complex, distributed process mediated by social interactions" (Varela, 1999, p. 62). Furthermore, identities are not enacted against a fixed background. A relational view contends that as an individual's identity evolves, the environment in which that individual is embedded evolves simultaneously. From this perspective, the unit of analysis in identity research would not be so much on the components of a system, such as participating individuals, but on "the *relations* that bind [the different components of the system] together in action" (Sumara & Davis, 1997, p. 415, emphasis original). By using enactivism as an interpretive and analytic framework, it is possible to be

attentive to "how individual and collective identities emerge, and how participation in any shared action contributes the very conditions that shape these identities" (p. 417).

Within mathematics education, Hall et al. (2018) use enactivism to provide "the theoretical depth needed to *understand* how students' mathematical identities might form through mutual co-specification with the environment of learning" (p. 186, emphasis added). In relation to methodology, however, the authors make use of the "listening guide" (Gilligan et al., 2003) to analyse an interview with a participating student, an approach they deem to be *compatible* with an enactivist framing since, the authors claim, enactivism does not specify particular analytical approaches.

A useful distinction in this discussion of identity are the enactivist concepts of *organisation* and *structure*. For enactivists, a living system is a closed system that maintains its organisation, that is, "those relations that must exist among the components of a system for it to be a member of a specific class" (Maturana & Varela, 1998, p. 47). In other words, class identity is conserved. To the observer, an individual appears as an autonomous unity distinct from its background and is characterised as having definite organisation. The structure of a living system, on the other hand, refers to its ever evolving form, and has been described as "the physically embodied, biological-experiential history of a system" (Davis & Sumara, 2007, p. 464). Changes in the structure of the system are triggered by the interactions that system undergoes (e.g. as part of a more complex system in which that individual is embedded) and determined by the existing structure of the system. These structural changes contribute to the ongoing *personal identity* of the individual. Thus, an individual is both a coherent unity and simultaneously part of other emergent unities (Davis & Sumara, 2007). As an individual and its environment interact, they experience a mutual history of evolutionary changes. This process of co-evolving happens through a bi-directional process of "structural coupling" (Maturana & Varela, 1998, p. 75, emphasis original). In brief, the organisation of a system is invariant and common across all members of a particular class (such as human beings), whereas the structure of that system is unique and constantly evolving. By specifying which interactions from the environment trigger structural changes, the individual "brings forth a world of significance" (Simmt & Kieren, 2015, p. 307).

A complex system such as a human being "couples structurally not only to its environment but also to itself, and thus brings forth not only an external but also an inner world" (Capra, 1996, p. 262). This bringing forth of such an inner world is intimately linked to language and is what some might associate more with their identity. Enactivists posit that we differ from other living beings since we exist in language. We inhabit a *relational domain* that arises in the relations between the dynamic structure of the individual and that of their surroundings which includes other people. Subsequently "the domains of discourse that we generate become part of our domain of existence and constitute part of the environment in which we conserve identity and adaptation" (Maturana & Varela, 1998, p. 234). To the enactivist, we cannot speak of our identity without also speaking of its surroundings, for its identity emerges in relation to its environment (Sridharan, 2015).

The relational domain in which we inhabit consists of conscious and unconscious dimensions. At any instant, what we are conscious of depends on what is accessible to our reflective awareness at that point in time (Maturana & Verden-Zöller, 2008), with much of what we do remaining below the level of awareness. "[I]t is in language that the self, the *I*, arises [...] in the network of linguistic interactions in which we move" (Maturana & Varela, 1998, p. 231, emphasis original). What we associate with our identity is "an ongoing flow of reflections" (p. 231) (i.e. consciousness) which is conserved in our histories. Language, therefore, does not *reveal* our identities, rather, our identities are *continuously brought forth* with others through the process of languaging (Maturana & Varela, 1998) and can be analysed as "arising out of our recursive linguistic abilities and their unique capacity for self-description and narration" (Varela, 1999, p. 61). We only become aware of ourselves in the domain of reflections.

Our reflections and our reasonings [...] are fundamental to the course of our living. Through reasoning and reflection we can braid our rational with our emotional awareness, and thus we can be responsible for both our emotioning and our reasoning as we contemplate these from the perspective of our desires.

(Maturana & Verden-Zöller, 2008, p. 102)

Our identities can be thought of as "our manner of relating and living with others" (p. 106) which, for the most part, happens without rational deliberation or conscious awareness. The emphasis that enactivists place on non-rational forms of knowing/doing and the relationship between conscious and unconscious modes of being is significant for identity researchers and offers an alternative focus to those methodologies that base themselves of the conceptualisation of identities as purely narrative constructs (i.e. the stories people tell about themselves and others). From an enactivist perspective, narratives are the *mechanisms* by which we can work on developing our identities, our manner of relating and living with others, in an intentional way, i.e. through engaging in the process of *identity work*.

# Conceptualising enactivist-informed (MTE) identity work

We see identity work as methodological in two ways which we cannot strictly separate. Firstly, as part of an ongoing *process of developing ethical expertise* in relation to pedagogical approaches and perspectives on mathematics education with prospective and practicing teachers, and secondly, in relation to the *process of researching* (MTE) identity. Informed by the enactivist theory of cognition and associated view of identity, we present our conceptualisation of (MTE) identity work as a set of seven methodological principles: *focusing on process (as opposed to outcome); orienting towards relationships and attending to the other; working from and with multiple perspectives; privileging non-rational forms of knowing; engaging in a process of becoming self-aware; facilitating the letting go of ego-centered habits;* and *enabling new ways of making distinctions*.

# Principle 1. Focusing on process (as opposed to outcome)

The notion of identity work fits well with an enactivist world-view where the focus is on the ongoing, ever-changing process of becoming (MTEs) as opposed to capturing descriptions of an individual's identity. MTE identity work is thus akin to a continual process of becoming. According to Simmt and Kieren (2015), "[e]nactivism as a methodological frame for mathematics education research is a form of research that is occasionally and multiversally incomplete" (p. 316), a claim that acknowledges the infeasibility of reaching a definitive set of findings that can be universally agreed upon and emphasises the nature of continuous change and evolution. It follows then, that there is "necessarily always more to be said and different grounds for the saving" (p. 316). In a process oriented paradigm, it is the work of identity work that takes precedence over the reaching/describing of a particular identity and so research questions informed by an enactivist perspective are most likely to be in the form of how-questions, such as "how am I becoming a mathematics teacher educator?" (Helliwell, 2021).

Principle 2. Orienting towards relationships and attending to the other Mason (2002) reminds us that "[s]tudying oneself can become solipsistic and even narcissistic, if gaze is always inward" (p. 174). It is important, therefore, to turn outwards to view identity in all its worldly relations and to understand the situated nature of identity which arises in, and is contingent on, the melee of ongoing interactions. Orienting research

toward the relationships between individuals and between individuals and their environments, the researcher's gaze is no longer upon the individual as it might be from more psychological perspectives. Instead, the focus is on the emergent activity within the various relations. MTE identity work is not an isolated introspective process, rather, it can be described as a process of researching *self-in-relation-to-other*. For instance, an MTE engaged in identity work will do so in relation to the identity work of the mathematics teachers whose learning they shape.

#### Principle 3. Working from and with multiple perspectives

In setting out enactivism as a methodology, Reid (1996) reminds us of "the importance of working from and with multiple perspectives" (p. 207). Since our structures determine what is possible to see, utilising multiple perspectives is a way to expand these possibilities, developing a way of seeing more than is possible to see from any single perspective. Reid offers four methodological ways that multiple perspectives can emerge, summarised here as: multiple researchers looking at the same data but through their own theoretical lens; multiple revisitations of data, using different theories; examination of a wide range of data; communicating research to others and inviting new interpretations. As a MTE engaging in identity work, the challenge of utilising multiple perspectives could be perceived as some-what amplified. Thus it is important to actively seek the perspectives of others, including other MTEs. It is through the process of opening ourselves up to others that we "also open the possibility of having our understandings of the world – and hence, our senses of *identity* which are cast against the background of that world – affected." (Sumara & Davis, 1997, p. 413).

#### Principle 4. Privileging non-rational forms of knowing

One corollary of embodied ways of knowing, is that they may never be fully articulated. Such implicit, intuitive knowing "embodies observations, distinctions, feelings, perceptual patterns and nuances that are too fine-grain to be caught accurately in a web of words" (Claxton, 2000, p. 36). The term tacit knowledge has been used to describe the type of knowing that cannot be explicated, what both Davis (1996) and Taylor (1995) have referred to as unformulated. Put simply, "*we can know more than we can tell*" (Polanyi, 1966, p.4, emphasis original). From an enactivist perspective, our embodied actions are foregrounded, and rational deliberation arises out of an awareness of action. Extension is the process by which knowledge and feelings that arise in a familiar situation, in which a particular action is considered correct, are extended in an appropriate way to other, analogous, but more complex situations, where the correct course of action is less clear. "To extend feelings is both to see that one situation resembles another and to have these feelings 'break through' into the new situation" (Varela, 1999, p.28). The process of extension, however, is neither passive nor presumed and Varela counsels the need for "some form of sustained, disciplined practice" (Varela, 1999, p.75) as a way of fostering a responsive and compassionate disposition. MTE identity work as a sustained discipline that involves deliberating over our (embodied, habitual, intuitive) actions as MTEs in such a way as to extend what is possible for us to do in the future. To foster a responsive and compassionate disposition (or MTE identity) takes disciplined commitment over a prolonged period, it is not a snapshot.

#### Principle 5. Engaging in a process of becoming self-aware

According to Maturana and Verden-Zöller (2008) we can "choose to be different through an emotional change that arises through our becoming aware of what we do and what we want to do" (p. 105). In terms of identity, we might say that changes in our identity can be observed through our changes in our emotional responses to situations or in our embodied actions (that include our discursive ones) and not only changes in the stories we and others tell. The enactivist focus on action does not preclude the importance of telling stories, in fact, since so much of what we do as MTEs is spontaneous, narrative tools, including creative analytical methods, can be used as a form of "deliberate analysis", a process that allows us to "reconstruct the intelligent awareness that justifies [our] action" (Varela, 1999, p. 32) that we see as a form of identity work. According to Varela, intelligent awareness is a "middle way" (p.31) between the two extremes of spontaneous action and rational deliberation and is thus intimately linked with the process of becoming self-aware. Brown and Coles (2011) build on Varela's notions of deliberate analysis and intelligent awareness in their framing of teacher development.

This post hoc deliberation provokes "intelligent awareness" as it allows experts to unpick, if necessary, the reasons an action was taken, and hence open themselves up to alternative possibilities in the future. (Brown & Coles, 2011, p. 862)

The process of deliberate analysis is akin to the kind of deliberation that Dreyfus & Dreyfus (1991) refer to as the "buttressing of intuitive understanding" (p. 241). This buttressing is what happens when an expert deliberates about the appropriateness of their intuitions and is an alternative to "detached, principle based, deliberation" that is "often incorrectly seen as the only alternative to intuition" (p. 241).

#### Principle 6. Facilitating the letting go of ego-centered habits

As Varela (1999) explains, ethical action (like that necessary for becoming MTEs working with teachers of mathematics) arises in an appreciation of the fluid, situated and enacted nature of identity as opposed to one that is pre-existent and independent of our everchanging environments. In other words, knowing that our identities are continuously emerging, both biologically and culturally, implicates us in our actions as MTEs given any situation at hand. To see more and differently requires us to open our habitual ways of seeing the world up to question, and by "creating the conditions for the emergence of the as-yet unimagined" (Davis, 2004, p. 184). For Varela (1999), intelligent awareness is developed through the process of deliberate analysis, which involves "disciplines that facilitate the letting-go of ego-centered habits and enable compassion to become spontaneous and self-sustaining" (p. 73). MTE identity work involves a process of developing intelligent awareness that supports the potential for new and different responses to otherwise habitual actions.

#### Principle 7. Enabling new ways of making distinctions

For enactivists, responding differently is linked to making distinctions. The finer the distinctions that our structure allows us to make, the wider the range of potential responses we might have in any given moment. As human beings, we perceive an object as separate from its background by "making an act of distinction" (Maturana & Varela, 1998, p. 40, emphasis original). What we attend to is conditioned by our existing structures meaning that many stimuli that could potentially trigger a response go unnoticed. By expanding the range and differentiation of interactions. those interactions, which a living system actually encounters (i.e. perturbations) will also expand. In return, new ways of acting and interacting in the future are made possible. What is possible to notice in the environment is literally enhanced, as is the potential for new and different responses, triggered by future interactions, opening new ways of acting up in a recursive process of learning. MTE identity work involves a process of developing new and more nuanced ways of seeing (and thus acting) in the moment of working with teachers of mathematics. One way to generate new, distinguishable categories is "through a descent into the detail of experience" (Brown & Coles, 2020, p. 88), where existing distinctions can be unpicked to reveal a more detailed description of the world (at the level of specific observable behaviours), where finer distinctions can be discerned. Through a process of labelling, these finer distinctions can become new categories and, as a result, we experience the world differently: we are more likely to recognise these distinctions again in the future (in the moment of teaching, sav).

# MTE identity work and learning

Rather than seeking to provide a description of MTE identity, the purpose of this paper has been in conceptualising the process of (MTE) identity work as informed by the enactivist theory of cognition. Based on our principle of *focusing on process* (*as opposed to outcome*) and recognising that enactivism is quite centrally a theory of learning, we conclude this article by briefly reflecting on the process of MTE identity work in relation to MTE learning.

In their seminal work, Sfard and Prusak (2005) make a distinction between actual identity and designated identity both of which, the authors contend, consist of the reified, significant narratives about a person. Actual identity reflects the "actual state of affairs" and the designated identity reflects "a state of affairs which, for one reason or another, is expected to be the case, if not now then in the future" (p. 18, emphasis original). The authors think of learning "as closing the gap" (p. 14) between these two collections of stories. We wonder how this description of learning fits with our own experiences of becoming MTEs, and although we are not convinced that we can ever really "close the gap" (since we imagine stories of our future practices as continuously evolving) we do see the relationship between current actions and our intended future actions as being key to developing our teaching practices as MTEs. The dissonance triggered by any disparity between our actions and those we might hope to enact in the future (i.e. our intentions) provides us with opportunities for learning. So, for us, it is not the closing of a gap, but the awareness of difference in what we do in relation to what we value that opens up new possibilities and creates "the conditions for the emergence of the as-yet unimagined" (Davis, 2004, p. 184) as MTEs. From an enactivist perspective, we could say that learning is a change in our embodied actions which include our discursive actions, not only in the stories we and others tell, and it is through deliberating on our actions, holding them up for question and considering alternative possibilities, where we are creating the conditions for any future changes in our teaching to take place. We see our own identity work as the process by which we, as MTEs, can open ourselves up to alternative possibilities in the future. a process of deliberate analysis in relation to our individual actions as a way of provoking the intelligent (self) awareness needed in the process of becoming expert practitioners.

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