Professional identities, conflicting positions, and the challenge of conducting mathematical discussions in professional learning communities

EINAT HEYD-METZUYANIM AND TALLI NACHLIELI

Our goal in this study is to examine the interweaving of identifying (identity authoring) and mathematizing of teachers in a *Professional learning community* (PLC) as they engage with a mathematical challenge. We rely on commognition and positioning theory to examine how storylines of teacher-students interfered with teacher-leaders' ability to hear mathematical ideas that were unfamiliar to them. We focus on one PLC session and show how the conflicts in positions, where the teacher-leader fought to maintain the identity of a mathematical leader, led to ineffectiveness of the mathematical discussion. We argue for the necessity of affording PLC participants alternative roles that would not threaten their professional identity as competent mathematics teachers.

For several decades, attempts around the world have been made to reform mathematics teaching towards more student-centered, problem-based and discussion-rich instruction (Munter et al., 2015; Pauli et al., 2007; Schoenfeld, 2014). In previous studies, we termed this instruction *teaching that affords explorative participation* (Heyd-Metzuyanim et al., 2020). Yet, for teachers to change their practice from the more familiar teacher-led, "talk and chalk" forms of instruction, their professional identity needs to change too (Bobis et al., 2020). For example, teachers need to be ready to release some of their control and authority in the classroom, at least during mathematical discussions. Moreover, teaching cognitively demanding tasks and letting students find their own ways of solving them puts the teacher in a vulnerable position where he or she may not know how to respond to a certain solution, or feel ill-prepared to answer certain questions coming from the students (Stein et al., 2008).

Einat Heyd-Metzuyanim, *Technion – Israel Institute of Technology*
Talli Nachlieli, *Levinsky-Wingate Academic College*

Heyd-Metzuyanim, E & Nachlieli, T. (2024). Professional identities, conflicting positions, and the challenge of conducting mathematical discussions in professional learning communities. *Nordic Studies in Mathematics Education*, 29 (3-4), 41–60.

Multiple studies have shown that an effective way to teach teachers explorative teaching practices is by engaging them in explorative activities as participants in professional development settings (Brodie, 2014; Horn, 2010). Yet the same challenges for teachers' identity embedded in leading cognitively demanding discussions in the classroom, reappear in professional development settings, where the teachers (or PD leaders) need to engage their fellow teachers with challenging mathematical discussions. These challenges may even be stronger in professional learning communities (PLCs) where the leaders are often practicing teachers themselves, without unique or more advanced mathematical education compared to their colleagues.

Despite the above challenges, surprisingly little is known about the ways by which explorative teaching practices challenge teachers' identities. Even less is known about how these challenges manifest in real-time conversations and discussions between teachers. This gap has probably to do with the general lack of studies focused on the intersection between identity and mathematics (Graven & Heyd-Metzuyanim, 2019). In this study, we aim to shed light on this intersection. Specifically, we wish to examine a case where identifying (namely, the construction of identities within discourse) interfered with mathematizing in a professional learning community. Through this close-up, discursive analysis, we wish to highlight the importance of taking teachers identity into consideration when engaging them with cognitively demanding tasks.

Literature review and theoretical background

Teacher learning through PLC discussions

One of the common ways to introduce teachers to explorative teaching practices, both in regular PD settings and in PLCs, is to involve them in mathematical discussions around problems that they are later expected to teach in a classroom (Smith, 2001). The leaders of these discussions, namely the PD leaders, are supposed to be sufficiently knowledgeable in the subject matter to be able to model the leading of such discussions. Yet, evidence from studies of teacher leaders shows that this expectation may be misaligned with the reality of PLCs and PDs led by teachers, who are not necessarily experts in mathematics more than their fellow teachers. For example, Borko and colleagues (Borko et al., 2014) examined novice teacher leaders who led workshops around the Problem-Solving Cycle, in their schools. They found that the novice teachers had difficulty connecting different solutions and discussing the constraints of various representations and affordances of various solution strategies.

Teachers' identity and its relation to teaching practices

The change needed for teaching to afford students explorative participation is often talked about in terms of skills or "practices" that teachers need to learn, accompanied by certain bodies of knowledge (Webb et al., 2019). However, recent studies have also shown that this change involves changes in teachers' identity. For example, Bobis and colleagues (Bobis et al., 2020) found that a year-long professional development intervention aimed at enhancing students' engagement, led to teachers shifting their identities, describing themselves as "facilitators" of knowledge rather than as the "Sage on stage".

Additional studies have pointed to the strong role of identity in teachers' experiences around reform in curriculum and in educational policies (Gellert et al., 2013). As Jong (2016) states: "the complex process of learning to teach mathematics necessitates explicit attention to teacher identity to support the enactment of reform-oriented teaching practices" (p.308). Ma and Singer-Gabella (2011) conceptualized the relation of teachers' identity and their practices in the classroom as drawing on a certain "figured world" where the roles and responsibilities of teachers and students are radically different than those to which they have grown familiar through their own educational histories, as students in traditional classrooms.

In a case-study of one middle school teacher who participated in a professional development program around explorative teaching practices, Heyd-Metzuyanim (Heyd-Metzuyanim, 2019) demonstrated the tight co-shaping of teaching identity and instructional practices. The focal teacher in this study, named "Mr. Morgan" started the PD by identifying himself as a teacher who firmly believes in the benefits of letting students discuss, enact authority, and come up with solutions to problems on their own. Yet, as the year of his participation in the PD progressed, and as he was faced with transcripts of his own teaching, Mr. Morgan started showing dissatisfaction with his own actions in the classroom, while at the same time expressing frustration that he was unable to change his old habits. Mr. Morgan's case illustrated that changing teaching practices sometimes involves a painful change in one's professional identity. His was a case of a teacher who expected himself to let students talk, yet still found it difficult to change his teaching routines in ways that would give space for students' ideas. Interestingly, many of the challenges that Mr. Morgan faced had to do with the mathematics involved in explorative teaching. Whereas he was used to focus his teaching on procedures, explorative teaching works best when the discussion revolves around mathematical objects. Mr. Morgan's change in professional identity was thus tightly linked to his ability (or inability) to change how he mathematized in the class.

Despite the general sense that we can get from the above reviewed studies regarding the tight relationship between identity and teaching actions in the mathematics classroom, there is still very little evidence of how this interaction plays out in mathematical discussions, particularly, in professional learning settings (Graven & Heyd-Metzuyanim, 2019). This has to do with a methodological obstacle: identity narratives are often elicited in out-of-class settings (such as interviews), whereas teaching mathematics and mathematizing take place in the classroom. For the above challenge, a discourse-analytic approach is useful, since it can capture both the identity construction (identifying) and the mathematizing activity as they occur concomitantly.

The commognitive framework

Commognition (Sfard, 2008) is a socio-cultural, discursive theory that is particularly useful for our purpose of understanding the interaction of identity with mathematical learning and teaching, for several reasons. First, it is a framework that uses discourse analysis, which enables examining the interactions between identity and mathematics as they occur ”on-line”, in the actual activity of the classroom (or PLC) discussion (Heyd-Metzuyanim & Sfard, 2012). Second, it is tailored specifically for mathematics, enabling the treatment of content, not just of social interactions in conversations and discussions. Finally, through connecting commognition with Positioning Theory (Harré, 2012), recent works within commognition have linked identifying actions with roles and positions available through certain ”storylines” (or scripts) that are common in a certain culture (Heyd-Metzuyanim & Cooper, 2022). This is particularly useful for examining conversations in professional settings, where roles and positions often play an important role. In what follows, we give a short introduction to this theory, with particular emphasis on the concepts that will be needed for our analysis.

The main assumption of commognition is that thinking is a form of communication (hence its name, which combines cognition and communication). Learning, according to this theory, is a process of becoming a participant in a certain discourse, or individualizing a socially existing discourse (Sfard, 2008). A discourse is defined by Sfard (2008) as ”a special type of communication made distinct by its repertoire of admissible actions and the way these actions are paired with re-actions” (p. 297). Mathematical discourses are made distinct by four main features: their keywords, their routines, the visual mediators and the narratives. These four features together constitute discursive objects, the most familiar of which are numbers and shapes. Mathematical discourses can, within themselves, be differentiated according to the main objects that they describe. For example, there is the discourse on natural numbers, the discourse on rational numbers, the discourse on functions, etc. Each of these discourses is hierarchically built on a more basic discourse, where the advanced discourse often subsumes the more basic discourse. The curriculum of today's mathematics learners often echoes the historical advancements of discourses. In this development of discourses, new objects are introduced, which are often a combination (or saming of) objects that were considered as different in subsumed (more basic) discourses. For example, Bourbaki's definition of a function introduced a discourse on functions which subsumed discourses on graphs and algebraic expressions that had existed earlier (Nachlieli & Tabach, 2012).

The movement of learners through individualizing of different discourses is not a simple matter. Different discourses have different *meta-rules*, which are rules about how to endorse narratives in the discourse, namely what is considered "true" in this discourse. Often, these new meta-rules are difficult for newcomers to detect (Sfard, 2007) and need to be explicitly taught. When they are not made explicit, a *commognitive conflict* often occurs. This is a situation where "interlocutors participating in incommensurable discourses try to communicate with one another" (Sfard, 2008, p. 296). In such cases, participants use the same word differently, which often appears to be a "disagreement" or an "argument" about ideas. However, when examined closely, the claims that appear to be conflicting are found to rest on different underlying assumptions. Therefore, they cannot be resolved (Nachlieli & Heyd-Metzuyanim, 2022).

For learning to be successful across meta-level shifts between different mathematical discourses, Ben-Zvi and Sfard (2007) suggested there needs to be a *teaching-learning agreement* between the parties, namely the expert (in the more advanced discourse) and the novice. The agreement needs to concern three aspects: (1) who is the leader; (2) what is the leading discourse; and (3) the nature of the expected change. In our study, we focus particularly on the two first points. This, since in teachers' discourse the nature of the expected change (from more basic to more advanced discourse) is probably not a big issue of concern, given that these teachers are familiar with the progress of mathematical discourses. In other words, given that they have previously learned the advanced discourses, they can detect which discourse is "supreme". However, the first point (who is the leader) around a particular mathematical contestation, may be quite in dispute, especially in PLCs, where the teacher-leader is not necessarily considered a more of a mathematical expert than the participating teachers. The second point, concerning what is the leading discourse, may remain hidden or uncontested when a task invites a more advanced discourse than that of school (or certain grades in school) and the teachers remain unaware of it.

The literature on commognitive conflicts treats the affective or emotional aspects of these conflicts only very tacitly (e.g. Sfard, 2007). However, such conflicts often raise strong emotions since the participants may assume they are arguing or need to prove themselves right. Such arguments may be particularly sensitive in teacher communities, where the stakes of "being wrong" may include harm to one's identity as a knowledgeable teacher.

Although we do not know of studies that tended to such identity conflicts between teachers, commognitive studies did attend to them in groups of students. In a case of four 7th graders faced with an unfamiliar problem, Heyd-Metzuyanim and Sfard (2012) found that the students were unable to proceed with solving the task, even though some of them were able to do so individually, due to the students being unable to agree on who was the leader, who should explain, and how to proceed. The analysis also revealed that these identity conflicts pertained to students' unwillingness to take upon themselves the roles of follower and leader. A later study (Heyd-Metzuyanim & Cooper, 2022) showed that also for adults (graduate students) highly implicit identity struggles around who is the mathematical leader and who is the follower could hinder the effectiveness of mathematical communication.

Identities, positions and storylines

Commognition treats identity through the well-known definition suggested by Sfard & Prusak (2005) of identity as a "collection of narratives ... that are reified, significant and endorsable" (p. 16). This definition is consonant with the commognitive view of learning being a discursive activity. Thus, the activity of *identifying* or identity authoring, is a discursive activity that always happens concomitantly with the activity of learning. Nevertheless, often this identifying activity is not made explicit, at least not verbally. Thus, participants in learning activities often communicate their own identity or how they identify others through non-verbal and implicit cues. We term this *implicit identifying*. By implicit identifying we mean communication about people that is not stated directly but is intended to elicit certain stories about an interlocuter. For example, an explicit identifying statement would be "I'm the teacher, so when I speak, you are quiet!". An implicit identification as a teacher would be "Could everyone please open the book on page 5?"

Heyd-Metzuyanim and Cooper (2022) supplemented the commognitive lens on identity authoring with some of the conceptual tools of positioning theory (Harré, 2012). Specifically, they defined positioning as acts of implicit identifications that align with certain socially recognized roles. Positioning theory helps us understand why the example above implicitly identifies the author as a teacher. It is because the statement implies that the author has certain rights (controlling his audience's actions, for example, opening the book), as well as *duties* (e.g. being sufficiently informative to allow accurate execution of the expected actions). These rights and duties draw on certain *storylines* – common narratives (or meta-rules of the identifying discourse) about who has the right and duty to do what in our society.

Goal and research questions

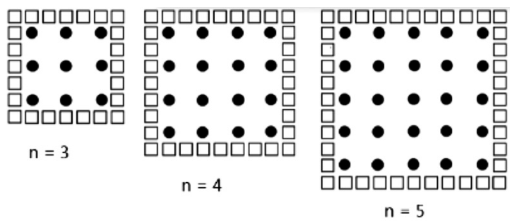
This study focuses on the role of implicit identifications in hindering the mathematical communication in PLC mathematical discussions. Our analysis was guided by the following research question: How did the PLC leaders and teachers implicitly identify each other in relation to a commognitive conflict?

Method

The context of this study was the TEAMS (Teaching Exploratively for all Mathematics Students) project, a professional development project aimed at introducing Israeli middle-school teachers to explorative teaching practices. The initial stages of this project were described in previous publications (Heyd-Metzuyanim et al., 2020). In the current stage, we shifted from a PD model, where we or graduate students from our research group led the PD, to a PLC model, where experienced teachers who participated in a PD (led by us) also led their own small PLC groups. Training these teacher leaders consisted of a 60-hour PD, half of which was conducted before they started leading their PLCs, and half during the first year of their leading a TEAMS PLC. The PLCs led during the first year by these novice teacher leaders were usually quite small (around 5-8 teachers) and often groups of teachers were leaders together, for extra support during their initial steps in leading TEAMS discussions.

Each TEAMS PLC meeting usually consisted of two parts. One was a mathematical discussion around an exploration-requiring task (called in the PLC jargon – a TEAMS task). The second was an introduction to pedagogical ideas such as "the five practices for orchestrating productive discussions" (Smith & Stein, 2011), Accountable Talk (Resnick et al., 2018), and the importance of explicit attention to concepts, balanced with giving students opportunities to struggle (Hiebert & Grouws, 2007). As part of the PLC, teachers, as well as teacher leaders, were expected to teach the TEAMS tasks that they had encountered in the PLC in their classrooms, to collect artifacts of students' work and to bring them back to the PLC for discussion.

The TEAMS PLCs project started at the beginning of 2020, at the height of the COVID pandemic. It therefore took place via Zoom meetings. (The general shift towards PD via Zoom meetings led to this staying the case for many PLCs also after the pandemic.) Several PLCs, in which all the participants signed consent forms, were recorded during their first year of discussions. In the initial stages of this study, we scanned these recordings fully, to see whether there were mathematical discussions which could be compared with each other. We found such in the occurrence of 5 different PLCs discussing the same task, named "The apple orchard task" (The first three sub-tasks are presented in figure 1).



1. *How many cypress trees are in an orchard of n lines of apple trees? Find 3 different ways to express the number of cypress trees.*
2. *Will the number of apple trees be greater than the number of cypress trees at a certain point? If so, when? If not – why not?*
3. *Look at the series of orchards that has the following [given] pattern. What changes at a faster rate - the number of apple trees or the number of cypress trees? Justify your reasoning.*

Figure 1. *The apple orchard task*

The five discussions around this task were fully transcribed and analyzed for possible conflicts. Conflicts were searched for by looking for disagreements, points where teachers signaled that they did not understand each other, and emotional expressions of puzzlement, frustration, or slight embarrassment. Importantly, these indicators are just hints that a commognitive conflict may be present. To discover the nature of the conflict, whether commognitive or not, there is a need for a much closer analysis.

The particular PLC discussion that we focus on was chosen since it included both a relatively long disagreement between the teachers around a mathematical issue, and there were indicators of some discomfort on the part of the teacher leader. We do not claim this discussion to be representative in any way. On the contrary, most mathematical discussions that we observed went without significant conflicts. However, it highlights the challenges of identity and the ways by which identifying can interfere with mathematizing.

The analysis will mostly be demonstrated in the findings section. It consisted of (a) carefully and repeatedly analyzing the mathematical words and procedures that teachers used; (b) mapping the identifications and positions that each of the participants authored; (c) identifying the main storylines according to which the participants positioned and identified themselves and each other.

Findings

Our study focuses on a discussion that occurred in one of our PLCs, which we will call here - the Sunnydale PLC, around the third sub-task of the apple orchard task: "Look at the following sequence of orchards. What changes at a faster rate – the number of apple trees or the number of cypress trees? Justify your reasoning".

The Sunnydale PLC was led by two teacher leaders (called here Inst1 and Inst2) via Zoom meetings. In this mathematical discussion, it was mostly Instructor 1 (Inst1) who talked while Instructor 2 prepared the upcoming pedagogical discussion. In addition to the two leaders, the session included five participating teachers. The discussion until the third sub-task proceeded fluently. The teachers established that one could use the linear function $y = 8x$ to describe the number of cypress trees in the series and the quadratic function $y = x^2$ to describe the number of apple trees in the sequence of orchards. They found algebraically that the number of apple trees will be greater than that of the cypress trees after the 8th orchard and noted that for $x = 8$ (point (8, 64)), there will be a point of intersection between the two functions. Satisfied with the answer thus far, Inst1 led the group to the 3rd task. She was inviting teachers to send solutions by WhatsApp, when Shalom suddenly said something that started an argument (see episode 2)[1].

Episode 2

11	Shalom	Perhaps we can add to more advanced tools that I write them (his students) [-- unclear, probably "derivative"] that in the square its 2n and then compare the 2n with 8, which is the slope of the straight line but that's already not tools of 9th grade, I think
12	Inst1	Why not?
13	Shalom	Derivation? That's not tools of 9th grade
14	Inst1	Why do you need derivation here? You have Desmos. You showed a linear function so that ...

15	Shalom	In Desmos you can see the slope, but it's an investigation of ... not investigation, not a definite investigation.
16	Inst1	no, but I can see rate of change that is constant and rate of change that is ...
17	Naomi	that's it, I did it with a table.
18	Shalom	right, but here you asked ehhh ... what changes at a faster rate, and I can see that the rate at the beginning is slower and then it intensifies and at a certain stage it changes, and for this I had to derive [to use derivation]... to see at which stage it changes.

The argument around whether derivation was necessary for answering the question went on for quite a while, with several misunderstandings and miscommunications around it. Before examining the identifications and positions around this argument, we start by elucidating the commognitive conflict underlying it.

The commognitive conflict underlying participants' positioning

Our analysis revealed that Shalom was using the discourse of calculus, talking about 2n and 8 which are the derivatives of the two functions previously discussed in the group [line 11]. Yet curiously, his talk about the derivatives 2n and 8 went unnoticed. Instead, Instructor1 quickly answered "why not?" [12] referring to Shalom's last claim that "it's not tools (learned in) 9th grade". Later, she claimed that "I can see rate of change that is constant and rate of change that is..." [16]. Although she did not complete her sentence, it is reasonable to believe that she meant to say "rate of change that is changing". By that, Inst1 was referring to rate of change as an attribute of functions that has a binary value (constant or changing). The argument continued (see episode 3).

Episode 3

19	Inst1	I can know that in a parabola, the rate changes and is not constant, while in a linear function, it's exact.
20	Shalom	and when is it exactly 8?
21	Inst1	excuse me?
22	Shalom	when is it exactly 8?
23	Inst1	what do you mean?
24	Shalom	you asked, you asked in the question what changes at a greater rate... and then there is a stage that the cypress tree is faster and there is a stage that the apple tree is faster and there is a stage that they are equal in rate.
25	Inst1	right, this relates to question 2, I think you got to it in question 2, to this point, didn't you?

26	Shalom	but there, there was the number of trees. Not the rate.
27	Inst1	ok, and the number of trees did not describe the growth rate?

Clearly, Shalom and Inst1 were not understanding each other. Shalom was referring to ”when is it exactly 8?” ([20], [22]) where the ”when” probably referred to a particular point on the function, while Inst1 was surprised by the question and did not seem to follow his reasoning.

It is important to mention at this point that we have no doubt that Inst1 had, at some point in her career, learned and participated in the discourse of calculus. We know this from her educational history and also from her later agreement with Shalom's claims. Our point is not that Inst1 ”did not know” calculus. Our point is that, at this particular moment in time, she was participating in a discourse that was different from that which Shalom was using. Inst1 was participating in a discourse of pre-calculus[1], where ”rate of change” (RoC) is a property of a function that can take on a binary attribute (changing or constant). The differences between the two discourses are presented in table 1.

Table 1. *Comparison of pre-calculus and calculus discourses used in the discussion*

Middle school pre-calculus discourse	Calculus discourse
RoC is a property of the entire function.	RoC depends on the specific domain of a function.
RoC of a function has a binary value (changes or constant)	
RoC of a quadratic function is greater than that in a linear function	RoC is related to the slope of the tangent to the function at a certain point. That is, it could be seen as a derivative at a point. RoC is also related to the derivative function.

We are now ready to explore the identifying activity in the Sunnydale PLC around the commognitive conflict.

Implicit identifications around the commognitive conflict

We found indications that the implicit identifying activity interfered with the mathematical communication to a point where Shalom, who was authoring the canonical narratives in the more advanced discourse (that of calculus) was nearly dismissed and ignored. It is only through his insistence, that the narratives of the calculus discourse became explicit in the episode.

Throughout most of the interaction up to the point where the argument started, the participants implicitly identified themselves according to the teacher-student storyline. Thus, Inst1 implicitly identified herself as a knowledgeable leader/teacher, while the participating teachers identified themselves as students/followers. Episode 1, from the very beginning of the discussion around sub-task 3, demonstrates these implicit identifications.

Episode 1

1	Inst1	[reads from the slide] "Let's look at the series of orchards that fulfill the given pattern. What changes at a greater rate – the number of apple trees or the number of cypress trees? Explain your answer." And let me add to this question, is there a relation between this question and the former one?
2	Shalom	I ... in the former question I drew a parabola and a linear graph. And from the parabola and linear graph you can see that the slope at a certain stage becomes bigger than the straight line, and there I also saw the 8 and the zero in the intersections between them.
3	Inst1	so I'll be glad if you raise that in the discussion, great solution. What about the left side of the parabola?

We see in this episode that Inst1 identifies herself as the leader [line 1] by reading the question from the slide, cueing that the group is moving on to this question. In addition, she states "and let me add to this question ..." a question of her own. By that, she implies that it is she who has the right to pose tasks to the group and it is the duty of the group members to answer her questions.

In response to Inst1's prompt, Shalom identifies himself as a follower/student. He answers Inst1's question, explaining what he did in "the former question", and what he saw as result of his doings ("I also saw the 8 and the 0 in the intersections"). His "also" hints that he is trying to answer Inst1's prompt not just to answer the question that she had just read, but also to link between the answer to question 2 and the answer to question 3. By these moves he communicates that it is his duty to answer the instructor's questions and to expose his thinking, as fits a student's role. He thus implicitly identifies himself as a student/follower.

Responding to Shalom, Inst1 continues to identify herself as the teacher. She does this through three speech acts. First, she directs Shalom's contribution to a later stage ("I'll be glad if you raise that in the discussion"), indicating his contribution was too early. By that, she communicates that she has the right to manage and direct participants' actions in the discussion. Second, she praises Shalom's solution ("great solution"), indicating that it is her right to evaluate the responses of participants and it is her duty to give positive feedback (even encouragement) to those who succeed. Finally, she issues another prompt to the group (not necessarily to Shalom, who she hints should wait till later in the discussion), asking ”what about the left side of the parabola?”. With this speech act, she indicates that it is her right to pose additional tasks to the group and to query their thinking. Moreover, with the specific focus on a part of the mathematical narrative that Shalom may have missed, she indicates that she knows what happens in ”the left side of the parabola”, fitting with her duty (as a teacher) to know the material before the students learn it.

Thus, in this first episode the implicit identifications of the Inst1 and Shalom (as well as the other participating teachers, which we did not focus on) aligned with the familiar storyline of teacher and students in the class. This was, in fact, the case during the whole episode up to the point where Shalom started questioning the necessity of derivation.

Episode 2 (presented at the beginning of this chapter) marks a change in the identifications of participants, in particular, the identification of Inst1 and Shalom. Instead of dutifully answering Inst1's questions, Shalom suddenly shifts to making autonomous suggestions (”Perhaps we can add...”). This happens while Inst1 is busy enacting her regular position of teacher, managing the working of others (Naomi), monitoring the photos sent through the WhatsApp group and praising Naomi for her great job.

In response to Shalom's remark in [11], Inst1 first queries his claim (”why not?”), then directly challenges it [14]. Her actions now shift from managing participants' contributions and giving feedback, to arguing about the mathematical content of the task. As Shalom and Inst1 continue the debate (15-17), Naomi enters the conversation, attempting to take up the position of a student that aligns with her teacher (”that's it”) by showing her work (”I did it with a table”). Yet Shalom ignores Naomi and continues the argument with Inst1 (”right, but here you asked...”). His ”you” is said in the singular 2nd person female conjunction, indicating that his challenge is directed at Inst1, not at Naomi and the other teachers or at a general group of authors who are responsible for the task. Thus, Shalom now identifies himself as a debater of Inst1, rather than as her follower.

There are various indications in the moments following this interaction that show Inst1 is unwilling to release her identity of a knowledgeable leader. For example, in response to Shalom's last challenge (line 26, episode 3), she says:

27 Inst 1	Right, that belongs to question 2. I think you (plural) got to it in question 2, to this point, haven't you?

By this move, Inst1 attempts to regain the rights to lead and control the whole group's actions (seen in the plural ”you”), as well as inquire into their working progress (”you got to it in question 2 ... haven't you?”). Yet

Shalom insists on continuing with the role of debater, rejecting Inst1's claim that the answer may lie in Q2. He says:

28	Shalom	But there it's the number of trees, not the rate
29	Inst1	OK, and the number of trees does not describe the rate of change?

This is an important moment for two reasons. First, because Shalom and Inst1 are now in a state where their identifications will be determined (retroactively) according to the mathematical veracity of the answer to Inst1's question. If the answer is "yes, the number of trees describes the rate of change", then Inst1 would be identified as the knower, while Shalom would be identified as the one who was mistaken (or confused). We note that according to the familiar teacher-student storylines of our culture, students have the right to err, while teachers do not. If, however, the answer to Inst1's question is "no, the number of trees does not describe the rate of change" then Inst1 would instantly be identified as wrong, and her identity as a knowledgeable leader would be severely threatened.

The second reason this moment is important is since it revolves exactly around the commognitive conflict that we identified in the first part of the analysis. As we showed earlier, the narrative that "the number of trees describes the rate of change" makes sense in the pre-calculus discourse, since both of them relate to the function object, and since there is no other significant object to which "rate of change" is ascribed. In contrast, in the calculus discourse which Shalom is using, the number of trees is described by the value of the function, while the rate of change is described by the derivative of the function at a particular point. We thus see that the identity conflict is intimately interwoven with the commognitive (mathematical) conflict.

Eventually, the conflict was resolved with Shalom convincing Inst1 and the other participants about his claims. This happened in episode 4, which is a result of around 70 turns (about 13 minutes long) and many back-and-forth responses between Inst1 and Shalom.

Episode 4

89	Shalom	So in the context of this question, between 1 and 4 the apples grow uh ... at a bigger rate, sorry the cypress at a bigger rate, and between 5 and up the rate of the apples is bigger
90	Inst1	OK. But they didn't ask from where. They asked which is bigger and why
91	Shalom	But there isn't a clear-cut answer
92	Inst1	No, all right. But from a certain point, which is first? If we go further on the number line toward the right

93	Shalom	But again, you said literacy, so let's consider the question. The question doesn't speak of a certain place in- at infinity. The question speaks of the entire range and this range is divided to two
94	Inst1	Alright I accept. Do you (plural, feminine) agree?

This episode marks the conclusion of the argument between Shalom and Inst1. Yet before it, there were still two instances where Inst1 attempted to reauthor her identity as the one who is right. Interestingly though, her arguments against Shalom were now only focused on the pedagogy (what the question asked)[90, 92]. Only after Shalom challenged these pedagogical considerations too, did Inst1 close the discussion with "alright I accept" [94]. However, she immediately attempted to reauthor her identity as leader by turning to the other participants and requesting them to state their agreement ("(Do you) agree?" [94]).

Discussion

Our goal in this study was to examine the ways by which teachers and teacher-leaders identify each other around a mathematical challenge in a PLC setting. This goal was driven by the gap in current knowledge on the relation between identifying and mathematizing in explorative teaching situations. For our analysis we chose a common situation in PLCs where, on the one hand, teacher-leaders need to engage teachers in productive mathematical discussions so that the participants become experienced in explorative discussions. On the other hand, the distribution of mathematical expertise in a PLC setting does not necessarily align with the official roles of the participants (PLC leader vs. teachers) due to the fact that, especially in secondary schools, teachers may have various mathematical experiences and educational backgrounds. We reasoned that this situation may give rise to identity conflicts, and particularly to places in which identifying will interfere with mathematizing, as previously found in student settings (Heyd-Metzuyanim & Sfard, 2012).

Our analysis of a particular PLC discussion where a mathematical argument occurred, showed indeed that identity conflicts between teacher leaders and participating teachers may hinder effective mathematical communication. Moreover, the identifications of the participants were interwoven with the power of the mathematics to determine the final answer. In the episode analyzed, the positions of interlocutors shifted from teacher-students to debaters precisely around the commognitive conflict, where the meta-rules of the mathematical discourse and the objects involved changed from middle-school pre-calculus discourse to calculus discourse. Around the shift between pre-calculus and calculus discourse, the achievement of a "teaching-learning" agreement was particularly challenging since within the familiar storylines of professional development settings, the teacher-leader is the one who is supposed to be the leader. However, in this particular case the teacher-leader authored narratives from the non-leading discourse, while the participating teacher (supposed to be *follower*) authored narratives from the more advanced discourse. It was the structure of these mathematical discourses that enforced, in a way, the shifts in positions of *follower* and *leader*. This ”stepping in” of the mathematics is illustrated in table 2.

Table 2. O*nly when the agreed-upon leading discourse changes, do we see movement in the roles (arrows)*

Identifying mathematizing	Leader	Follower
Calculus (leading discourse)		Shalom
Middle-school pre-calculus	Instr1	

From this case study, many teacher educators would probably conclude that teacher-leaders should be better prepared mathematically (Borko et al., 2014). Yet we wish to question this widespread assumption, especially since PLCs are settings where teacher-leaders can mostly receive training in *pedagogical* ideas and there is no guarantee that their mathematical education would be superior to that of their participating teachers.

Instead of ”placing the blame” on teacher leaders and their mathematical experience, we wish to offer that storylines of what it means to successfully lead a mathematical discussion in a PLC setting need to change. After all, if Inst1 had not been so protective of her identity as the knowledgeable leader (around the mathematical discussion), there are good chances that (a) the discussion would have proceeded with much fewer hurdles of communication; and (b) the other participating teachers would have better access to Shalom’s claims. In this regard, we note that once Inst1 accepted Shalom’s claims, she hastily moved on to the next subject. She never halted to ensure all four other participants fully understood Shalom’s claims. Thus, other teachers’ opportunities for learning can be missed when identity issues overbear mathematical discussions.

The case we analyzed in this study may not necessarily represent the discussions in our TEAMs project. In particular, we note that the data was taken from the first year of the project, when teacher leaders were still relatively inexperienced in conducting mathematical discussions with teachers. Still, we believe that identity-related issues, and in relation to them, ”face saving” acts (Vedder-Weiss et al., 2019) are quite common in any mathematical discussion between teachers. As long as entrenched implicit assumptions about mathematical ability and knowledge are not challenged, teachers will continue to hide their insecurities when discussing mathematics with their colleagues. If we wish to challenge such fixed notions of ability in the classroom (Boaler, 2013), if we wish to instantiate norms in the classroom that erring is part of learning and should not be a source of shame, we probably need to start by educating our teachers in the same way.

References

Ben-Zvi, D. & Sfard, A. (2007). Ariadne's thread, Daedalus' wings and the learners autonomy. *Éducation et Didactique*, 1 (3), 73–91. https://doi.org/10.4000/educationdidactique.241

Boaler, J. (2013). *Ability and mathematics: the mindset revolution that is reshaping education*. Forum. https://doi.org/10.2304/forum.2013.55.1.143

Bobis, J., Khosronejad, M., Way, J. & Anderson, J. (2020). "Sage on the stage" or "meddler in the middle": shifting mathematics teachers' identities to support student engagement. *Journal of Mathematics Teacher Education*, 23 (6), 615–632. https://doi.org/10.1007/s10857-019-09444-1

Borko, H., Jacobs, J., Seago, N. & Mangram, C. (2014). Facilitating video-based professional development: planning and orchestrating productive discussions. In Y. Li, E. A. Silver & S. Li (Eds.), *Transforming mathematics instruction* (pp. 259–281). Springer. https://doi.org/10.1007/978-3-319-04993-9

Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics*, 85 (2), 221–239. https://doi.org/10.1007/s10649-013-9507-1

Chauraya, M. & Brodie, K. (2018). Conversations in a professional learning community: an analysis of teacher learning opportunities in mathematics. *Pythagoras-Journal of the Association for Mathematics Education of South Africa*. https://doi.org/10.4102/pythagoras

Escudero-Avila, D., Montes, M. & Contreras, C. (2021). What do mathematics teacher educators need to know? Reflections emerging from the content of mathematics teacher education. In M. Goos & K. Beswick (Eds.), *The learning and development of mathematics teacher educators* (pp. 23–40). Springer. https://doi.org/10.1007/978-3-030-62408-8_2

Gellert, U., Espinoza, L. & Barbé, J. (2013). Being a mathematics teacher in times of reform. *ZDM*, 45 (4), 535–545. https://doi.org/10.1007/s11858-013-0499-1

Graven, M. & Heyd-Metzuyanim, E. (2019). Mathematics identity research: the state of the art and future directions. *ZDM*, 51 (3), 361–377. https://doi.org/10.1007/s11858-019-01050-y

Harré, R. (2012). Positioning theory: moral dimensions of social-cultural psychology. In J. Valsiner (Ed.), *The Oxford handbook of culture and psychology* (pp. 191–206). Oxford academic. https://doi.org/10.1093/oxfordhb/9780195396430.013.0010

Heyd-Metzuyanim, E. (2019). Changing teaching practices towards explorative mathematics instruction – the interweaving of teacher identity and pedagogical discourse. *Teaching and Teacher Education*, 86. https://doi.org/10.1016/j.tate.2019.06.016

Heyd-Metzuyanim, E. & Cooper, J. (2022). When the problem seems answerable yet the solution is unavailable: affective reactions around an impasse in mathematical discourse. *International Journal of Research in Undergraduate Mathematics Education*, 9, 605–631. https://doi.org/10.1007/s40753-022-00172-1

Heyd-Metzuyanim, E., Nachlieli, T., Weingarden, M. & Baor, R. (2020). Adapting a professional development program for cognitively demanding instruction across shifting contexts. *Educational Studies in Mathematics*, 104 (3), 385–403. https://doi.org/10.1007/s10649-020-09967-y

Heyd-Metzuyanim, E. & Sfard, A. (2012). Identity struggles in the mathematics classroom: on learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, 51–52, 128–145. https://doi.org/10.1016/j.ijer.2011.12.015

Hiebert, J. C. & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Information Age.

Horn, I. S. (2010). Teaching replays, teaching rehearsals, and re-visions of practice: learning from colleagues in a mathematics teacher community. *The Teachers College Record*, 112 (1), 225–259. https://doi.org/10.1177/016146811011200109

Jong, C. (2016). Linking reform-oriented experiences to teacher identity: the case of an elementary mathematics teacher. *The Journal of Educational Research*, 109 (3), 296–310. https://doi.org/10.1080/00220671.2014.947398

Ma, J. Y. & Singer-Gabella, M. (2011). Learning to teach in the figured world of reform mathematics: negotiating new models of identity. *Journal of Teacher Education*, 62 (1), 8–22. https://doi.org/10.1177/0022487110378851

Munter, C., Stein, M. K. & Smith, M. S. (2015). Dialogic and direct instruction: two distinct models of mathematics instruction and the debate(s) surrounding them. *Teachers College Record*, 117 (11), 1–32. https://doi.org/10.1177/016146811511701102

Nachlieli, T. & Heyd-Metzuyanim, E. (2022). Commognitive conflicts as a learning mechanism towards explorative pedagogical discourse. *Journal of Mathematics Teacher Education*, 25, 347–369. https://doi.org/10.1007/s10857-021-09495-3

Nachlieli, T. & Tabach, M. (2012). Growing mathematical objects in the classroom – the case of function. *International Journal of Educational Research*, 51–52, 10–27. https://doi.org/10.1016/j.ijer.2011.12.007

Pauli, C., Reusser, K. & Grob, U. (2007). Teaching for understanding and/or self-regulated learning? A video-based analysis of reform-oriented mathematics instruction in Switzerland. *International Journal of Educational Research*, 46 (5), 294–305. https://doi.org/10.1016/j.ijer.2007.10.004

Resnick, L. B., Asterhan, C. S. C. & Clarke, S. N. (2018). *Accountable talk: instructional dialogue that builds the mind* (Educational practices series 29). IAE & IBE.

Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Educational Researcher*, 43 (8), 404–412. https://doi.org/10.3102/0013189X14554450

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Sciences*, 16 (4), 565–613. https://doi.org/10.1080/10508400701525253

Sfard, A. (2008). *Thinking as communicating*. Cambridge University Press. https://doi.org/10.1017/CBO9780511499944

Sfard, A. & Prusak, A. (2005). Telling identities: in search of an analytic tool for investigating learning as a culturally shaped activity. *Educational researcher*, 34 (4), 14-22. https://doi.org/10.3102/0013189X034004014

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics*. NCTM.

Smith, M. S. & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. NCTM.

Stein, M. K., Engle, R. A., Smith, M. S. & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, 10, 313–340. https://doi.org/10.1080/10986060802229675

Vedder-Weiss, D., Segal, A. & Lefstein, A. (2019). Teacher face-work in discussions of video-recorded classroom practice: constraining or catalyzing opportunities to Learn? *Journal of Teacher Education*, 70 (5), 538–551. https://doi.org/10.1177/0022487119841895

Webb, N. M., Franke, M. L., Ing, M., Turrou, A. C., Johnson, N. C. & Zimmerman, J. (2019). Teacher practices that promote productive dialogue and learning in mathematics classrooms. *International Journal of Educational Research*, 97, 176–186. https://doi.org/10.1016/j.ijer.2017.07.009

Notes

1 The episodes in this paper are numbered according to chronology. Thus, the first episode to appear in the paper is episode 2, as it actually followed episode 1 which appears later in the paper.

2 The pre-calculus discourse we refer to in this paper is often used in middle school to relate to analysis of function focusing on continuous change, before learning the notion of derivation.

Einat Heyd-Metzuyanim

Einat Heyd-Metzuyanim is an Associate Professor at the Faculty of Education in Science and Technology, at the Technion – Israel Institute of Technology. Her research interests include mathematical identity, classroom discourse, teaching for explorative participation and processes of change in teaching practices and in teachers' professional identities. Her studies rely mostly on socio-cultural and discursive frameworks.

einathm@technion.ac.il

Talli Nachlieli

Talli Nachlieli is an Associate Professor at the mathematics education department at Levinsky-Wingate College of Education. Her research focuses on communication and discourse in mathematics classrooms, identifying and formulating teaching practices that aim to teach explorative mathematics to all students, and examining teachers' learning of these teaching practices.

tallin@l-w.ac.il