Analyzing teachers' narratives to identify potential impediments of professional development in mathematics

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This study explores two experienced teachers' identifying narratives on teaching mathematics with problems in professional development. Data is from the *Partners in practice* project, which aims to support transition from traditional to progressive educational practices. The study investigates narratives of these teachers and how they align with dominant discourses of teaching in the field. Analysis suggests that both teachers are squeezed between their specific pedagogical discourses and more official pedagogical Discourses. Secondly, there is an unclear connection between the teachers' discourses as learners and as teachers of mathematics.

In a recent professional development initiative, the authors of this paper worked with a group of primary mathematics teachers for three semesters. We invited the teachers into a partnership with the aim of collaboratively exploring what a reform-oriented practice that focuses on student engagement in discussions of mathematical problems might look like in their mathematics classrooms. Although the teachers appeared skeptical of the ideas we discussed, we experienced the discussions in the professional development sessions as positive. When observing their teaching, however, we did not see much sign of what we had discussed. Toward the end of the initiative, we felt that something had not worked right, but we were not sure what it was. The more traditional approaches to consider possible development of the teachers' knowledge (e.g., Bell et al., 2010; Hill, 2011), beliefs (e.g., Calleja, 2022), or practice (e.g., Dash et al., 2012; Gee & Whaley, 2016), did not seem promising for our case. We already knew that the teachers had not changed their practice, and it did not seem like their knowledge or beliefs had changed either. To better understand what had happened, we decided to consider the teachers' identifying

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narratives instead. We were familiar with a recent study by Heyd-Metzuyanim and Shabtay (2019), which had a promising approach. In that study, the authors considered the pedagogical discourses of teachers who participate in professional development – distinguishing between "exploration pedagogical discourse" and "acquisition pedagogical discourse" – and their careful analysis of teachers' narratives provided insight into how and why some teachers align with the reform ideas (exploration pedagogical discourse), where others do not. Drawing on the work of Heyd-Metzuyanim and Shabtay (2019), this study aims at exploring the narratives of two teachers who participated in all of the sessions to identify possible impediments to our professional development efforts.

Theoretical background

Our literature review starts with theoretical perspectives on mathematics teacher identity followed by a narrative approach, insisting that identities are not *reflected* or *represented* by narratives, but identities *are* narratives (Sfard, 2019). The section ends with a presentation of our analytic framework.

Theoretical perspectives on mathematics teacher identity

Research on mathematics teacher identity has increased in recent decades (Lutovac & Kaasila, 2018; Zhang & Wang, 2021) – often within efforts to develop teaching from a traditional to a more reform-oriented approach. These are often seen as two conflicting paradigms of teaching and learning (Gage, 2009). In a traditional approach (cf. Dewey, 1938), the teacher is active and does most of the talking, aiming at demonstrating and explaining content to students. In a reform-oriented approach, students shift from being passive receivers to taking a more active role in explorative discussions with the teacher. The distinction between traditional and reform-oriented teaching can be described in terms of teachers' pedagogical discourses (Heyd-Metzuyanim & Shabtay, 2019).

In mathematics education, identity studies often include teachers' experiences of engagement in professional practices, and these studies often aim to understand the influence of professional development on participating teachers' classrooms (Skott, 2019). Researchers refer to the identity of mathematics teachers with different terms – including "mathematical identity", "(professional) mathematics teacher identity", and "mathematics-related teacher identity" – and different theoretical and methodological approaches. Many define identity as, "who teachers understand themselves to be in relation to mathematics and its

teaching" (Lutovac & Kaasila, 2019, p. 506). Identities are considered dynamic and multiple rather than static and stable, they are constructed in situated interactions, and they include broader discourses on what it means to teach. Finally, identity involves teachers' agency and professional decision-making (Skott, 2019).

Our study focuses on mathematics teacher identity with a social stance reflected in social practice theory (e.g., Holland et al., 1998), with emphasis on "how learning, knowledge, and identity are inherently linked to practices as they unfold in a particular situation" (Skott, 2019, p. 471). Within social practice theory, identity is considered as dynamic and continually negotiated as people engage with one another (Skott, 2019) within communities of practice (Wenger, 1998). Research on mathematics teacher identity rarely operationalizes what identity is and how it can be studied empirically (Graven & Heyd-Metzuyanim, 2019). One promising attempt is Sfard and Prusak's (2005) operationalization of identity as the narratives people tell – others or themselves – about who they are (cf., Holland et al., 1998).

We apply a narrative approach to mathematics teacher identity, and we follow Heyd-Metzuyanim and Shabtay (2019) who state that, "mathematics teachers construct their identity based on narratives of what mathematics should be taught, how it should be taught, when certain topics should be taught and to whom" (p. 541). We unpack this below.

A narrative approach to identity

In their attempt to operationalize the concept of narrative identity, Sfard and Prusak (2005) considered Gee's (2001) work as a promising starting point, stating that identity is being recognized as a certain kind of person in a specific given context. They also build on Holland et al. (1998, p. 3), who proposed that "people tell others who they are" and "they tell themselves … who they are." According to Sfard and Prusak (2005), these approaches to identity are promising in that they foreground the individual's own narratives of who they are, and in that identity is defined as a communicational practice.

Building on this definition of identifying narratives, Heyd-Metzuyanim and Shabtay (2019) highlight that identities draw on certain Discourses (Gee, 2014). Discourse here refers to distinctive ways of communicating, which relate to ways of acting, valuing, feeling, thinking, and believing – all of which contribute to enacting identities (Heyd-Metzuyanim & Shabtay, 2019). Identifying narratives thus belong to the official Discourses people draw upon. Heyd-Metzuyanim and Shabtay (2019) emphasize that such narratives are not formed in a vacuum, but they

belong to Discourses that people draw upon, often without being aware of how they appropriate such narratives to themselves. For instance, teachers who describe themselves as successful in mathematics construct their identities based on the available narratives around what successful or effective teaching entails.

We make a similar distinction between discourse and Discourse. A *discourse* is something to engage in (like a mathematical discourse), and we use it to denote the two experienced mathematics teachers' personal pedagogical or mathematical discourses while participating within the professional development sessions (see method section). *Discourse* with a capital D denotes pre-existing texts formed by "discrete acts of communication by specific people" (Heyd-Metzuyanim & Shabtay, 2019, p. 542). In the present study, these wider and pre-existing narratives are related to the themes that we identified from the transcriptions of the professional development sessions (see method section): Narratives about problem solving related to a) the new national syllabus, b) national exams, and c) work plan. According to Heyd-Metzuyanim and Shabtay (2019), the distinction between Discourse and discourse is not always straightforward, but we aim at tracing the two teachers' talk to the pre-existing narratives about problem solving in national exam, syllabus, and locally in the school's work plans. Pedagogical Discourses thus orient teachers toward explorative teaching, whereas the personal pedagogical discourse is extracted from the two teachers' talk in the professional development sessions about how to teach, what to teach (Heyd-Metzuyanim & Shabtay, 2019), and other issues related to teaching.

Graven and Heyd-Metzuyanim (2019, p. 364) emphasize that "identities *are* narratives. Identities are not *reflected* or *represented* by narratives." In their review of research on identity in mathematics education, they highlight one study (Heyd-Metzuyanim & Shabtay, 2019) that provides "a useful analytic frame for understanding individual teacher identities as the nexus of narratives adopted from different pedagogical discourses" (Graven & Heyd-Metzuyanim, 2019, p. 373). In the next section, we elaborate on how we have adopted this frame in our study.

Exploration versus acquisition pedagogical discourses

Heyd-Metzuyanim and Shabtay (2019) distinguish between two forms of pedagogical discourse. One emphasizes the *acquisition* of ideas that characterizes traditional or teacher-centered instruction (referred to as *Acquisition pedagogical discourse*), and another emphasizes students' participation and *exploration* of ideas, which is rooted in progressive instruction (referred to as *Exploration pedagogical discourse*). They explore how

teachers' pedagogical discourse aligns with the Acquisition pedagogical discourse and with the Exploration pedagogical discourse, attempting to explain how teachers adopted certain narratives from the latter contrasted with the former. The teachers were interviewed and presented four different teaching vignettes that represent different types of instruction. In the final part of the interviews, the teachers were introduced to a mathematical problem, which was chosen to focus on the teachers' mathematical discourse. When discussing this problem, they were positioned as learners or users of mathematics rather than as teachers. In the present study, we have also positioned the teachers as learners of mathematics when they were challenged to reflect on their own problem solving. Including the problem in the interviews allowed the researchers to identify the teachers' explorative or ritual mathematical discourse in their problem-solving process. Heyd-Metzuyanim and Shabtay (2019, p. 543) apply "the commognitive distinction between ritual and explorative participation in mathematical discourse." Whereas ritual participation is rigid and pays attention to procedures and rules, explorative participation highlights flexibilities and different approaches for solving problems. Although ritual participation might be a prerequisite for exploration, these authors emphasize the danger of being stuck in a ritual stage.

All teachers in that study aligned some parts of their teaching identities with student-centered or reform-oriented ideas, but the level of alignment varied. Although the teachers adopted from the official narratives, their descriptions of what they would do in a lesson leaned toward acquisition. The authors also indicate a connection between the teachers' own ritual participation in problem solving and their alignment with the Acquisition pedagogical discourse. In line with Sfard (2016), the authors suggest that "for teachers to be explorative in their teaching, they must themselves be explorative participants in mathematical learning" (Heyd-Metzuyanim & Shabtay, 2019, p. 553).

Our study adopts this frame and examines individual mathematics teacher identity in relation to multiple pedagogical discourses. We explore the narratives of two experienced teachers, discussing the teaching of mathematics with problems in professional development and approach the following research questions.

1 What characterizes the narratives of two experienced mathematics teachers when participating in professional development?

2 How are their narratives aligned with the pedagogical D/discourses of exploration or acquisition?

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Methods

This study is part of a larger Norwegian project, *Partners in practice*, which seeks to establish a collaborative space for continuous development of practice. We analyze data from a group of mathematics teachers from the same primary school who participated in six professional development sessions – each with a duration of approximately two hours – that were held over a period of 18 months. The first sessions aimed at joint exploration of key concepts related to mathematical problem solving. The teachers got firsthand experience with solving mathematical problems before trying out customized problems with their own students. The focus then shifted toward presenting problems, facilitating group discussions around solving problems, and finally toward facilitating whole-class discussions. In between sessions, the teachers worked on different tasks, like observing their own students' mathematical thinking with a given problem.

The collaboration involved five teachers, sometimes accompanied by the school principal or another person from the school administration, and three researchers (the authors of this paper). The researchers were responsible for facilitating the professional development sessions. We are aware that the presence of persons from the school administration might impede on the teachers' full participation in the discussion. The school principal signed up the school for the project, and he expressed a wish to develop explorative mathematics teaching and better mathematical understanding among their students. We do not know if the teachers aligned with that wish. Although we experienced the discussions in the professional development sessions as positive, we also observed some skepticism among the teachers to the ideas we discussed.

We selected two experienced male teachers (pseudonyms: "Roy" and "Ted") as cases, because they were present in all six sessions. They both taught students from Grade 5 to 7 (aged 10–12 years). Audio and video recordings from the professional development sessions were transcribed verbatim into Norwegian for further analysis. We analyzed data in several steps. First, we identified core themes in the data by applying topic modeling (Blei et al., 2003). Topic modeling is a type of machine learning that applies probability models to identify "topics" in texts. We used topic modeling to identify three such topics in the utterances of Roy and Ted. After a process where we went back and forth between the transcripts and the topics to see how these words were used by Roy and Ted, we ended up describing the topics as: "Doing and finding", "Problem and task" and "Students and class." Findings from such a summative approach to content analysis "are limited by their inattention to the broader meanings present in the data" (Hsieh & Shannon, 2005, p. 1285). We thus continued

with conventional content analysis (Hsieh & Shannon, 2005), where we first marked all words related to the three themes from the summative analysis in separate colors throughout the transcripts. Next, we carefully examined each theme throughout the sessions. This examination led to a deeper understanding of the teachers' talk within each theme. The talk about students and class mainly reiterated students' discourses, which was less relevant for our analysis of the teachers' identifying narratives. We thus decided to leave out this topic from further analysis and focused on the main themes and sub-themes important for characterizing the narratives of two teachers as presented in table 1.

Table 1. *Main themes and sub-themes*

Main theme	Sub-theme
1 Doing and finding	a Problem solving in the new syllabus
	b National exams and problem solving
	c Problem solving versus work plan
2 Problem and task	a Reflection on experience from own problem solving
	b Understanding the problem, relating it to a familiar context
	c Working on problems in own class

After having identified these themes in our data material, we applied the theoretical perspectives of Heyd-Metzuyanim and Shabtay (2019) in a theory-driven content analysis (Hsieh & Shannon, 2005). We considered how the teachers' narratives within the two themes aligned with *acquisition* and *exploration* pedagogical discourses. In this part of the analysis, we coded the data material related to these two broad categories. By this theory-driven approach to data analysis, we also identified possible discrepancies in the official pedagogical Discourses and the personal pedagogical discourses of Roy and Ted by illustrating how their own narratives drew pedagogical discourse of exploration or of acquisition that conflicted with acquisition or exploration pedagogical Discourse or vice versa. This will be highlighted in the results section.

Results

Through the conventional content analysis, we identified three sub-themes within each of the two main themes (see table 1). Below, we unpack what characterizes the narratives of the two teachers within each of the six sub-themes, and how their narratives align with the pedagogical D/discourses of exploration or acquisition.

Doing and finding

Problem solving in the new syllabus

In Norway, a new syllabus was introduced in 2020. According to the two teachers, the verbs in this syllabus differ from previous syllabi in that "there is a lot the students should do" (Ted, Session 1, S1). Ted continues that what the students should do – such as time-consuming problem-solving tasks – is something they do not associate with mathematics.

Roy likes problem solving, but he finds it challenging to teach. His interpretation of the syllabus is that students "hardly write anything" and that "there shouldn't be any algorithms" (S3). Roy finds this challenging, because his students work a lot on algorithms, and they have "written math assignments every week" (S3) where the ultimate goal is to learn algorithms by heart. Roy adds that solving mathematics tasks by using algorithms was the reason why he "loved math" (S6), thus highlighting ritual participation from his own experience as a learner of mathematics.

Ted also finds the problem-solving approach to teaching, as presented in the syllabus, challenging. However, he considers the students' lack of focus to be the main challenge. He adds that students' view of mathematics as a "production subject" (S1) challenges their motivation; they want to produce "a certain number of math tasks every lesson" to confirm their ability (S1). Based on this, Ted does not find enough time for problem-solving tasks in his lessons.

National exams and problem solving

Both Roy and Ted appear to focus on problem solving in their teaching because of the national exam: "When students enter Grade 5 and are having national exams in math, they are shocked to find that there are hardly any calculation tasks" (Ted, S1). Ted adds that they practice problem-solving strategies before the exam, since students are required to "transform their previous knowledge when they face a new [problem]" (S6). It was easier for students before, Ted continues, "when math tasks were math tasks." Roy points to "the mundane work of automization" (S6) as the basis for problem solving, and he thus values this ritual work with algorithms. Yet, he acknowledges that national exams have "a type of task that they have never faced before" (S6), and this is a challenge.

Both teachers find the problem-solving tasks in the national exams to be challenging for their students, but in different ways. Roy considers algorithms as a basis for problem solving, and he thus highlights these over problem-solving strategies in his teaching. Ted emphasizes problem-solving strategies as preparation for the exam, but students still find problem-solving tasks challenging because of lack of time for solving problems in lessons.

Problem solving versus work plan

In this school, students get a weekly "work plan", which mostly includes textbook tasks that students should do that week. Roy describes the purpose of this to provide "training in automization" (S2). Some tasks are solved during the lessons, and the rest is homework. The teachers say that the result of having such a work plan is that students want to complete all the tasks on the plan rather than working on problem-solving tasks outside of the work plan. There is thus a tension between the ritual aspects of the automization tasks in this plan and the explorative aspects of the problem-solving tasks that teachers facilitate in the lessons. Roy stresses that "both are important." Ted highlights the challenge: "the math tasks of the plan are tasks they are supposed to master", but problem-solving tasks demand more and require collaboration. They agree that there is a tension between repetition and developing understanding by collaboration. Ted adds that some students refuse to spend time on problem-solving tasks and want to work on tasks from the book, "which enables them to relax a bit" (S1).

Our conventional and theory-driven analyses of the two teachers' narratives related to this first theme, "Doing and finding", and the two first sub-themes – the national syllabus and national exams – indicate that both teachers' narratives to a lesser extent draw on official pedagogical Discourse of exploration than on pedagogical Discourse of acquisition. Yet, both are visible in the narratives. When the teachers discuss the syllabus and the national exam, we see signs of the explorative approach emphasized in the national syllabus and concretized by the problem-solving tasks in the national exam. The tension highlighted by both Ted and Roy between tasks used to train automization (as most of the tasks in their weekly work plan) and developing understanding by collaborating on exploring mathematical problems (necessary for managing the tasks on the national exam), might be seen as an indication of conflicting narratives. Roy's narrative illustrates this. He values algorithms as a foundation for exploration. Making room for automating algorithms is therefore important for his mathematics lessons. For Ted, there is not enough time to practice for the problem-solving tasks highlighted in the national syllabus and included in the national exam. When they in their narratives relate the syllabus and tasks included in the national exams to their own work as teachers, it seems like they are both squeezed between the national pedagogical Discourse of exploration, as visible in the syllabus and the national exams, and the challenges they face in the classroom. This is true even though Roy explicitly says that he likes the explorative approach to teaching mathematics. His narrative about the national syllabus and national exams is thus aligned with the official pedagogical

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Discourse of exploration, but also with his personal pedagogical discourse of acquisition when he talks about his own teaching, highlighting the importance for his students to learn algorithms by heart. Ted's narrative about the national syllabus and national exams aligns with the official pedagogical Discourse of exploration, but also with his personal pedagogical discourse of acquisition when he talks about not having enough time for problem solving in his own lessons.

In their narratives related to the sub-theme "Problem solving versus work plan", both teachers draw on the more local pedagogical Discourse of acquisition. The weekly work plan is something all teachers use in their school and can thus be seen as an official or school-based pedagogical Discourse. Both teachers experience a conflict between exploration and the more traditional tasks used to develop algorithmic fluency as presented in the work plan. The conflicting narratives seem to stem from *exploration* pedagogical Discourses as in the official Discourse (Gee, 2014) of the syllabus and the exam, and acquisition pedagogical Discourses as represented by the more local work plan. Summing up this far, it seems like their narratives are aligned with both pedagogical D/discourses of exploration or acquisition and are conflicting. From their talk, and even if their narratives seem to align with the national pedagogical Discourse of exploration, it seems like the local pedagogical Discourse of acquisition is what they end up valuing the most in their teaching.

Problem and task

Unlike Heyd-Metzuyanim and Shabtay (2019), who gave teachers a specific problem in an interview situation that could cause embarrassment if the teachers failed to solve it, our teachers were invited to work on two specific mathematical problems during the first two sessions. These problems might have been too demanding for the teachers to solve individually. By facilitating collaborative explorations of these two problems, we hoped that it would be less stressful for the teachers to participate and engage in the mathematical discourse. Below, we illustrate how Ted (the geometry problem) and Roy (sums of consecutive numbers) engaged in the mathematical discourse.

Reflecting on experience from own problem solving

One problem required construction of an equilateral triangle such that P is an interior point, where the distance from P to the sides of the triangle is 3, 5 and 7 cm respectively. The teachers were then asked to choose an arbitrary equilateral triangle ABC with a given interior point P in which da, db, dc are the distances from P to the sides of the triangle (da is the

distance from *P* to the side opposite of *A*, etc.). From this, the teachers should work on the following subproblems.

a   Choose different positions for *P* and measure *da*, *db*, *dc* each time. Make a table and look for patterns. Try to formulate a conjecture.

b   Try to prove the conjecture in a.

c   Try to generalize the problem above.

Ted reflects on his experience working on the first parts of this problem: "This is a kind of task that irritates me, since I know that I have seen it before, I just can't remember!" Ted has tried to make sense of the problem by reading it carefully, analyzing the problem by highlighting that, instead of beginning the construction with the equilateral triangle, "you have to start the other way," and focus on point *P*. He adds that you must focus on the lengths of the line segments from *P* to the intersections with the sides of the triangle. Ted then expresses his attempts to look for patterns and make a table to find a conjecture.

> But isn't there a connection between these things? Like, really, when you say that, if you can make a table, you can make a table with distance, sides, and then you get some kind of connection [...] and then you can make a hypothesis (S2).

The first session involved the following problem: "If we consider the sums of consecutive numbers, what numbers can we never get?" When trying to make sense of the problem, Roy puzzles over what the word "consecutive" means. He thinks the interpretation of this word is crucial for solving the problem. When invited to share his interpretation, Roy talks about *two* consecutive numbers, for instance 700 and 701. In the discussion that follows, the teachers are asked to consider if it can only be two consecutive numbers. Roy says:

> The more I think about it, the more ... I basically lost the problem. I mean, it is likely because one is used to looking for the solution and not the problem, that one kind of thinks that the problem has been understood, and then: No, it wasn't like that! And then the problem kind of slips away! (S1)

Roy reflects on the challenge of considering more than two consecutive numbers, indicating that he would then "lose" the problem. He suggests that the problem would then be too overwhelming, emphasizing that he is more used to looking for a solution than trying to spend time to understand the problem. This indicates a tendency toward ritual participation.

From these reflections from own problem solving, a general and interrelated theme across corpora illustrates that the teachers are also concerned with relating problems to a familiar situation for use in own classroom. The two teachers' own pedagogical discourses are extracted from this specific theme and from the theme when the two teachers express how they work on problems in their own class.

Understanding the problem, relating it to a familiar context
Roy and Ted agree that understanding the problem is vital to finding a solution.

Roy: Yes, I experienced in my class that if they don't understand the problem, they are not going to find a solution to it (S1).

Ted: Yes, it is kind of like a problem they know, exactly like that, more or less. But that's what they used as a starting point in the video [refers to a problem seen in a video clip], that, kind of, I think that the teacher started from something specific that the students knew (S1).

Ted explains that students' challenge of understanding a problem is to identify the mathematics: "And that's the clue, in order to hook them on. I mean, that's the problem, to find the math inside of these word problems" (S1).

Working on problems in own class
Roy explains how he works on problems in his classrooms:

> No, but what I normally do is to say that this is the problem, and then they must discuss in pairs, and they are not likely to raise their hands or anything like that, but now it is their turn to respond. They take turns (S1).

Ted adds:

> What is kind of cool, and we have done it in my class a lot, where we have activities where, okay, when you win, you must face another winner. Is it a coincidence that you win, or are you consciously doing something, right, and then they suddenly understand why (S3).

From these two extracts, we learn that Roy often uses the talk move "turn and talk" (Kazemi & Hintz, 2014), giving students opportunities to discuss a problem in pairs – thus facilitating explorative participation. The students must also be prepared to contribute to a class discussion when Roy challenges student pairs to find a solution. In Ted's class, a problem is often related to a competitive activity, indicating elements of student engagement and motivation.

Both teachers have been inspired to try out ideas in their own class based on discussions from the professional development sessions. For instance, Roy has been inspired to use open questioning with word problems.

> Yeah, I can say that the way of asking questions, that's something I fancied a lot. So, I kind of took that with me to the classroom to test it out when we worked on word problems (S2).

When unpacking the general theme of considering the problem or task, we have exemplified both Roy and Ted's narratives concerning issues like reflecting on experience from their own problem solving. Both Roy and Ted's narratives are mostly aligned with *exploration* mathematical discourses when it comes to their own problem solving as learners of mathematics, and they both try to make use of problem-solving strategies that could help them to make progress to reach solutions. They have both highlighted the importance of making sense of and understanding a mathematical problem. More specifically, Ted shows some affective involvement, being concerned with trying to relate the geometry problem to an analogous problem he has solved earlier, but without succeeding in applying this problem-solving strategy. Roy seems, on the other hand, to be satisfied with one solution to the problem with the sum of two consecutive numbers, indicating that the problem kind of slips away if he tries out the problem with more than two consecutive numbers. This part of Roy's mathematical discourse is therefore more aligned to a ritual mathematical discourse since he indicates some avoidance of looking back on the solution, making further explorative attempts to elaborate on and refine the solution.

Analysis of data from all six sessions indicates that both Ted and Roy consider a mathematical problem to be a word problem. They express this throughout the sessions without making further reflections or expressing any change in their narratives over time. We wonder why this is the case based on their experience of working on the two rich problems presented above and based on their participation in the mathematical discourse without showing signs of stress or discomfort (Heyd-Meyzuyanim & Shabtay, 2019), but we lack data to dig deeper into this issue from their participation in the mathematical discourse.

We see signs of the official *exploration* pedagogical Discourses when the two teachers emphasize the understanding and relating of a problem to a familiar situation for use in their own classroom. They seem to have good intentions to facilitate exploration of problems in their own class by trying out ideas based on discussions from the sessions. Roy also reports that he provides his students with opportunities to discuss problems in smaller groups. Ted often relates a problem to a competitive

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situation where it is important to quickly finish a task and move to another one. Yet, when reflecting on how students work on tasks, both of their narratives align with an *acquisition* pedagogical discourse in which it is important for students to quickly find a solution and move on to another task. By applying the theoretical perspectives of Heyd-Metzuyanim and Shabtay (2019), we have identified how the two teachers' narratives align with ritual and explorative mathematical discourses when working on two mathematical problems as learners of mathematics. Their narratives align with both *acquisition* and *exploration* pedagogical discourses when they talk about how they work on problem solving in their own classroom.

Discussion and conclusion

Based on our findings from analysis of two experienced teachers' narratives, we make two claims. The first claim concerns the alignment of their narratives with the pedagogical D/discourses of exploration and acquisition. The second claim relates to teacher education and professional development and considers the lack of connection between teachers' discourses as learners and teachers of mathematics.

First, we claim that teachers' identifying narratives – and their practice – are pulled between two conflicting discourses, and this has implications for efforts to change. Our analysis indicates that Roy and Ted have adopted narratives from both exploration and acquisition pedagogical D/discourses (cf. Heyd-Metzuyanim & Shabtay, 2019). They are eager to work exploratively in their own classroom, thus aligning with the broader pedagogical Discourse of the Norwegian syllabus and the national exams. On the other hand, their practice is also influenced by their personal acquisition pedagogical discourse – including their preference for ritual routines, through algorithms – and the local school-based acquisition pedagogical Discourse, which becomes visible in their narratives about the work plan. Heyd-Metzuyanim and Shabtay (2019) emphasize that identity formation is related to how teachers navigate between certain pedagogical D/discourses, and they argue that this will "necessitate a slow and wide-encompassing change in the pedagogical Discourses to which teachers get exposed and with which they are engaged" (p. 552). Students might prefer the less challenging work of acquisition to the more time-consuming and demanding work of exploration. Change is therefore not just a matter for teachers to switch from one Discourse to the other, but it requires a substantial transformation in the Discourse of all participants in education.

Second, there is an unclear connection between the teachers' discourses as learners and teachers of mathematics, and this has implications for teacher education and professional development. In the first professional development sessions of our project, the teachers engaged with mathematical problems as learners rather than teachers. Both teachers showed signs of an exploration mathematical discourse during problem solving. For instance, they looked for analogies and were thus engaged in important problem-solving strategies, and they emphasized the process of making sense of and understanding a mathematical problem. They also revealed intentions to work exploratively in their own classrooms, indicating possible alignment between the exploration pedagogical Discourses and their own explorative participation in mathematical learning, which would confirm previous research (e.g., Sfard, 2016). Previous studies suggest that there is a connection between teachers' alignment with exploration or acquisition in their teaching and their own explorative or ritual participation as learners of mathematics (Heyd-Metzuyanim & Shabtay, 2019; Sfard, 2016). Yet, in our study, we observed that the two experienced teachers were pinched between their own specific pedagogical discourses, the mathematical discourses of their students, and the more official pedagogical Discourses. We therefore suggest that there is not always a straightforward connection between teachers' mathematical discourse as learners and their pedagogical discourse. This should be considered in teacher education and professional development, where experiences that aim at challenging participants' own mathematical discourses as learners are often deemed to stimulate change in pedagogical discourses.

We are aware that the aims of the Partners in practice project to establish a professional community as a collaborative space for continuous development of practice towards deeper learning, critical thinking, and exploration with a focus on student participation are ambitious. By focusing on the teachers' narratives, this study helped us identify how the two teachers seemed to be squeezed between their own specific pedagogical discourses and more official pedagogical Discourses. This seems to be a potential impediment to the professional development. We have also pointed out that the pre-sence of the school principal might be an impediment to the teachers' participation in the discussion throughout the six professional development sessions. Knowing about this will be useful for future professional development efforts. In their review of methods for studying mathematics teacher identity, Lutovac and Kaasila (2019) called for studies that differ from the mainstream approach. We studied teachers' narratives in professional development sessions instead of in an interview setting, and we believe that this setting was important

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for our efforts to investigate the teachers' narratives on problem solving. If we had invited these teachers to work on the mathematical problems in an interview setting, we fear that they would have felt intimidated, and we might not have had the same opportunity to explore their narratives. Finally, we suggest that further efforts should be made to explore the combination of quantitative and qualitative approaches to analyze teachers' narratives in identity research.

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