Do students need to learn how to use their mathematics textbooks?

The case of reading comprehension

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The main question discussed in this paper is whether students need to learn how to read mathematical texts. I describe and analyze the results from different types of studies about mathematical texts; studies about properties of mathematical texts, about the reading of mathematical tasks, and about the reading of mathematical expository texts. These studies show that students seem to develop special reading strategies for mathematical texts that are not desirable. It has not been possible to find clear evidence for the need of a specific "mathematical reading ability". However, there is still a need to focus more on reading in mathematics teaching since students seem to develop the non-desirable reading strategies.

When discussing students' use of mathematics textbooks one can ask many questions: *Do* they use the textbook? *How* do they use the textbook? *Why* do they use the textbook? Are they *able* to use the textbook? How *should* they use the textbook? Do they need to *learn* how to use the textbook? Unfortunately, these questions seem to be somewhat neglected in the research community of mathematics education [1]. Several authors note that the use of mathematics texts in a learning situation seems to be not well-researched (Fenwick, 2001; Laborde, 1990; Love & Pimm, 1996). Generally, mathematics education research seems to focus on cultural and social factors or interactions of different kinds (Sierpinska, 1998, p. 42). Sierpinska points to a risk that one thereby neglect important activities by the individual, such as "silent speech and thinking to one self, not out loud; listening to others; reading mathematical texts".

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In this paper, I discuss one specific aspect of students' use of textbooks, namely reading comprehension. Since using a textbook ought to involve reading in some form and to some extent, this aspect is of central importance. One can distinguish between at least two main forms of reading when discussing mathematics textbooks; the reading of tasks on the one hand and the reading of texts that describe and explain something (i.e., expository texts) on the other. Since mathematics textbooks seem to focus on exercises, questions and tasks of different kinds (Valverde et al., 2002) one could expect students' experiences with reading in mathematics to come mainly from the reading of tasks.

Although Valverde et al. (2002) noticed a general focus on tasks in mathematics textbooks, their study (i.e., TIMSS) also shows that there is a gradual increase in the use of narratives compared to tasks, from lower to higher grades. Their study covers grades up to the end of the secondary level. When comparing secondary and tertiary mathematics education one can in general notice many differences (de Guzmán et al., 1998; Tall, 1991), which of course will be reflected in textbooks. In addition, there can be differences in the way textbooks are used at these two levels:

> At the secondary school level it is assumed that students are instructed verbally and that reading is only really important when students are given written problems in homework assignments or tests. However, at the tertiary level students need to be able to read lecture notes, handouts and textbooks as part of the learning process itself. (Hubbard, 1990, p. 265)

However, it can still be the case that students at the tertiary level mainly focus on the tasks and seldom use any other parts of textbooks, as was the case in a study by Stephens and Sloan (1981). In this case, for some reason the students seem to view other parts of the textbooks than tasks as less useful for their learning, something that could be due to that they do not know how to use these other parts in a productive way. In line with this, Gelfman et al. (1999) highlight the need to teach students how to use their textbooks. Particularly for reading there seems to be a general agreement, although not explicitly based on research results, that students at all educational levels need to be taught how to read mathematical texts (Burton & Morgan, 2000; Cowen, 1991; Fuentes, 1998; Konior, 1993; Krygowska, 1969). There also exist examples of university courses that among others aim to develop this kind of ability among students (Esty & Teppo, 1994).

These aspects of reading in mathematics education show the importance of studying reading comprehension as one aspect of students' use of mathematics textbooks, and the main question that I will discuss in this paper is:

> Do students need to learn to read mathematical texts in a special way, that is, is there a need to develop some kind of ”mathematical reading ability”?

A need for a special type of reading ability for mathematical texts could be caused either by certain properties of mathematics itself or by how one chooses to present the content in texts (Brunner, 1976). One way to study the main question could therefore be to examine mathematical texts in order to look for some properties of either mathematics itself or mathematical texts that could create a need for a special kind of reading ability. Another way of approaching the main question is to study how students read mathematical texts: If and why students use special strategies for reading mathematical texts. This approach could lead to a discussion whether the students' reading behavior is desirable (and thereby can be seen as part of a reading ability to teach) or not desirable (and thereby can be seen as something to avoid when teaching about reading in mathematics).

In this paper, only mathematical texts created for educational contexts are of interest. Since students' use of textbooks is of interest, focus is on such texts that are part of mathematics textbooks. In particular, I will discuss the two mentioned main aspects of reading in mathematics, the reading of tasks and expository texts. In the present paper I describe two literature surveys and three empirical studies from my doctoral dissertation (Österholm, 2006b) regarding (a) properties of mathematical texts, (b) students' reading of tasks, in particular word problems, and (c) students' reading of expository texts. In the final section, the results from these three parts will be discussed in relation to the main question of this paper. However, first there is a need for a theoretical perspective on the notion of ”mathematical reading ability”.

Content literacy

In this paper, content literacy refers to the ability to read, understand and learn from texts in a specific subject area, as it is also defined by McKenna and Robinson (1990). They distinguish between three components of content literacy: general literacy skills, content-specific literacy skills, and prior knowledge of content. Both the general and the content-specific literacy skills can be assumed to refer to some more general type of knowledge that is not dependent on the detailed content of a specific text. The third component of content literacy, prior knowledge of content, refers to knowledge that is connected to the content of a specific text. Thus, the general literacy skills refer to an ability to read that is common for all types of texts and the content-specific literacy skills refer to an ability to read that is common only for texts within some type of genre or content area. Excluded in both these types of literacy skills is the utilization of prior knowledge specific for the content of a given text. Thus, the research question in the present paper addresses the issue whether there is a need for students to develop some sort of content-specific literacy skills for mathematics.

Properties of mathematical texts

The use of symbols in mathematics is one of the subject's most essential distinguishing features (Pimm, 1989; Woodrow, 1982). This feature may create a need for content-specific literacy skills, for example because symbolic expressions and natural language do not follow the same syntactical and grammatical rules (Ernest, 1987). It might even be the case that in order to understand a text written purely with mathematical symbols one does not need general literacy skills, that is, one might not need to know how to 'read' (in the sense perhaps most commonly used, i.e., reading texts written in a natural language). The symbolic language in mathematics, which can be seen as a separate language (Drouhard & Teppo, 2004), is a huge topic in itself and will not be discussed here in more detail [2]. Instead, I will focus on the use of symbols in conjunction with the use of natural language in mathematics, which is sometimes called the mathematical register (Halliday, 1975). This notion does not refer to a special kind of language but a special way of using an existing natural language [3], which could create such properties of mathematical texts that require content-specific literacy skills.

A literature survey was conducted in order to find studies that deal with special properties of mathematical texts (Österholm, 2006b, section 3.1). In total 31 references were included in this survey. In this literature, one finds many beliefs and claims about special properties of mathematical texts, which are thought to affect the reading in particular ways. In addition, the literature includes claims that comprehension of mathematical texts requires special ways of reading. However, not many of these claims come as a result from specific empirical or theoretical research studies, but often they are taken for granted without motivation or explanation. In this survey, only three *empirical* studies focused directly on mathematical texts in some way. Two of them used texts as empirical data (Burton & Morgan, 2000; Konior, 1993) and one studied college students' reading of different kinds of mathematical texts (Watkins, 1979). Neither of these studies compared mathematical texts with other kinds of texts, which makes it difficult to find any special properties of mathematical texts from their studies. However, Burton and Morgan (2000) noted both some common properties among mathematical research articles but also a great variety within this specific genre. This variety points to the difficulty of discussing properties of mathematical texts in general. Instead of trying to pinpoint some specific structure of mathematical texts to focus on in the learning of mathematics, Burton and Morgan (2000, p. 451) point to a need to develop a more multifaceted linguistic awareness, focusing on "knowledge of the forms of language that are available".

The general description of mathematical texts that is revealed from the survey focuses on complexity, that such texts are complex in many different ways. For example, this can concern single words in mathematical texts (e.g., that certain words are used in a complex way), single sentences in mathematical texts (e.g., that the sentence structure is complex), mathematical texts in general (e.g., that the texts have low redundancy), and the reading of mathematical texts (e.g., that one needs to read the texts several times). As mentioned, these characterizations mainly do not come from any structured analyses of texts, but claims are often made without any motivation or exemplification. In addition, mathematical texts are most often compared with 'ordinary' texts. No references were found that compare mathematical texts with texts from some other subject area. Many of the claims about mathematical texts might be relevant when comparing with more ordinary texts but not when comparing with other subject areas. For example, mathematical texts could be included in the genre *technical prose*, which includes texts from many subject areas: "Most textbooks are technical prose. They present densely packed complex information that is usually highly novel to the reader" (Kieras, 1985, p. 89). In line with this, Harmon et al. (2005) note that problems with 'difficult' texts are discussed within many subject areas. Thus, a possible property of complexity does not seem to be specific for *mathematical* texts.

Many of the properties of mathematical texts found in the survey do not deal with specific aspects of mathematics, but are applicable to texts in general. For example, it seems reasonable that a text can be more or less redundant or compact regardless of content domain. Also, even if mathematical texts can be characterized as being less redundant and more brief than texts from other domains, these properties seem primarily to demand a higher level of a more general reading ability (i.e., general literacy skills) and not necessarily some content-specific literacy skills. Furthermore, as already mentioned, technical prose in general seems to be associated with complexity, and this strengthens the view of a need for more general literacy skills. However, from the survey one can notice some properties of mathematical texts that more directly can be connected to the subject area itself, for example, the use of quantifiers, the fact that logical implications (if ... then ...) always should be interpreted strictly logically, and nominalization. Laborde (1990, p. 57) gives an example of a (double) nominalization: ”the composite of rotations”, where the activities ”to compose” and ”to rotate” have become objects. The relationship between processes and objects within mathematics (Gray & Tall, 1994; Sfard, 1991) can be connected to nominalization. This property of mathematical texts can thereby be ’derived’ from a property of mathematics itself. Of course, such properties of mathematics lie close to knowledge and learning in mathematics in general, and are thereby not specific to *reading* in mathematics. Thus, these special aspects of reading mathematical texts could be seen as being ’reduced’ to dealing with activating and using suitable types of prior knowledge, which can be viewed as a valid characterization of reading in general, and thereby mainly referring to the first and third components of content literacy (general literacy skills and prior knowledge of content). However, this phenomenon could also be seen as pointing to a problematic aspect of the theory of content literacy; to distinguish the three components in practice. Some more developments of this theory might therefore be necessary, but this lies outside the scope of the present paper.

From this survey, which has excluded specific aspects of symbols, it is difficult to find any distinct properties of mathematical texts that directly reveal a need for content-specific literacy skills. A more detailed linguistic analysis of texts from different subject areas could perhaps give some more insight into this issue together with an analysis of how the mentioned properties of mathematics (e.g., nominalization) can affect reading comprehension of mathematical texts.

Reading mathematical tasks

Since it had been noted that much of the research concerning reading in mathematics has focused on problem solving (Hubbard, 1990), a literature survey was conducted in order to examine the results from studying students’ reading of mathematical tasks[4]. Österholm (2007) gives the details of this survey and here focus is on the results, described through illustrating examples. The survey includes 199 references about word problems and studies that somehow focus on reading in relation to problem solving. Very few studies were found that in a direct manner focus on the aspect of reading comprehension in relation to problem solving. However, a number of studies examine how variations in text properties or reading ability affect student performance when trying to solve the problem. From such studies results can emerge that could be interpreted in relation to reading.

It is perhaps not surprising that the performance when solving problems can be negatively affected by a higher complexity of the language used in the problem text (Abedi & Lord, 2001) as well as by a relatively lower reading ability among students (Jordan & Hanich, 2000). Other studies also show a correlation between reading ability and performance on a mathematics test, for example in PISA (Roe & Taube, 2006). Much of the common variation between reading ability and solving mathematical tasks can be explained by a more fundamental common variable, such as intelligence (Aiken, 1972). Besides intelligence, an existing remaining correlation could be interpreted as showing the need for general literacy skills also when reading and solving problems in mathematics. However, studies of correlations do not *explain* much by themselves, and the relations between reading ability and mathematical performance can be complex. For example, a longitudinal study by Lerkkanen et al. (2005) shows that the ability in reading and the ability in mathematics were correlated in both school years one and two, but ”mathematical performance predicted subsequent reading comprehension during the first year rather than vice versa" (p. 121).

Several results seem to show that students are using special kinds of reading strategies when reading a mathematical task. One of these strategies is that students often seem to ignore realistic considerations when solving mathematical problems (Yoshida et al., 1997), that is, they seem to disregard certain types of prior knowledge. Another type of strategy is to focus on numbers and keywords in the problem text (Hegarty et al., 1995), that is, focus is on certain parts of the text. Both of these strategies seem to be specific to mathematics since other studies have let students solve the same task in different situations, which resulted in students giving more realistic answers in a non-mathematical situation (e.g., Wyndhamn & Säljö, 1997). In addition, Bilsky et al. (1986) show that students' reading strategies can be influenced by making them read a text either as a mathematics problem or as a telling of a story. When read as a problem, the text was read with a focus on quantitative aspects and as a story, it was read with a focus on more qualitative and temporal aspects. Thus, students seem to develop special types of reading abilities for mathematical tasks. Some authors also highlight the need for such special reading strategies for comprehension of these types of texts. De Corte et al. (1985) describe a word problem as a quite peculiar type of text that can include ambiguous statements, which in the given situation need to be interpreted in a particular way. For example, the statement 'a person has $5' could in general be interpreted as either that the person has exactly $5 or that the person has at least $5.

Although, as Siegel and Borasi (1992) point out, reading is perhaps often not in focus in a mathematics classroom, the mentioned types of reading strategies might nevertheless be introduced more informally. For example, regarding the strategy about focusing on keywords, Draper's study of methods textbooks for pre-service teachers shows that this strategy is discussed in this type of textbooks (Draper, 2002). In addition, Metsisto (2005, p. 9) has seen examples of teachers trying to help their students by saying "such things as '*of* means *times*' and '*total* means you probably have to add something'". However, this type of strategy can function as an interfering factor when solving problems (Nesher & Teubal, 1975), and it can even be characterized as being used in order to avoid reading altogether. Figure 1, which is a graphical interpretation of the discussions of Nesher and Teubal (1975, p. 42), distinguishes between two main strategies when trying to "translate" natural language into mathematical representations: "(a) a direct sequential translation, and (b) a grasping of the whole problem structure, making use of auxiliary representations and then proceeding to translate". The strategy of focusing on keywords tends to become a strategy of type (a), going directly from text to text. In principle, this type of strategy does not involve reading comprehension since it does not focus on comprehension of what is described in the text, but on the text itself (i.e., a surface feature).

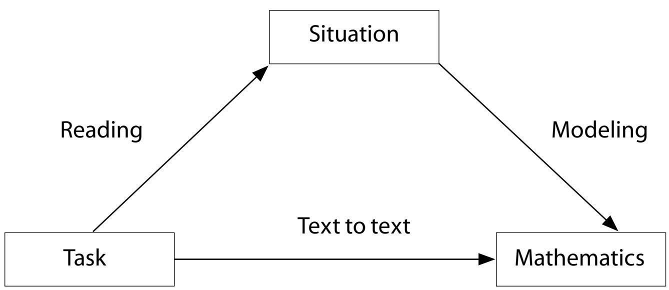

Figure 1. *Model describing two main strategies for "translating" a text in a natural language into a mathematical representation.*

The studies included in this survey treat reading comprehension mainly in an indirect manner, and observations about students' solutions and solution strategies have here been interpreted in the light of reading. The survey revealed very few studies that try to examine both students' reading comprehension and solutions of the same task. The study of Knifong and Holtan (1977) is one exception, which shows that the first step in main strategy (b), the upper part of figure 1, seems not to be the main problem when solving word problems. Their results showed that students who were interviewed almost always (in more than 90 % of the cases) could read the text correctly and also describe the situation in the text and what was asked for in the task, but seldom (36 %) could they describe a correct manner in which to solve the task. This result highlights the need for more thorough investigations when dealing with the relations between reading and solving a task.

In conclusion, studies about word problems and reading mathematical tasks seem to show that students use both general literacy skills and also content-specific literacy skills when reading and trying to solve tasks. It is not necessarily the case that the special types of reading strategies are used for mathematics texts in general, but that they could be limited in use for mathematical *tasks*.

Reading mathematical expository texts

Here I will summarize and discuss the results from three of my empirical studies regarding students' reading comprehension of mathematical expository texts. Some aspects of the methods used in these three studies are summarized in table 1.

Table 1. *Summary of methods for empirical studies.*

Study	Participants	Texts	Data collection
A	95 upper secondary and university students	Math with symbols Math without symbols History	Test: prior knowledge Read text Test: reading comprehension
B	91 upper secondary students	Math with less symbols Math with more symbols Math about concept Math about procedure	Read text Test: reading comprehension
C	9 university students	Math with symbols	Read text Judge own comprehension Motivate judgment

The two studies A and B used the same type of procedure for collection and analysis of data. The voluntarily participating students worked individually with self-instructed material in which the students gave all answers in writing. In the test of prior knowledge the students gave associations to selected words and phrases, which were taken from the texts. These associations were analyzed with respect to organization of knowledge (Langer, 1984), where associations can be categorized according to whether prior knowledge is highly, partially, or diffusely organized. A quantitative measure of prior knowledge was then created by giving points to the different categories and summarizing the results for different words (five words were given for each text read). The test of reading comprehension consisted of several questions that could be answered with some explicit information given in the text. In order not to measure pure memorization, the questions were not formulated in ways that resemble specific formulations given in the texts. A quantitative measure of reading comprehension was created by giving points on each question according to how complete and correct the answer was in relation to the content of the text, and then summarizing the results from the different questions.

Study A (Österholm, 2006a) focused on the use of symbols in mathematical texts and relations between the reading of texts from different domains. This was done by comparing reading comprehension of three different texts; two mathematical texts and one historical. The two mathematical texts both present basic concepts of group theory, but one does it with use of mathematical symbols while the other only uses natural language. For the participating students, the only new symbol introduced in the text using symbols is one for a general operation in a system or group (which in the text is described as a 'rule of combination', for which '~' is used as a symbol). Besides that, the symbols used are familiar to the students ('=', '+' and variables). By comparing the mathematical texts to the historical, a possible specificity of reading comprehension of mathematical texts could also be studied. The participants read one of the mathematical texts and the historical text. By looking at correlations between measures of reading comprehension, the results revealed a similarity in reading the mathematical text without symbols and the historical text (interpreted as general literacy skills at work), but no such similarity between the mathematical text with symbols and the historical text. In addition, the level of comprehension of the mathematical text with symbols was lower than the level of comprehension for the text without symbols. These results suggest that mathematics in itself is not the most dominant aspect affecting the reading comprehension process, but the use of symbols in the text is a more relevant factor.

When comparing students from different levels (upper secondary and university) there was no significant difference regarding their reading comprehension for the mathematical text with symbols. The courses at university level (in algebra and analysis) had thereby not affected the students' reading ability for this type of mathematical text. A similar result appeared in study B (Österholm, 2006b, chapter 8) but this time for the most advanced mathematics course at the Swedish upper secondary school (course E). For a text about partial fraction decomposition, a topic new for all participating students, there was no difference regarding reading comprehension between those students who had studied course E and those who had not. This study also focused on the use of symbols in mathematical texts. This time two texts were used, both describing definitions and properties of absolute value and both using symbols. The difference between the texts is in the amount of symbols used. This variation did not affect the reading comprehension among students. Therefore, the earlier result showing a clear difference between texts either using symbols or not using symbols at all, might essentially depending on the mere existence of symbols in a text, which is causing the students to read the text differently.

In study B (Österholm, 2006b, chapter 8), the reading of two other mathematics texts was also compared. A main difference between these texts is that one focuses on procedural knowledge (partial fraction decomposition) and the other on conceptual knowledge (absolute value). Studies of correlations showed that a noted co-variation between results on reading comprehension of these two texts could be explained by a common background factor (prior knowledge). Therefore, any direct effect of a use of common literacy skills for these texts could not be detected. Whether the focus on different types of knowledge in these texts also is the main factor affecting the students' reading comprehension needs to be examined in more detail. One conclusion that can be drawn is that there is a variation among mathematical texts regarding students' reading comprehension, that is, reading comprehension in mathematics is not a one-dimensional or homogenous aspect.

In study C, metacognitive aspects of reading comprehension were studied, where focus was on how students decide whether they have understood a text or not (Österholm, 2006c). The students read a mathematical text, which explained something that was new for them, and then orally stated to what extent they had understood the text in question. They were also asked to explain and give reasons for why they felt that they had or had not understood (some part of) the text. Besides studying specific metacognitive aspects, this procedure also gave the opportunity to examine qualitative aspects of the students' reading strategies in general. The clearest result from this study was that the students had difficulties to talk about their own reading comprehension in this manner. The students sometimes did not to talk about details of their own comprehension but on properties of the text that *could* make it difficult or easy to understand. However, some comments deal with the content of the text in more detail and can be said to reveal some types of strategies the students tend to use when reading mathematical texts:

> *Checking surface coherence*: The decision if you have understood something is based on whether a specific part of the text has been mentioned before. This strategy was in particular used for symbolic expressions, where the included symbols were checked to be mentioned in the text prior to the expression.
>
> *Accepting statements*: The acceptance refers to when one regards a statement as true without thinking about or testing whether it is true or not. This strategy was in particular used for texts in natural language and seems to lie close to memorizing.
>
> *Focusing on symbols*: This strategy refers to the occasions when comments about the level of comprehension focused on the symbolic parts of the text. Statements in natural language were mainly commented on when these were presented separately and not together with symbols.

The last type of strategy gives support for the conclusion discussed earlier; that it is the mere existence of symbols that affect students' use of specific reading strategies. Also, some more general beliefs were noted among the students, for example that reading mathematical texts for learning is very difficult, or even impossible, and that one needs to try to solve some tasks in order to know if one has understood a mathematical text or not.

The results from these empirical studies about mathematical expository texts show that students seem to use a more general reading ability for mathematical texts without symbols, but that a special type of strategy is used when mathematical texts contain symbols. When using this strategy, students focus on symbolic parts of the text. In addition, this special reading strategy can affect reading comprehension negatively compared to when more general literacy skills are used. Exactly how beneficial the general reading ability is for texts with symbols need to be further examined since this cannot be decided from the results about the students' spontaneous use of reading strategies.

Studying reading comprehension and the use of textbooks

Before drawing some more comprehensive conclusions from all the different studies, I will here discuss some properties of the type of studies that have been referred to.

Most of the studies discussed in the present paper have tried not to influence how participating students read given texts, in order to examine students' spontaneous use of reading strategies. While this type of research methodology could be seen as beneficial in order to study students' reading abilities, there are also some research questions that are more difficult to examine with this type of studies. For example, it becomes difficult to decide if some other type of reading strategy than the ones used by students could have been more beneficial for certain texts. Some studies about the reading of mathematical tasks have deliberately influenced the reading process, where they have made students read a mathematical text in different ways (e.g., Bilsky et al., 1986). This type of studies could be a starting point in a broader investigation of reading strategies in relation to different properties of mathematical texts, such as the use of symbols, where also the reading of expository texts could be included.

The focus on students' spontaneous reading strategies can be seen as a naturalistic property of research, and in the last paragraph I noted a need for a larger experimental component in order to examine the breadth of different types of reading strategies. However, while most of the studies discussed here have this type of naturalistic property, at the same time most of them are to a large degree experimental in nature. The situations used for data collection in these studies have been created specifically for the research study and are not part of a natural classroom situation. This experimental situation could of course cause the participating students to act differently than they would in other, more ordinary situations. On the other hand, the benefit of studies that are of a more experimental type is that you have more control over the situation where you can create and choose suitable material to use, such as texts, instructions, and questions. As a complement to my own studies about the reading of expository texts and to most of the studies in the field, there is a need for more naturalistic studies about reading comprehension of mathematical texts. Such studies could have a similar purpose as the one in the present paper; to describe and analyze students' reading strategies and reading comprehension. Another purpose of such studies, which is more complementary to the type of studies discussed in the present paper, could be to examine how the reading of mathematical texts is treated at the different educational levels. Such studies could for example focus on the questions if and how different reading strategies are discussed, formally or informally, by teachers and developed by students.

Most of the studies discussed here have been of an experimental type, but a strength of the present paper is that results from different types of studies have been covered, in particular regarding the reading of different types of mathematical texts. A comparison of results from these different types of studies is a way to strengthen possible conclusions about reading comprehension of mathematical texts.

Conclusions and discussion

Some comparisons can be made between the two kind of reading situations, that is, the reading of expository texts on the one hand and the reading of tasks on the other. Regarding the reading of tasks it has been noted that students read the text differently depending on the situation when the text is read, whether the situation is conceived as mathematical or not. A similar effect has been noted for expository texts, when students read a text differently if it includes symbols or not. Since the use of symbols is a central aspect of mathematics, a text not using symbols is perhaps not primarily seen as a mathematical text. Thereby, students might read expository texts in different ways depending on whether the text is conceived as mathematical or not. The main conclusion that can be drawn from these two types of empirical results is that the students do develop special types of reading strategies for mathematics. Are these strategies desirable? The strategy of focusing on key words when reading a mathematical task has been described as a potential interfering factor when solving problems (Nesher & Teubal, 1975) and can be regarded as a strategy not focusing on comprehension of content of the text but on surface aspects. In addition, when reading expository texts it has been shown that the strategy of focusing on the symbolic parts of texts can affect reading comprehension negatively compared to when more general literacy skills are used (Österholm, 2006a). Thus, it seems like students develop strategies that are not desirable, at least not from the perspective taken here.

Why then, do students develop these strategies? If reading is never dealt with in the mathematics classroom, students will develop strategies themselves from what demands they experience when it comes to reading mathematical texts. If their experiences with reading are limited to mathematical tasks, this can of course create special kinds of strategies that possibly could be generalized to all types of mathematical texts. The strategy of focusing on symbols when reading expository texts could be a generalized form of the strategy of focusing on numbers and keywords in mathematical tasks. The way that reading is dealt with in mathematics education also seems to be a possible explanation for the strategies developed by students. For example, it has been noted that within methods textbooks for pre-service teachers, discussions about reading is not only scarce but also that reading is sometimes described negatively, as a less suitable choice of teaching method by associating it with old-fashioned methods (Draper, 2002). Also, Metsisto (2005, pp. 9–10) notices that teachers *do* guide students about reading, but that "most strategies are still procedural – *follow this recipe* – rather than about helping students to read for understanding". If students experience these aspects of reading in mathematics; that it is not important and that it is separate from reading other types of text where focus is on understanding the content of the text, it is perhaps not surprising that the mentioned types of reading strategies are developed. Even in the research community of mathematics education, reading has somewhat been reduced to a potential obstacle for learning (Borasi & Siegel, 1990, 1994), for example by focusing on how limitations in reading ability affect learning or on readers' misunderstanding of a written task and how this can influence the solving of the task. In addition, the literature survey discussed earlier shows that there seems to be a strong agreement in the literature that mathematical texts are complex and difficult in different ways. Such a view of mathematical texts could explain the mentioned tendencies among teachers to try to guide the students past the reading of mathematical texts. However, the survey also revealed a need for in-depth studies about these types of properties of mathematical texts.

Regarding the question whether students need to learn how to read mathematical texts it seems like the answer could be both yes and no. The fact that the students seem to develop undesirable reading strategies highlights the need for more explicit teaching of reading in mathematics education, that is, to teach students how to read mathematical texts. This conclusion is strengthened by the results showing that mathematical courses do not always affect students reading ability for mathematical texts. On the other hand, some results have shown the benefits of using general literacy skills also when reading mathematical texts. One aspect of learning to read mathematical texts might therefore not be to read these texts differently (i.e., to use some content-specific literacy skills) but to use more general literacy skills also in mathematical situations. At the same time, some properties of mathematical texts have been discussed that could create a need for content-specific literacy skills. However, since no studies have been found that relate these properties of texts to reading comprehension it is not yet possible to determine their impact on reading.

Instead of focusing on *the language* of mathematics and what special properties it might have, in order to investigate if the reading of mathematical texts might demand some type of content-specific literacy skills, it has been highlighted that the variety of texts and reading situations might be a more suitable starting point. For example, this has been shown by the fact that even within a specific genre of mathematical texts there is great variety regarding properties of texts (Burton & Morgan, 2000) and that results regarding reading comprehension among students show differences depending on the type of mathematical text that is read (texts focusing on conceptual or procedural knowledge). Furthermore, the results among students showing difficulties when reflecting on their own reading comprehension, and sometimes focusing on surface aspects of texts, together with an existing belief that reading mathematical texts for learning is virtually impossible, show a need for students to experience different purposes with reading. For example, this could include focusing on a text itself and the comprehension from reading it, and not only using it for solving problems.

From what has been noted about the variety of mathematical texts and the limited view of reading among students, it seems like students of all ages would benefit from more varied experiences with reading mathematical texts. Such experiences could include the reading of different kinds of mathematical texts, not limited to tasks, and different types of needs for reading, where texts are used for different purposes. These types of experiences could cause the students to see the need for different forms of reading strategies and to see the benefits of using general literacy skills also for mathematical texts.

In conclusion, if one expects students to be active and competent users of mathematics textbooks, including all parts of textbooks, there seems to be a need to focus on reading and reading comprehension in mathematics education, where a varied experience of texts and reading is offered to students.

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Notes

1 For example, a search in the MathDi database (http://www.emis.de/MATH/DI/) on October 1, 2008 resulted in 44 references which had 'textbook*' and 'student*' in the title (of which 13 written in English) compared to 350 references when the word textbook was exchanged for computer (of which 280 written in English).

2 See Österholm (2006b, section 3.3) for a discussion focused on reading comprehension of symbols and symbolic expressions.

3 This includes, but is not limited to, the usage of special terms and terms with special meaning. Other types of linguistic properties can also be included in the mathematical register, such as special ways of structuring texts or the usage of the passive voice.

4 Sometimes it can be of importance to distinguish between tasks and problems, but here these two words will be used interchangeably, referring to all types of tasks.

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Sammanfattning

Huvudfrågan som diskuteras i denna artikel är om studerande behöver lära sig att läsa matematiska texter. För att besvara denna fråga beskriver och analyserar jag resultat från olika typer av studier kring matematiska texter; studier om egenskaper hos matematiska texter, om läsning av uppgiftstexter och om läsning av förklarande texter. Studierna visar att studerande verkar utveckla speciella lässtrategier för matematiska texter som inte är önskvärda. Det finns inga tydliga bevis för att man behöver utveckla någon sorts ”matematisk läsförmåga”. Dock är det i alla fall nödvändigt att behandla läsning i matematikundervisning eftersom studerande verkar utveckla de icke önskvärda lässtrategierna.