Prospective mathematics teachers' collective identity work: navigating failure experiences and concerns about relating to students

SONJA LUTOVAC AND JOHANNA HAVIA

The present study explores the collective identity work of prospective secondary mathematics teachers as they engage with their experiences of failure and success. The findings show how they navigate concerns about relating to students, resulting in failure-provoking or failure-reducing collective identity work. The prospective teachers' perspectives arising from these distinct types of collective identity work are discussed as they might impact their development and future instruction, especially due to the absence of self-development strategies. This study highlights the need to build prospective mathematics teachers' ability and confidence to relate to students unlike themselves and to problematise their seemingly unproblematic learning experiences and perspectives they co-construct in the collective identity work.

Teacher identity, which is broadly understood as "how teachers define themselves to themselves and to others" (Lasky, 2005, p. 901), is a prominent line in mathematics education research (Skott et al., 2013; see also Graven & Heyd-Metzuvanim, 2019). Discussions on the development of mathematics-related teacher identity¹ often include the concept of identity work (Lutovac & Kaasila, 2018a), which typically refers to the various processes through which identity "is created, expressed, sustained and modified" (Snow & Corrigall-Brown, 2015, p. 177). Identity work has often been understood through discursive and argumentative perspectives (Alderton, 2017; Chronaki & Matos, 2014; Gomez, 2018; Hossain et al., 2013; Neumayer DePiper, 2013), revealing, for example, how prospective teachers argue their awareness of teaching constraints and assert ownership of identity to the rational other (Gomez, 2018) as well as how they "disrupt dominant pedagogies and authoritative discourses of mathematics in school" (Alderton, 2017, p. 155). The concept of identity work has also been framed through a narrative-biographical

Sonja Lutovac, University of Oulu Johanna Havia, University of Oulu

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perspective (Lutovac & Kaasila, 2018b), revealing the points of "continuity and change in a teacher's identity over time" (p. 253) and the factors that shape teacher identity in pre- and in-service phase.

The intentional facilitation of identity work in teacher education has also been emphasised in the literature. Neumayer DePiper (2013) advocated for identity work in mathematics courses for prospective elementary teachers, linking it to understanding the social and political aspects of teaching and highlighting the importance of prospective teachers' critical self-reflection. Lutovac and Kaasila (2011, 2014) demonstrated how identity work through narrative pedagogies, involving purposeful reflection and meaning-making from experiences, empowered prospective primary teachers to reframe their past experiences with mathematics, significantly influencing their future-oriented teacher identities.

Thisstudy addresses two research gaps. First, toour knowledge, research has not been conducted on the collective identity work of prospective teachers. Although individual teacher identities have been synthesised to understand their collective identity (Darragh & Radovic, 2019) and the collective identities of teacher educators have been explored (Osborn et al., 2021), teacher education typically emphasises individual prospective teachers' identity work, overlooking the potential benefits of collective engagement with experiences and identities to facilitate mutual learning and broaden perspectives (Lutovac, 2020). The present study examines prospective mathematics teachers' collective identity work in group discussions, responding also to the calls for the exploration of group interactions among teachers in identity research (Bobis et al., 2020).

Second, there is limited research into the formative role of positive and negative prior experiences for prospective mathematics teachers' identities (Xenofontos & Andrews, 2023, see also Losano & de Costa Trindade Cyrino, 2017). Although attention to generalist teachers' negative learning experiences is warranted (Machalow et al., 2022), neglecting mathematics specialists' prior experiences could overlook the identity development challenges arising from their seemingly unproblematic prior experiences. For example, prospective mathematics teachers may be unaware of the sociocultural factors influencing their success in school mathematics, leaving them unprepared to teach for diversity (de Freitas, 2008). Therefore, this study situates identity work within the context of prospective mathematics teachers' experiences of failure (and success), which are crucial in shaping how they perceive themselves in relation to mathematics and the students they will teach (Xenofontos & Andrews, 2023; Lutovac, 2019, 2020).

This paper aims to initiate a discussion about prospective mathematics teachers' collective identity work, as conceptualised via the narrative-biographical perspective. We regard prospective secondary mathematics teachers as a collective whose members typically "hold common stories about where they come from, who they are, and who they will or want to be" (Rappaport, 2000, p. 6). We begin with the assumption that, in this cohort, there exists a "shared sense of 'one-ness' or 'we-ness' anchored in real or imagined shared attributes and experiences [...] and in relation or contrast to one or more actual or imagined sets of 'others'" (Snow & Corrigall-Brown, 2015, p. 175). Instead of exploring collective identity, we focus on collective identity work as prospective teachers collectively make meaning of their biographical experiences of failure (and success) and co-construct new perspectives in this process. Our research question is: *What characterises prospective secondary mathematics teachers' collective identity work as they navigate their experiences of failure and success*?

Conceptual framework

Impact of learning experiences on developing teacher identity

Positive and negative mathematics learning experiences significantly influence the development of mathematics-related teacher identities (Lutovac & Kaasila, 2018a, 2018b). These experiences contribute to lasting relationships with mathematics (e.g. Black et al., 2009), and self-identifications as mathematical victors or victims (Kaasila et al., 2012; Xenofontos & Andrews, 2023), or as math people or not math people. These learner identities shape "all aspects of a teacher's professional identity", including mathematical knowledge, competence and instructional strategies (Xenofontos & Andrews, 2023, p. 11; see also van Putten et al., 2014). In a study of the experiential construction of teacher identity among primary teachers, Xenofontos and Andrews (2023) found that positive experiences resulted in confidence and pride in one's mathematical competence and flexible instructional planning, while negative experiences led to a lack of confidence and a focus on emotional safety in instruction.

Though limited, the research on secondary mathematics teachers' learning experiences has demonstrated that their school years revolve around "their ability to learn new concepts easily, feelings of boredom because of what they regarded as unchallenging exercises and finishing the work much faster than their peers" (Prescott & Cavanagh, 2006, p. 426). Furthermore, their experiences shape their mathematics mastery identities (de Freitas, 2008). Meanwhile, the transition from secondary to university mathematics is often perceived as challenging, shifting

individuals' perceptions of themselves as strong mathematics students and making them believe in effort over innate ability (Liang et al., 2023). Additionally, prospective mathematics teachers may not feel adequately prepared for the challenges of university-level mathematics, and they may struggle and experience frustration as they approach the limits of their competence (Goulding et al., 2003).

In a series of narrative studies on prior experiences of failure, the first author found that prospective mathematics teachers predominantly experienced failures at university (Lutovac & Kaasila, 2022) and displayed resilience in the face of failure as they continued to identify as math achievers (Lutovac, 2019). Lutovac (2020) compared the future-oriented teacher identities of generalist and mathematics specialist teachers and found similar aspirations and concerns. However, self-development strategies among prospective secondary mathematics teachers were missing, as they confidently asserted their competence in teaching mathematics but lacked confidence in noticing student difficulties, a concern that was directly linked to their limited failure experiences. Lutovac (2020) highlighted the relational qualities of the evolving teacher identities; drawing upon their past failures, they anticipated the needs of students in coping with failure.

Lutovac and Flores (2021) found that prospective teachers across different subject disciplines, including mathematics, viewed student and teacher failure as inseparable; student failure meant that the teacher had also failed. In alignment with Lutovac and Kaasila (2022), the present study conceptualises failure as an autobiographical experience that typically concerns prospective teachers' "real or perceived underperformance and underachievement" in a range of ordinary situations, such as making mistakes (p. 13). Importantly, no definition of failure was imposed on the participants; instead, it was left to them to bring their subjective understanding of failure into the discussions (Lutovac, 2019).

Collective identity work: narrative-biographical conceptualisation

In the present study, identity work is understood through a narrative-biographical perspective. Narrative perspective is underpinned by the notion that identity develops as individuals interpret and attribute meaning to experiences within personal and social contexts, with a narrative serving as a unique means of conducting this meaning-making process (Georgakopoulou, 2006; Ochs & Capps, 1996). We view a narrative as a form of talk-in-interaction (Georgakopoulou, 2006) that is expressive, formative and capable of empowering or impeding, providing a direction for future selves (Rappaport, 2000; see also Lutovac & Kaasila, 2014; Lutovac, 2020). Importantly, as narratives are always partial and incomplete (Ochs & Capps, 1996), so too are the identities emerging from identity work. We add a biographical perspective to highlight the temporal aspect of identity (Ricoeur, 1991) in which past experiences and future anticipations contribute to one's self-understanding over time. With this addition, we highlight the notion of the experiential construction of teacher identity (Xenofontos & Andrews, 2023) and distinguish the narrative perspective in this study from others that emphasise discourses over biographies. Moreover, we view identity work as an individual and a collective process (Snow & Corrigall-Brown, 2015).

Employing the narrative-biographical perspective, we see that, when prospective teachers collectively narrate and interpret their mathematics-related experiences (Lutovac & Kaasila, 2018b), including those of failure, they engage in collective identity work. We conceptualise collective identity work as an intentionally facilitated engagement with identity involving collective meaning-making of biographical experiences by a group of prospective teachers. Through interactions with others and constructing narratives of past experiences and future-oriented aspirational narratives (Lutovac & Kaasila, 2011, 2014), collective identity work gives rise to prospective teachers' shared meanings regarding their roles as mathematics learners and future teachers, respectively. Additionally, collective identity work involves co-constructing new perspectives, which we view as facets of individual and collective identities.

Individual identities typically encompass perceptions of mathematics, its learning and teaching and roles as learners and teachers (Kaasila et al., 2012; Machalow et al., 2022; Marschall, 2022). In contrast, collective identity stems from shared interests, experiences and solidarity among members (Whooley, 2007) and is characterised by, for example, shared perceptions and emotions (Snow & Corrigall-Brown, 2015). Collective identity, like individual identity, is multiple instead of unitary and evolves through interactions, however, it is often more transient in comparison (Snow & Corrigall-Brown, 2015). Additionally, individual and collective identities are intertwined, with individuals perceiving collective identity as integral to their sense of self (Gamson, 1991).

Methodology

Study context and participants

This study is part of a larger project (Lutovac, 2022) and situated within the subject teacher master's degree in a course on the didactics of mathematics, physics and chemistry for prospective teachers² preparing

to teach these subject combinations in secondary³ education. While prospective teachers in Finland typically study these three subjects, they often teach two of them and occasionally all three. However, other subject combinations are also possible.

A total of 22 prospective teachers participated in this study. Eighteen participants majored in mathematics and minored in 1–2 of the following subjects: physics, chemistry or computer science. One participant minored in music. Four participants majored in physics or chemistry and their minor was mathematics or physics (or both). Of all participants, only one did not select mathematics as a major or minor. Most participants were 22–25 years old; a few were older. Although we did not directly inquire about gender, we inferred from our knowledge of the participants that there were 14 males and 8 females. Most participants were enrolled in their third year of study and had thus far undergone basic and intermediate courses in their major and minor subjects as part of their respective degree programmes. Furthermore, they were at the beginning of one year of pedagogical studies in the *Faculty of education and psychology* and had not yet had their practicum.

Two course instructors (including the second author) and two facilitators (including the first author) designed a 1.5-hour teaching session to engage the participants in group discussions about their experiences of failure and success in mathematics. This session, which was aligned with the course learning outcomes and the research aims of the wider project, was presented to the participants as part of the course as well as a site of data collection. The first part of the session introduced the larger project, explained the research and pedagogical purposes and provided the participants with instructions for the group discussions. The main part of the session was the group discussion activity, serving as a site for data collection (see next section). The end of the session was reserved for a discussion with all the participants in which they shared the insights they had gained. The topic of failure in mathematics was elaborated, incorporating findings from the first author's prior research on identity and experiences.

The first author led most of the session and the instructors aided the discussions, as needed. The session was held in a LeaF infrastructure at the Faculty which was designed to allow for the flexible formation of small groups and equipped with audio- and video-recording technology.

Data collection

Data were collected from the audio- and video-recordings of the group discussions in the teaching session. Six groups, each consisting of 2–4

participants who were familiar with each other, were formed. The participants chose the group in which they worked and were given a set of questions (see appendix) to guide them as they discussed memorable failure experiences, elaborated on their significance and explored connections to their development as mathematics teachers and their future work. The discussions were unmoderated and lasted 45 minutes. The participants were instructed to consider as many questions as possible.

Data collection through students' coursework and activities is a common practice in Finnish research-based teacher education. For example, the participants had previously participated in a study conducted by the second author (one of their instructors). Although participation in the course module was mandatory, involvement in the research was voluntary, and students could opt out without affecting their completion of the course. At the end of the session, the first author led a detailed discussion on confidentiality and anonymity. The consent form was thoroughly explained, and the participants were able to raise questions or concerns. All participants signed the consent for the use of their contributions to the research. The study adhered to the *Finnish national board on research integrity* (TENK) guidelines and did not require an institutional ethical review.

Although we acknowledge the potential for asymmetrical power relationships between the researchers (instructors and facilitators) and the participants (Alderton, 2017), such dynamics were not evident in the collected data. On the contrary, the participants seemed at ease discussing their perspectives without concern for our judgment. For instance, their choice of language, which occasionally involved the use of less formal words, such as bungler or stupid, suggested that they may have forgotten about the recordings.

Data analysis

To analyse the data, we applied Braun and Clarke's (2006) thematic analysis approach. We reviewed the data from the six group discussions and identified the subtext to enable a more detailed examination. In terms of the participants' evolving teacher identities, we found that the discussions were most intriguing when the groups considered the following questions: "How can your failures and successes help or hinder you in becoming a teacher? What could teachers do to handle failures?" (see appendix). This subtext was transcribed verbatim, yielding about 15 single-spaced pages (with 12-point font size) and faithfully translated into English to preserve the original content and tone. Next, we coded the subtext descriptively, assigning codes to single utterances or longer passages. Although the coding was conducted inductively, without a predetermined frame, our familiarity with prior research on prospective teachers' experiences of failure and their connection to identity development (Lutovac 2019, 2020; Lutovac & Kaasila, 2022) may have influenced our initial coding decisions. The codes were then clustered together according to their similarities to arrive at the themes within and across the group discussions. The themes that emerged in all group discussions included 1) failure as a learning opportunity, 2) prioritising effort in learning and teaching, 3) students' strategies in the face of failure, 4) teaching approaches to address failure in the classroom, 5) school as a safe place to fail and 6) navigating concerns about relating to students. Memoing was used throughout the coding process to note any observations, including reflections on the data's meaning and possible responses to the research question (Bingham, 2023).

Based on the initial observations captured during memoing, the focus of the analysis was narrowed to two groups that discussed the theme: navigating concerns about relating to students. Given the limited attention to this theme in the literature and its position as the key theme in the two groups, based on the space devoted to it (Braun & Clarke, 2006), we opted to offer a detailed account of this theme. In addition, the two groups - an all-male group and an all-female group - were chosen because they engaged in reflective and in-depth discussions. Rather than simply taking turns expressing individual thoughts to answer the guiding questions, the participants built on each other's views to arrive at shared understandings, thereby truly engaging in collective identity work. This stage of the analysis, guided by our operationalisation of collective identity work, focused on identifying shared meanings derived from the participants' experiences to inform their anticipations of their future as mathematics teachers. We paid special attention to new perspectives co-constructed in the process of identity work, such as suggestions that the participants made in terms of their actions as future teachers, e.g. "as a teacher, you should ...". In particular, we focused on the perspectives accepted by most or all group members. These shared meanings and new perspectives were interpreted as potential instances of a shared sense of one-ness or we-ness, i.e. their collective identity.

The cross-group comparison revealed that the groups navigated concerns about relating to students differently, leading to the identification of two types of collective identity work. The findings section presents a vignette from each group to illustrate the types of collective identity work. We broke the two vignettes into smaller pieces to discuss them. We refer to the participants with pseudonyms.

Findings: navigating concerns about relating to students

Failure-provoking collective identity work

At the beginning of the discussion, Tuomo, Jere, Lasse and Konsta shared their failure experiences. They experienced failures in university exams, often anticipating this outcome based on their awareness of inadequate preparation and study. The group members had few early failure experiences (Lutovac & Kaasila, 2022; Prescott & Cavanagh, 2006). For example, one participant mentioned a careless attitude towards school in general and succeeding in mathematics due to talent and without investing significant effort. All labelled the emotional impact of their failures as minimal. Consequently, they collectively acknowledged that failures are foreseeable and attributable to an individual's actions, correlating with the amount of work invested (Lutovac, 2019; Liang et al., 2023).

The group discussion then expanded to reflections on the varying levels of effort required for success. One participant described challenges in adapting to the demands of the university as he was not used to having to invest effort and study (Goulding et al., 2003). Others agreed with this idea, suggesting that "We could set up a peer support group", which very much resonated with our initial assumption that there would be a shared sense of we-ness, i.e. a collective identity, among the participants.

The group then discussed how their prior experiences might affect their role as teachers and their practices as well as how they could improve how they relate to "good" students.

Tuomo:	The way my own schooling was, will have a huge impact on what kind of teacher I will be.
Jere:	Yeah, I've never been bad in relation to math and never really failed, so I will understand the good students better. And understand that
Tuomo:	That they are not disturbing the lessons because they are not inte- rested, but because they are bored because everything is too easy.
Jere:	That you should give [them] more stuff to do
Lasse:	And more challenging stuff.

These prospective teachers focused on "good" students and ensuring that such students get the learning opportunities they need. They expressed a desire to keep students engaged with challenging tasks to ensure they are not bored (Prescott & Cavanagh, 2006). Furthermore, the excerpt above gives a sense of collective identity, as the participants kept finishing each other's sentences. The discussion continued with Konsta presenting the following viewpoint.

- Konsta: I'm afraid that I might develop a delusion in perspective since I have always been talented in math, studied it for five years at university and will then teach it. So, I might not even understand how something can be difficult for someone. So, if I start teaching too fast, it's not fair, of course.
- Tuomo: I couldn't understand in upper secondary school how it can be so difficult for some people.

Konsta described a delusion in perspective based on a fear that he might not be able to relate to students who struggle or fail due to a lack of failures during his own education (see also Lutovac, 2019, 2020). Konsta thought that an inability to relate might manifest, for example, through a tendency to quickly cover course content. Tuomo joined the conversation, remembering how, as early as upper secondary school, he struggled to understand how some of his peers experienced difficulties in mathematics.

The discussion then turned to what could be done to address the issue of not understanding struggling students, and Jere joined the conversation.

Jere:	Maybe it's that [] School should, in the early stages, provide possibilities for everyone to fail.
Konsta:	[laughing] What a beautiful thought, why not?
Jere:	No, but if you never fail, you feel you know it all.
Tuomo:	There is a good point there. So that we can ensure that everyone will fail sometimes, we really should provide challenges for the good students.
Konsta:	That is true. In an educational setting, it will add strength to the character [of the students] so that people wouldn't become martyrs as we are here at the moment and just blame the system for never experiencing failures.
Jere:	Providing challenges [] so that students would need to challenge themselves and the way they comprise themselves and their talent. That there always would be something that you don't know how to do.

When Jere suggested providing opportunities for students to fail, Konsta, unsure of this suggestion, laughed. Jere explained his thoughts further and convinced Konsta of his views, and the discussion continued around the metaphor of a martyr. Using this metaphor, Konsta labelled the entire group martyrs because they all lacked failure experiences. Concerned that they had not personally experienced sufficient failures, they wished to avoid situations in which students like them did not have any failure experiences in mathematics. To that end, they wanted to ensure that the "good" students received sufficient opportunities to fail and were able to challenge their perceptions of themselves and their talent. Next, the group touched upon differentiation, highlighting a prevailing focus on weaker students in educational discourse and neglect pertaining to high-performing students who may lack the necessary support. However, as future teachers, they also want to provide a supportive environment where everyone can experience failure and foster resilience. One participant quoted Star wars' Master Yoda, stating, "Greatest teacher failure is". They concluded that they need to be compassionate towards themselves and their own failures at university and agreed that it is sufficient to adequately understand mathematics, given their prospective role as teachers rather than mathematicians.

In summary, based on their prior experiences, especially their limited failures during early schooling, the meaning that this group collectively made was that they would relate better to mathematically high-performing students. In their collective identity work, limited attention was placed on mathematically weaker students. Concerns about not having experienced failure and relating better to high-performing students led to the following new perspective: prospective teachers aspire to frame mathematics instruction around the needs of high performers. The participants expressed a desire to add complexity and challenges to their future teaching to ensure that high performers had the opportunity to experience failures in mathematics. Hence, we labelled their collective identity work failure-provoking.

Failure-reducing collective identity work

Another group of four prospective teachers - Sara, Elisa, Noora and Anna - discussed failures and successes. One participant pointed out her challenges with probability calculations, and another talked about making occasional negligent errors in upper secondary school mathematics. Despite regarding mathematics as an easy subject (Prescott & Cavanagh, 2006) and not recalling specific instances of failure, they acknowledged experiencing setbacks in other subjects, particularly languages. The transition to university brought about new challenges, including difficulties understanding certain content (Goulding et al., 2003). However, the group reframed these encounters as non-threatening, attributing them to their limitations and perceived lack of invested effort (Liang et al., 2023), e.g. "laziness". The group also talked about avoiding failure through means such as refraining from providing answers during lectures and hesitating to present exercise solutions on the board. The participants collectively acknowledged that they have lowered their academic expectations: they now accept lower grades and feel more responsible for learning from failures.

The group discussed how these experiences can help or hinder them in becoming teachers. They felt that experiencing failures would help them relate better to students.

Sara:	How can your failures and successes help or hinder you in becoming a teacher?
Elisa:	It helps so that you don't require too much from your students.
Noora:	You understand them, and you don't emphasise it too much if someone makes a mistake.
Sara:	Yeah, you turn it into a victory; they can learn from their mistakes.

The excerpt above conveys a sense of collective identity in terms of the we-ness among these prospective teachers as they finish each other's sentences. They pointed out that prior experiences of failure could help them be less demanding towards their students and employ teaching approaches that use failure as a learning opportunity instead of highlighting students' mistakes. Elisa continued the discussion as follows by expressing her fear of teaching students.

Elisa: [Laughing] I am a bit afraid if I have to teach a bungler⁴.

Unidentified voices: Me too!

Anna: If the students question you, if you don't have an answer to something, then just say, "Let's think about this together". It is not that serious if the teacher doesn't know everything.

Sara: We are only human, too.

Elisa seemed embarrassed about her use of the word bungler [*tunari* in Finnish]. However, her fear of teaching struggling students received support from the group. The group continued discussing how to deal with difficult questions from students and concluded that a teacher does not need to be a know-it-all.

The focus of the discussion shifted to the positive and negative influences of prior successful experiences as follows.

- Noora: But then if you think about how the successes help you or hinder you in becoming a teacher [....] Maybe the success can hinder so that you can't recognise the parts where someone who has weaker initial skills, how they understand like ...
- Elisa: Well, I have an example on this, about function, just a basic equation for this University of Applied Sciences student of mine. It was really difficult for him to understand what this f(x) means. For us, it is just the name of the function ...
- Sara: Yeah, it's like, if you have always been really good at something, then how can you understand how someone else just can't do it? [...] The most important thing is that you understand the basics because you

will never teach at the level you are studying here. The point is to understand where it comes from and to go to the level of the students and explain it so that they understand it. Because if you explain it as they teach it here [....] It would not happen. They [lecturers at university] are so in it somehow. Maybe, as a subject teacher, you shouldn't be that developed; it might even be a bad thing somehow.

Anna: Yeah, true, if you can't relate.

Sara: If you can't think of some simple examples. Because there are a lot of students who just simply don't understand, like, if suddenly there are letters. It is really difficult for many students.

Noora noted that success might hinder teachers' understanding of mathematically weaker students (Lutovac, 2020). Elisa supported this view with her recent experience tutoring a student who encountered a similar challenge. In Elisa's utterance, "For us, it is ...", she insinuated that there might be a collective identity among the group members. Sara agreed and emphasised the importance of understanding fundamental concepts when teaching mathematics and connecting with students at their level of understanding rather than delving too deeply into advanced subject matter. Sara then implied that strong subject knowledge might hinder teachers' ability to explain complex ideas in a way that students can grasp. Anna agreed, acknowledging the significance of teachers' ability to relate to students.

The participants continued to share their concerns related to their prospective teaching roles. These concerns included not understanding simple exercises and unease regarding difficulties with performing rapid mental calculations. One participant articulated a concern regarding failing as a teacher, stating, "I would feel I have failed somehow if the students didn't learn anything". This concern appeared to be linked to the fear of teaching struggling students, which Elisa brought up earlier in the discussion. The concerns around failing as a teacher (see Lutovac & Flores, 2021) could explain the use of words such as bungler; by definition, a bungler consistently fails, raising doubts about how a teacher could succeed.

In summary, this collective identity work placed much attention on relating to low-performing students and how to teach them, and brought forth a fear of teaching such students. Shared concerns about not relating to low performers made the participants want to simplify mathematics instruction and reduce the demands placed on students. In their simplification, a new perspective emerged as the participants concluded that their subject matter knowledge might be detrimental to their ability to relate to and teach all students. We labelled their collective identity work as *failure-reducing*.

Discussion

The collective identity work in both groups was characterised by minimal encounters with failure as mathematics learners. They construed failure as simple mistakes, negligence or less than desired performances on university mathematics exams; these characterisations are in line with Lutovac and Kaasila's (2022) findings. Nevertheless, encountering failures at the university level did not seem to undermine the participants' positive relationships with the subject (Black et al., 2009) or their identifications as math achievers or talented, which were forged based on early educational experiences (Lutovac, 2019). Furthermore, both groups continued to identify as what Xenofontos and Andrews (2023) termed mathematical victors. These features provided a sense of one-ness and we-ness – i.e. a collective identity among the participants (Snow & Corrigall-Brown, 2015).

The core of the groups' collective identity work revolved around concerns about relating to students, especially those deemed mathematically weaker or low-performing. While these concerns aligned with prior research (Lutovac, 2019, 2020), inquiring into collective identity work allowed us to observe how the groups navigated and diverged in navigating these concerns, leading to two distinct types of collective identity work. First, failure-provoking collective identity work focused more on revisiting past experiences related to learner identities and less on the future teacher role (Lutovac & Kaasila, 2014). Drawing on their past experiences, the participants advocated the inclusion of challenges in mathematics instruction to ensure opportunities for students like themselves to experience failure. Second, failure-reducing collective identity work extended beyond the focus on experiences as learners to consider the role of teachers more extensively. The participants advocated for the reduction of failures and suggested that subject knowledge might hinder their future teaching.

Examining collective identity work, we were also able to uncover additional fears and concerns among these prospective teachers which remained hidden in the first author's (Lutovac, 2020) previous research into the identity narratives of individual prospective teachers. More importantly, we observed the new perspectives co-constructed in this process, which we find concerning and which indicate a need for problematising the meanings prospective mathematics teachers' make of their experiences. While research has acknowledged the potential of productive failure under certain conditions (e.g. Kapur, 2008), the participants' desire to frame instruction as challenging and failure-provoking revealed that they did not recognise that repeated failure is precisely the reason why some students struggle. For those already overwhelmed, adding more failure as a learning opportunity is hardly an appropriate teaching strategy. The inclination to tailor instruction to high performers is not conducive to effectively teaching diverse student groups, requiring prospective teachers to reflect on "the contrast between their identities and experiences and those whom they will teach" (Gomez et al., 2000, p. 731) and underscoring the need to challenge sole reliance on personal experiences.

The perspective that teachers' advanced subject knowledge might hinder future teaching is also concerning. Subject knowledge has been regarded as a key aspect of mathematics teachers' identities (van Putten et al., 2014), and extensive evidence has highlighted its crucial role in effective mathematics teaching (e.g. Loewenberg Ball et al., 2008) as well as its importance in the development of pedagogical content knowledge (Kleickmann et al., 2013). Given that participants' experiences of university and school mathematics differed (Liang et al. 2023) and that they had just begun their pedagogical studies, this perspective is somewhat understandable. Nevertheless, this perspective requires challenging for these prospective teachers to appreciate and feel empowered by the mathematics knowledge they hold (Fish & Persaud, 2012).

Both types of collective identity work in the present study lacked self-development strategies, corroborating the findings of prior research (Lutovac, 2020). Neither group considered addressing identified concerns or weaknesses; in contrast, Lutovac and Kaasila (2011, 2014) found that prospective primary school teachers often recognised, for example, the need to enhance subject knowledge for effective teaching. The absence of self-development strategies is concerning, as bridging the gap between current and future aspects of identity is crucial for identity development (Lutovac, 2020; Marschall, 2022). This finding may suggest that prospective mathematics teachers do not perceive the need for development, perhaps because their teaching know-how has not yet been challenged in their pedagogical studies.

We found that instances of a shared sense of one-ness or we-ness (Snow & Corrigall-Brown, 2015), such as shared experiences as successful mathematics students and future aspirations, facilitated deep group discussions among the participants. Without needing to explain "where they come from" (Rappaport, 2000, p. 6), there was a sense of mutual understanding in the group discussions, allowing for greater depth of meaning-making and richer insights than individual interviews. Additionally, collective identity work provided therapeutic value, allowing prospective teachers to recognise shared concerns about their future identities. These concerns could be challenging to bring up in some contexts. However, people who share experiences and interests – and among whom a collective identity exists – typically understand each other.

Conclusion and implications

Our study aimed to initiate discussion on the concept of collective identity work in mathematics education research, and explore prospective mathematics teachers' concerns about relating to students (Lutovac, 2019, 2020) and how they navigated these concerns. This study highlighted the value of collective identity work as a theoretical concept in mathematics education research and a process that requires intentional facilitation in teacher education. Building on these findings, two key conclusions emerge.

First, the ability to relate to students is important as teachers are expected to interact in and through their teaching with students "both like and unlike themselves" (Gomez et al., 2000, p. 731) and in this manner successfully teach all children. Thus, more attention must be devoted to this in teacher education so that prospective mathematics teachers build the ability and confidence to relate to students unlike themselves. Second, although prospective mathematics teachers typically have positive experiences of learning mathematics, the meanings they make of these experiences can give rise to hindering perspectives. Since collective identity work can empower or impede (Rappaport, 2000), it is crucial to challenge these perspectives in order to disrupt the reproduction of inadequate mathematics teaching portraved in the literature on learners' identities (e.g. Towers et al., 2017). We thus suggest the urgent need to facilitate collective experiential construction of mathematics teacher identity (Xenofontos & Andrews, 2023) among prospective mathematics teachers to explore and problematise their prior experiences and the perspectives constructed based on those experiences.

While this study was conducted within a specific context and focused on the particular experiences of prospective teachers, we refrain from generalizing our findings. We thus acknowledge potential variations in collective identity work across different teacher education contexts and highlight that the types of collective identity work we identified are not necessarily exhaustive. This study provides one example of how collective identity work may be implemented in teacher education through group discussions, yet we acknowledge that we recognised the characteristics of the prospective teachers' collective identity work only after engagement with the data, hindering timely intervention. The reported activity thus requires follow-up pedagogical activities.

To conclude, an emphasis on mathematical and methodological competencies often overshadows an emphasis on how individuals and groups construct themselves in relation to mathematics. Nevertheless, identity work whether individual or collective is essential for understanding oneself and others, which is crucial for teaching and thus deserves a place in mathematics teacher education research and practices.

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Notes

1 Lutovac and Kaasila (2018a) coined the term "mathematics-related teacher identity" as a more inclusive term that is applicable to all teachers who teach mathematics, such as mathematics specialist and generalist teachers. Therefore, this term covers those who identify as mathematics teachers and those who identify as teachers of mathematics.

- 2 The term "prospective mathematics teachers" is used in this study, even though the participants studied two or three subject combinations. This choice was made because the participants were asked to reflect on their experiences in the context of mathematics.
- 3 Secondary education in Finland encompasses lower secondary school (with learners aged 13–15 years) and upper secondary school (with learners aged 16–18 years).
- 4 The Finnish word *tunari* refers to a person who does things wrong. The word can also have a mocking connotation and serve as a nickname for those who fail a lot. We note that such language was uncommon during the course, and we assume that, in conversing with their peers (some of whom were also friends), Elisa and others may have felt comfortable using such language. Additionally, we speculate that the word choice could be an expression of frustration when dealing with students who do not understand seemingly straightforward content.

Appendix

Instructions for the group discussion

Begin the group discussion by sharing your most memorable failures related to mathematics. These experiences can be from your time in primary school, upper secondary school or at university. Describe the emotions surrounding your experiences and your actions in those situations.

Continue the discussion by focusing on the following questions:

1. What connects your experiences? What distinguishes them?

2. How did you understand and handle failures when you were younger? And how about now, as an adult? What significance have these experiences had for your identity?

3. How can your failures and successes help or hinder you in becoming a teacher? What could teachers do to handle failures?

4. If you could go back in time to a moment of failure and change one thing, what would you change? What can individuals do regarding handling failure?

5. Based on your discussion, what conclusions did you come to regarding failure? What does it mean to succeed or fail?

Sonja Lutovac

Sonja Lutovac is an associate professor at the Faculty of Education and Psychology, University of Oulu, Finland. Her research interests include the development of teacher identity in initial teacher education, focusing on how biographical experiences shape prospective teachers' identities, and on pedagogies in teacher education that assist and support pre-service teachers in this process.

sonja.lutovac@oulu.fi

Johanna Havia

Johanna Havia is a university lecturer working at both the Faculty of Technology and Faculty of Education and Psychology, University of Oulu, Finland. She received her PhD on inorganic analytical chemistry and her current research interests are in the fields of mathematics, physics and chemistry teacher education. She is especially focused on the concept of interest in these subject domains.

johanna.havia@oulu.fi