Analyzing mathematical classroom discourse

Initiating elaborations on the usefulness of the dialogical approach

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The dialogical approach has been introduced for studying mathematical classroom discourse in a growing body of studies conducted by researchers from the Nordic countries. However, since it is developed for analyzing human action, communication, and cognition in general, it is important to explicitly discuss how it could be developed and complemented for serving the purposes of mathematics education research. In this article I initiate such a discussion by drawing on theoretical analysis as well as my own experiences of using the dialogical approach. By relating it to a framework of criteria for research in mathematics education it is shown that the dialogical approach could be a useful tool for fulfilling several aspects of relevance for mathematics education research. The article concludes by suggesting further aspects that need to be discussed and elaborated on in the project of making it even more useful for understanding mathematical teaching and learning.

In the field of mathematics education there is a growing interest in studying communication and face-to-face interactions in mathematics classrooms. According the Kieran, Sfard, and Forman (2001) this interest in communication has both a theoretical strand originating from sociocultural and interactional perspectives as well as a practical strand referring to changes of how the work in the mathematical classroom could be arranged. While the interest in mathematical classroom interactions [1] is high, the analytical approaches used for conceptualizing and analyzing

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them need to be developed (Bartolini Bussi, 1998; Cobb, 2007; Kieran, Forman & Sfard, 2001; Ryve, 2006a; Silver & Herbst, 2007). One approach introduced for examining classroom interactions in mathematics education is the dialogical approach (e.g., Linell, 1998, in press), used by several researchers from the Nordic countries (see, e.g., Bjuland, 2002, 2004, 2007; Carlsen, 2005; Cestari, 1997; Cestari, Daland, Eriksen & Jaworski, 2005; Ryve, 2006a; 2006b; 2007). Given the fact that the dialogical approach includes many ideas and analytical focuses (see, e.g., Linell, in press) and that it is not particularly developed for studying mathematical communication and cognition, it is natural that mathematics education researchers not only use different aspects of the dialogical approach (DA^2) but also relates it to different kinds of mathematical frameworks. The aim of this article is to introduce the dialogical approach, make explicit how it could be used for analyzing mathematical teaching and learning, and introduce aspects that need to be further considered for making the dialogical approach even more useful for mathematics educators. In the spirit of dialogism I regard the process of developing the dialogical approach as a collective endeavor and therefore hope that this article could initiate a scientific communicative project among researchers in mathematics education interested in interactional approaches of how to adjust, complement, and use the interactional theories in general, and the dialogical approach in particular, for analyzing mathematical teaching and learning.

The article is divided in four main parts. Firstly, historical roots, principles, and analytical constructs from DA are outlined by reference to, first and foremost, the work of Per Linell. Secondly, an excerpt of an interaction sequence in a classroom is introduced to show how the dialogical approach could be used for analyzing empirical data. Thirdly, I reflect on how DA could be useful in mathematical education research by relating it to criteria of quality in mathematics education research. Finally, the article is concluded by an initiation of several aspects that need to be discussed regarding the relation between the dialogical approach and mathematics education research.

Introducing the dialogical approach

The transcript below is taken from a study reported in Ryve (2006a; 2007) and the interaction is entered when a prospective teacher (PT) is presenting a solution, at the chalkboard, to the following mathematical task:

> Three friends are carrying a 19-meters-long flagpole in a block where each and every street is 7 meters wide and perpendicular to

each other. Could they turn around a street corner if the flagpole is carried in a horizontal position?

The teacher educator (TE1), and in [44] also the audience (A) consisting of prospective teachers, contribute to the solution process. In this article the transcript serves the purpose of illustrating and helping concretizing aspects of DA such as principles, frameworks, and analytical constructs and how those could be used for analyzing data.

[43] PT ... square root of 98, I need help with that.
[44] TE1 Yes but it is an even (A – it becomes 9.899) ne, ne, ne no decimals.
[45] PT How much was it?
[46] TE1 No, look at the number under the square root sign and say something about it.
[47] PT It is 98 (TE1 – Yes) yes it is smaller than 10.
[48] TE1 eeh say something sensible about it.
[49] PT It is close to 10.
[50] TE1 98 is much bigger than 10.
[51] PT Yes, yes 98 is bigger than 10 but it is close to 100 and 100.
[52] TE1 Yes and not just close to 100 say something precise about it.
[53] PT It is an even number.
[54] TE1 Yes! Divide it by two then, what do you get?
[55] PT Yes okey then I get 49.
[56] TE1 And 49.
[57] PT Yes the square root of 49 is 7.
[58] TE1 Exactly, thus it is equal to.
[59] PT 7 times the square root of 2.
[60] TE1 Yes then it is exact, 7 square root of 2, okay.
[61] PT Is this junior high school mathematics.

Historical roots and dialogical principles

The concepts of the dialogical approach and dialogism are used by many scholars in partly different ways and should therefore not in general be seen as one well defined and coherent approach to the study of communication and cognition (Linell, in press). This implies that it is essential to specify which strand of them one refers to. Here, as well in the Nordic studies referred to above, dialogism and the dialogical approach are used in accordance with what the Bad Homburg Study Group on 'The Dynamics of Dialogue' has developed and especially what is presented in

Linell (1998). Furthermore, in line with Bjuland (2002) I choose to denote the analytical approach developed in Linell (1998) a dialogical approach while dialogism refers to a general and broad theoretical perspective on human cognition, communication, and sense-making.

Linell (1998) is primarily focused on ways of conceptualizing and analyzing face-to-face interactions. It is here important to stress that DA is not developed for analyzing any particular activity and therefore does not guide the researcher about, for instance, how to track mathematical changes in an interaction. This does not, however, imply that the development of DA does not draw on empirical studies. Instead, the project of developing the dialogical approach is based on empirical studies taken from, for instance, courts, police interrogations, doctor-patient consultations, and dinner conversations.

The theoretical ground of dialogism and the dialogical approach includes influences from four perspectives: phenomenology, pragmatism, symbolic interactionism, and sociocultural theory. From *phenomenology* dialogism has adopted the insight that the world is always seen from some particular perspective which implies, among other things, that individuals try to develop mutual perspectives and intersubjectivity in communications. An important idea from *pragmatics* that dialogism builds on is that individuals typically do not possess well-articulated intentions that they are ready to transmit to other individuals, but instead that individual cognition could be conceptualized as flow of thoughts. Such a conceptualization of mind results in an interest in how individuals co-construct the interactional focus and interactional projects instead of how individuals' intentions determine the focus of the interaction. Linell (1998) draws on *symbolic interactionism* (Mead, 1973) and the idea that there need to be at least three turns in an interaction to be able to develop intersubjectivity. Schematically speaking, agent A wants to communicate meaning ***m*** to B and therefore pronounces an utterance ***u1*** and B responds to that by ***u2*** indicating his/her understanding of ***m*** followed by some kind of indication of A's reaction on ***u2***. Finally, dialogism is inspired by *sociocultural theory* including the work of Vygotsky, Wertsch, Bakhtin, and Leontèv in that Linell (1998) views interactions as situated in wider sociocultural contexts.

While Linell (in press) is focused on elaborating on the broad theoretical perspective of dialogism, Linell (1998) stresses three basic principles of a dialogical view on interaction: *sequential organization*, *co-construction*, and *act-activity interdependence*. First, all interactions have a sequential organization. That is, the meaning of each discursive component is to a high degree determined by its position in a sequence. This implies that the discursive components could not be fully understood outside the

sequence forming one of its contexts. Take a turn from the transcript above, such as [56] "And 49", as an example. It is obvious that [56] must be understood in relation to what is said before and after that turn and this holds for each and every turn.

Second, interactions are jointly constructed, even though they could be asymmetrical. This implies that a researcher using DA should stress intersubjective features of dialogues and not solely regard turns as expressions of individuals' intentions. In other words, individuals always adjust their ways of expressing themselves in relation to whom they are speaking to. Notice in the transcript how TE1 and PT together accomplish the project of discussing how to transform $\sqrt{98}$ into $7\sqrt{2}$ and how TE1 adjusts his/her way of expressing himself/herself in, for instance, [46], [48], and [52]. Hence, TE1 and PT together construct the interactional sequence and adjust their way of speaking in relation to each other.

Third, discursive components should be understood in relation to, and at the same time as constituting, the activity. Hence, there exists an act-activity interdependence. In relation to the transcript above the act-activity interdependence could be illustrated by conducting a thought experiment: Could this be a dinner conversation among friends? A number of features indicate that this is not the case; the one who knows the answer asks most questions, special tools such as chalkboards, tasks as well as mathematical symbols are used. That is, the activity to a certain extent determines ways of talking and the roles of the participants. On the other hand, the activity is developed through communicative acts and in this activity, for instance, being precise about $\sqrt{98}$ becomes established as an important feature of the activity. Hence the acts contribute to the establishment of the activity.

To sum up, Linell (1998) draws on ideas from phenomenology, pragmatism, symbolic interactionism, and sociocultural theory as well as detailed examinations of tape-recorded interactions in formulating the three principles of DA. While it is important to understand the basic principles if one is about to use DA they, alone, are of little help in analyzing interactional data. Therefore, Linell has derived both a theory of contexts and a number of analytical constructs of usefulness in analyzing interactional data. I now turn to those frameworks.

Contextual resources and contexts

In the introduction to the transcript few contexts were presented. Different traditions engaged in analyzing interactions such as conversation analysis (e.g., Schegloff, 2007), discursive psychology (Wetherell, 1998), and interactional sociolinguistics (Kress, 2001) hold different opinions on

whether, and in which ways, contexts should be included in the analysis of interactions. It is out of the scope of this article to dig deeper into this debate (see for instance, Schegloff, 1997; Wetherell, 1998). But for Linell (1998) it is clear that "a theory of discourse needs a theory of contexts" (p. 127). This statement follows naturally from the third principle of act-activity interdependence which explicitly postulates that interactions are dependent on the activity. Further, from a dialogical point of view nothing becomes a context in and by itself but instead becomes relevant through the activities of the participants in the interaction (cf. Goodwin & Duranti, 1992). It is therefore helpful to make a distinction between *contextual resources* and *contexts*, where the former serves as a starting point for the analysts to structure possible contexts while the latter is used for denoting aspects that actually become relevant in the interactions. Linell introduces ten *contextual resources* as a framework supporting the process of understanding what is happening in interactions.

(1) *Surrounding concrete situation*: As the name indicates, this contextual resource accounts for what is happening on the spot. The prospective teacher (PT) is standing at the chalkboard solving a mathematical assignment in front of the teacher educator (TE1) and the other prospective teachers (A). As will be shown below the fact that the prospective teachers in the audience have got calculators and that PT does not turns out to be an important context for understanding the transcript.

(2) *Co-text*: This contextual resource refers to the interaction immediately prior to the piece of talk presented. For instance, turns [43]–[50] constitute the co-text to [51]. This contextual resource is important in relation to the first principle of sequentiality.

(3) *Actors' knowledge and beliefs about topics*: In mathematics education this is often the object of study and not a context. Thus, what is regarded as a contextual resource in interactional research could be the focal event in mathematics education research. In the current setting it is reasonable to assume that both the prospective teachers and the teacher educator could be seen as knowledgeable in solving the mathematical task. That is, if the topic is defined as being able to solve the mathematical task it seems reasonable to assume that the participants of the interaction are knowledgeable, but if the topic is defined as solving the task and explaining the solution in an appropriate manner for junior high school students one could not directly assume that all actors could be regarded as knowledgeable. In fact, this discussion raises issues of to what extent it is possible for the analyst to state something about contextual resources without carrying out some kind of a priori analysis of

contextual resources. This question holds for many contextual resources and will be discussed further below.

(4) *Actors' knowledge and beliefs about communicative projects*: In relation to the former contextual resource, this contextual resource refers to actors' knowledge and beliefs of more specific topics of the interaction. In the present case, the prospective teachers' and teacher educator's knowledge of the process of transforming $\sqrt{98}$ is of significant relevance in relation to the transcript. The prospective teachers' knowledge and beliefs about the necessity of avoiding transforming $\sqrt{98}$ into a rounded decimal number may in itself be an interesting object of study but in Linell's framework it should be seen as a contextual resource.

(5) *Actors' knowledge of each other*: In relation to the transcript, it may be of importance if PT at the chalk board is regarded as mathematically strong or weak in relation to his/her fellow students. That is, one could imagine that if PT is regarded as strong in mathematics it is more natural for TE1 to ask, what I regard as, tough questions such as [48] " eeh, say something sensible about it". Furthermore, it may affect the discussion if the prospective teachers are used to presenting solution at the chalkboard in front of each other and to what extent the teacher educator and prospective teachers know each other.

(6) C*ommunicative genre*: This is a contextual resource that accounts for interactions' embeddedness in different activity types. The empirical example above could be contextualized as a course for prospective mathematics teachers especially directed towards problem solving. Research (e.g., McHoul, 1978) has shown that classrooms could be seen as a specific activity type that to a certain extent determine ways of speaking. On the other hand, research in mathematics education (e.g., Cobb and colleges) has tried to change and develop norms and genres of mathematics classrooms including what counts as a mathematical justification and who has the right or responsibility to justify mathematical claims. Therefore, it seems to be a complex relation between how activity types influence the interaction and how interactions establish activities. From a dialogical point of view it is important that the analyst acknowledges this relation. In the transcript it can be noticed that, for instance, TE1 is about to establish norms on how to handle decimal numbers within solution processes, and also that it is not necessary for TE1 to justify why PT should avoid decimal numbers. Linell (1998) views communicative genres as one of the most central concepts in communicative theories and therefore introduces it as an analytical construct.

(7) O*rganizational premises*: This contextual resource refers to aspects such as working conditions and division of labor. In this specific course the prospective teachers were working in groups preparing the presentations of the assignments before they presented their solutions at the chalkboard. One may also include aspects such as the amount of time the teacher educator gets for preparing the class or aspects of what other courses the prospective teachers are taking for the moment. Such aspects may affect how much time the prospective teachers have had to prepare the presentations and therefore influence the interaction.

(8) *Sociohistorical contexts*: This resource could include gender or socio-economic issues as well as discussions of the status of teacher education in society. For an analyst interested in conducting a priori analyses of that contextual resource there exist studies in mathematics education conducted from sociocultural perspectives that may be helpful (cf, e.g., de Freitas & Nolan, 2007). One may ask questions such as, does it make a difference if TE1 and PT are men or women? Are their ethnical backgrounds of relevance for understanding the interaction?

(9) *Actors' knowledge of language, communicative routines, and action types*: Here one could notice that Linell (1998) seems to distinguish between activity types (see the context of communicative genre) and actors' knowledge of activity types. This distinction could generate interesting discussions about where contexts are located: in the mind, in the interaction, or somewhere out there? Part of the project in becoming knowledgeable in mathematics is to be able to participate in mathematical discourses (Sfard, 2008) so the contextual resource is both a means and an end in the classroom discussion. It is reasonable to assume that the TE1 could be seen as more knowledgeable than the prospective teachers in many aspects of the discussion which influence his/her role in the discussion. Along another dimension, in the data connected to this study, examples could be found in which prospective teachers with a different mother tongue than the language of the discussion engaged in the discussion. Such aspects also belong to this context and may be consequential for the co-construction of interactions.

(10) G*eneral background knowledge*: This contextual resource is related to common-sense as well as to cultural specific ways of viewing the world. It refers to cultures of larger size than activity types such as the 'Euro-American culture'. For example Setati and Adler (2000) show how both knowledge of language (cf. former contextual resource) and general cultural background knowledge could influence the students'

access to mathematical classroom discussions. In the present study I did not get strong evidence about how such aspects impinged the classroom interactions.

The contextual resources introduced above should be seen as a framework structuring the analyst's process of noticing relevant contexts in analyzing the interactional data. In addition to the ten contextual resources, Linell (1998) elaborates on constructs used for structuring and ascribing meaning to the data. These analytical constructs will be introduced below.

Analytical constructs

Linell (1998) suggests two different basic building-blocks of talk, namely *turns* and *idea units*. A turn is basically a period of time when one speaker holds the floor; while an idea unit refers to, as the name indicates, chunks of ideas. The choice of elementary contributions has to be connected to the aim of the analysis. If the researcher is focused on the interaction between individuals, a turn becomes a natural unit to start the process of structuring the analysis. That is, using the turn as a building block helps the researcher to focus on how individuals take turns and adjust their turns to the other participants in the interaction. If the researcher is more interested in the individuals' ways of building arguments and rhetorically constructing turns, or if the turns become longer, the idea unit may be preferred as a building block.

In line with the principle of sequentiality, each turn should be interpreted and understood in relation to the prior interaction as well as seen as creating conditions for the ongoing interaction. Initiative and responsive characteristics are therefore seen as ubiquitous aspects of turns. Researchers should search for and understand how a turn is connected to both what has been said immediately prior to it and immediately after it. Take TE1's turn [50] "98 is much bigger than 10" as an example. [50] is a direct response to [49] "It is close to 10", but also directs PT towards how to express [51] "Yes, yes 98 is bigger than 10 but it is close to 100 and 100".

In creating a coherent interaction, the participants must be attentive to each other, establishing a common focus and tying "their contributions together, thereby sustaining and developing foci of attention and discourse topic over sequences of interaction" (Linell, 1998, p. 179). Topics should not be seen as something the interaction is about, but rather as the activity of the interaction. Hence, topics are constituted and transformed by the participants through the interaction and the researcher should analyze not only which common foci are established, but also

how topics are co-constructed by the participants of the interaction. The transcript above shows how the topic of transforming $\sqrt{98}$ into $7\sqrt{2}$ is a collective process which TE1 and PT together construct by adjusting their turns towards each others' turns. Linell uses the concept of *topical episode* to denote a piece of talk in which the participants co-construct a common focus.

While turns and topical episodes are closely connected to psychical units[3] of the interaction the construct of *communicative project* is used for going beyond talk and ascribe meaning to it (Linell, 1998). Therefore, communicative projects are units of meaning rather than physical units of expressions. Several communicative projects could therefore be ascribed to the same piece of talk since the projects could vary in character (e.g. social, mathematical) and size (a scale from local to global). For instance, the transcript above could be seen as a communicative project of a more social character in relation to [61] "Is this junior high school mathematics" in which PT comments on the struggles he/she went through in front of his/her fellow students. On the other hand, it could be seen as a micro mathematical project of transforming $\sqrt{98}$ into $7\sqrt{2}$. Furthermore, a communicate project of more global character could be ascribed to the transcript, for instance, the project of educating prospective teachers in Western societies.

Communicative projects are defined in relation to which communicative problems they aim at solving. Attempts to solve communicative problems are necessarily collective even though different interlocutors could have different interpretations of the problem and different purposes participating in the discourse. Linell (1998) argues that "we must therefore assume that some purposes and projects are collective and cooperative, whereas other goals and projects are more tied to role incumbents with different responsibilities and/or pursued by actors in competition." (p. 220). In addition, communicative projects are always collective but often with an asymmetry between the participation, especially in institutional settings. In fact, if there was not any kind of asymmetry there would be little reasons to communicate at all. This observation holds for the transcript where it could be seen that TE1 to a large extent controls the interaction even though TE1 must adjust his/her turns in relation to both PT and the audience.

Contextual resources were introduced to explicate how different settings and contexts affect the interaction. In further stressing such aspect, as well as taking the principles of act-activity interdependence seriously, Linell presents the analytical concept of *communicative genre*. Communicative genres are seen as ways of interacting where certain historical and cultural norms, routines, and interactional patterns have been

established. These routinized ways of interacting have been established, according to Luckmann (1992), to solve recurrently occurring communicative problems[4]. In the transcript relevant communicative genres may be school genre, teacher education genre, or mathematics teacher education genre helping the researcher to understand, for instance, why TE1 asks a lot of questions even though he or she apparently knows the answer (cf. Mercer, 1995), or why transforming $\sqrt{98}$ into $7\sqrt{2}$ is regarded as important. Linell accentuates the homogeneity of communicative genres but also notices that interpretations of tasks and utterances could vary among participants depending on how they (implicitly) define the communicative genre. This implies that different participants may talk about the same issue, but from different perspectives, leading to "contrasting and competing versions of the 'same' events in the world" (Linell, 1998, p. 256). Below, I will elaborate further on how individuals interpret communicative genres.

To conclude this section, within DA a number of analytical constructs have been developed that correspond to the three basics principles. In the next section I will continue the discussion by illustrating how constructs from DA could be used for analyzing the transcript. This introduction would also serve as a background for elaborating on how DA could be complemented for serving the special needs of mathematics education.

Using the dialogical approach

Let us return to the transcript above. In [43] PT states 'square root of 98, I need help with that'. At least one prospective teacher in the audience contextualizes turn [43] as a request for a numerical value (cf. the answer "It becomes 9.899" from one of the prospective teachers in the audience). I want to raise three issues in relation to turn [43] that are of relevance from a dialogical viewpoint. First, it is obvious that the turn cannot be interpreted solely by focusing on the lexical meaning of "I need help with that". In fact, while at least one prospective teacher in the audience interpreted it as a request for using the calculator to present a decimal number, TE1 wants it to be a request for help in transforming $\sqrt{98}$ into $7\sqrt{2}$. Second, to understand [43] it is important that we take the first contextual resource of the surrounding concrete situation into account. That is, it is of relevance that the prospective teachers in the audience have got calculators and the PT has not. This strengthens the interpretation that PT is interested in a numerical value. Third, and related to the first comment, as an analyst one needs to have a fairly good knowledge about the institutional learning of mathematics to realize that the issue of numerical contra exact values within processes of calculations could

be of significant importance in mathematics education. Therefore, it would certainly be helpful to relate this observation to explicit theories of, for instance, prospective teachers' beliefs about mathematics. Such additional theoretical support is in harmony with the dialogical approach since the framework of contextual resources enables the researcher to include a variety of theoretical backup. We can therefore conclude that the dialogical approach offers possibilities to include mathematical aspects within the analysis, and that in order to better understand what is happening in the interaction such inclusions are necessary.

Let us continue by studying turn [46] 'look at the number under the root and say something about it'. While one of the prospective teachers in the audience gives the numerical value, TE1 initiates the communicative project of transforming $\sqrt{98}$ into $7\sqrt{2}$. Two aspects of [46] are of relevance in this context. First, the turn must to a high degree be understood in relation to the contexts, and especially the communicative genre, in which it is produced. That is, to say something about a number could include very different things such as "it's beautiful" or "it's black" and the communicative genre to a certain extent guides PT about what is a reasonable thing to say about the number. Second, PT nevertheless seems to have vague ideas about TE1's contextualization of doing 'something' with $\sqrt{98}$. The second statement could be supported by the fact that TE1 needs to perform some communicative strategies, such as specifying the expression of 'something' by turn [48] 'say something sensible about it' and by turn [52] 'say something precise about it'. We could thus see that the process of transforming $\sqrt{98}$ into $7\sqrt{2}$ is a collaborative project, just as Linell (1998) stresses, but also that it could be characterized as asymmetrical.

In [54] TE1 seems delighted about PT's contribution in [53] 'It is an even number'. Notice though that in [44] TE1 was about to introduce the fact that 98 is an even number. The interaction connected to turns [53]–[60] proceeds without complications and PT and TE1 continue the communicative project of turning $\sqrt{98}$ into $7\sqrt{2}$. In [60] TE1 indicates that the communicative project is ended both by saying "then we have" indicating that they have reached a sub goal and by "okay", here serving as an indication that the project is finished and it is time to move on. In [61] 'Is this junior high school mathematics.' PT performs a reflective meta-turn referring back to the communicative project just ended. This turn could have been the starting point for another communicative project about the mathematics involved in the problem, especially the relevance of not introducing inexact decimal numbers within a solution process. Instead TE1 chooses not to comment on [61] and continues the main communicative project of solving the mathematical problem.

What has been produced so far in terms of analysis is a fairly micro-oriented turn-to-turn study of how the participants co-construct a communicative project. By using conversational analysis (e.g., Schegloff, 2007) or ethnomethodological approaches (e.g., Garfinkel, 1967), the analysis of the sequence could have been made even more detailed. On the other hand, conversation analysis and ethnomethodology offer less theoretical support for widening the analytical focus. Linell's (1998) third principle of act-activity interdependence and the related constructs of activity types and communicative genres open up for a broader focus. There are of course many research questions that could be formulated and examined within such a theoretical frame. For instance, why is TE1 anxious that PT should be involved in the turning $\sqrt{98}$ into $7\sqrt{2}$? Why is PT standing at the chalkboard solving a mathematical junior high school problem? Both questions could be connected to the activity type of educating prospective mathematics teachers. The important point here is that the dialogical approach includes theoretical support for initiating such examinations.

The dialogical approach and mathematics education

The historical roots of DA, its three principles, its contextual resources, and an empirical example have been presented so far. From this background I now turn to the discussion of how DA could be useful in mathematics education research by relating it to the aspects of the framework of criteria for the quality of mathematics education research presented in Lester and Lambdin (1998). The framework consists of seven criteria including: Worthwhileness, coherence, competence, openness, ethics, credibility, and other qualities of good research reports. Since the framework is designed to capture all aspects of the research process, only some of the criteria are of interest to discuss DA in relation to mathematics education.

In mathematics education researchers (e.g., Sfard, Nesher, Streefland, Cobb & Mason, 1998) as well as curricula and standards (NCTM, 2000; Swedish National Agency for Education, 2000) emphasize social aspects of mathematical teaching and learning, including communication. This indicates that research directed at analyzing mathematical communication is worthwhile (cf. Lester & Lambdin, 1998). From such a perspective DA could be useful for conceptualizing collaborative and communicative aspects of the mathematics classroom.

Lester and Lambdin (1998) further note that researchers should provide the reader with a clear sense of how the data were analyzed, an aspect of the criterion of openness. As Niss (2007a, 2007b) makes clear,

explicit use of theories for analyzing data is rare in mathematics education. Linell's explicit introduction of analytical constructs facilitates the process of actually using theories for analyzing data. Besides the analytical constructs discussed above, some researchers (e.g. Bjuland, 2002; 2004; Carlsen, 2005) in mathematics education have also introduced and used Initiative-Response analysis (I-R) (Linell & Markovà, 1993; Markovà & Linell, 1996) as a coding scheme from DA. I-R is especially developed to understand how participants build on each other's turns. This type of analysis has turned out to be useful for understanding students working in groups solving mathematical problems. In short, DA supplies the analyst with several analytical constructs and methods of analysis that are useful in analyzing interactions and classroom discussions. One may ask the question: Are there not already such analytical approaches developed for analyzing mathematical classroom interactions?

There exist several analytical approaches developed within mathematical education for studying mathematical discourse (e.g., Sfard & Kieran, 2001; Cobb, Stephan, McClain & Gravemeijer, 2001) and approaches borrowed from other research traditions including ethnomethodology (see, Cobb & Bauersfeld, 1995), sociolinguistics (see, Morgan, 2006), sociocultural studies (see, Lerman, 2000). While it is out of the scope of this article to compare DA to every existing interactional approach used in mathematics education research, it could be mentioned that the approach of Sfard and Kieran (2001) is more useful in analyzing group discussions than whole-class discussions (see Ryve, 2007), and the approach of Cobb and collaborators is especially developed for *designing*[5] classroom interactions and establishing norms. When applied on the transcript above I found these two approaches of limited help, while DA allowed me to analyze how the participants co-constructed communicative projects in the mathematics classroom and how those projects were related to different kinds of contexts.

Concerning analytical approaches imported from other traditions it is clear that they need to be adjusted to the special demands of mathematics education (Bartolini-Bussi, 1998). This is also true for DA. As Lester and Lambdin (1998) stress "worthwhilness has to do with the potential of a research study for adding to and deepening our understanding of issues associated with mathematics teaching and learning" (p. 420). Since DA includes neither a theory of learning in general nor a theory of mathematical teaching and learning, DA needs to be supplemented with theories of mathematics teaching and learning in order to be useful for researchers in mathematics education. In (Ryve, 2007), Pólya (1945), Schoenfeld (1992), and Wyndhamn, Riesbeck, and Schoultz (2000) are examples of frameworks which all are especially developed for

analyzing and understanding mathematical problem solving. In different articles in mathematics education using DA examples could be found of how other theories of mathematical teaching and learning are used in a combination with DA (see, e.g., Bjuland, 2002, 2004, 2007; Carlsen, 2005; Cestari, 1997; Cestari, Daland, Eriksen & Jaworski, 2005). To conclude, DA needs to be combined with frameworks accounting specifically for mathematical aspects of the classroom interaction.

Another aspect that is missing from DA is a construct taking account of individual's contributions and interpretations of interactions. The reason for that such a construct is missing seems natural in relation to Linell's (1998) interest in studying in which ways interactions are co-constructed. Yet, Linell keeps coming back to the discussion that individual's projects and intentions influence the interaction and he states "how an individual utterance [...] is understood is dependent on how the overall activity is implicitly defined, and sometimes parties to an interaction may have discrepant views on this" (p. 256). Since mathematics education research is not only interested in interactions per se but also how interactions are related to collective as well as individual learning (cf. Lester & Lambdin, 1998) it may be argued that DA needs to be complemented with theories capable of capturing such aspects. In Ryve (2007) individual's different interpretations or definitions of the overall activity turned out to be very consequential for the whole-class discussions. For instance, TE1's and PT initial different framings or contextualizations of doing something with $\sqrt{98}$ are consequential for the discussion above. Therefore it was helpful to introduce the theory of contextualization (e.g. Halldén, 1999; Nilsson, 2006; Scheja, 2002) to account for such individual differences. It is important to notice that while individual's contextualizations affect the whole-class discussions, the whole-class discussions affect individuals' contextualizations, hence they are dialogically connected.

As introduced above Linell (1998) presents a theory of contexts. An explicit discussion of contexts could facilitate a carefully conceptualized, designed, and reported research study (cf. Lester & Lambdin, 1998). However, the framework of Linell is not straightforward to use. Linell indicates that this framework should be seen as preliminary and it seems reasonable that the framework needs some adjustments and specifications in relation to studies in mathematics education. That is, contextual resources could serve as a framework for facilitating the conceptualization of mathematical classroom interactions but the researcher must reflect on which aspects of the framework of conceptual resources are relevant, and to what extent it is possible to say something sensible about those contextual resources without carrying out some a priori investigations of those.

To conclude, DA provides researchers with analytical constructs to conceptualize classroom discourse as well as to relate those conceptualizations to contexts, both features stressed as important in mathematics education research (see, e.g., Morgan, 2006). Analytical constructs and the framework of contextual resources also facilitate the process of actually using the theory for analyzing data, which seems to be rare in mathematics education research (Niss, 2007a, 2007b). While DA was useful in the above mentioned aspects to analyze the empirical material connected to Ryve (2007) an analytical construct for taking account of individual's contextualizations of the classroom discourse were missing. In addition, DA does not guide the researcher to specific mathematical aspects of the discourse. Therefore, theoretical support for addressing aspects needs to be included in expending DAs analytical approach for the purpose of better understanding mathematical classroom discourse.

Concluding comments

By relating DA to the criteria for research it has been shown that DA could facilitate the process of fulfilling several of the criteria. However, as described above, DA is not developed for analyzing any particular communicative activity but instead a general conceptualization of face-to-face interactions. Although Ryve (2007) and other Nordic researchers (see, e.g., Bjuland, 2002, 2004, 2007; Carlsen, 2005; Cestari, 1997; Cestari, Daland, Eriksen & Jaworski, 2005) have elaborated on several aspects of how to complement DA, I regarded it as productive to continue the discussion since there are several questions that have not yet been fully answered.

Which kinds of theories of mathematical teaching and learning are suitable to combine DA with? As mentioned above, Ryve (2007) uses Pólya (1945), Schoenfeld (1992), and Wyndhamn et al. (2000) to account for aspects of mathematical problem solving. Bjuland (2007) also uses the work of Pólya and Schoenfeld as well as Sfard and Kieran (2001) to stress problem solving and aspects of mathematically productive discussions. Other examples could be found in the work of Carlsen and Cestari. Hence, there are examples of how to combine DA with frameworks accounting for mathematical aspects of the interaction. However, should it be complemented by other kinds of frameworks or theories to be able to account for aspects of mathematical *learning*, rather than just mathematical *interactions*? Should it be complemented by a theory of learning from the participation metaphor or is it more reasonable to combine it with a theory of learning situated in an acquisition metaphor (cf. Sfard, 1998)?

From my perspective, DA will play different roles in the research process depending on which metaphor is used for conceptualizing learning. If DA is combined with a theory of learning in harmony with the acquisition metaphor, DA will be useful for analyzing *conditions* for learning, such as classroom interactions and communicative genres. On the other hand, if DA is combined with a theory of learning in harmony with the participation metaphor, Linell (1998) could possibly serve as an operationalized approach for analyzing how individuals become socialized into practices. Researchers using learning theories in harmony with the participation metaphor, such as situated cognition (cf., Watson & Winbourne, 2008), are on their way of developing "finer tools which still recognize the immense power of the social and cultural contexts of learning but also express differences between learners and difference in the nature of mathematical participation" (p. 6). DA could serve as a tool for accomplishing this endeavor but, as stressed above, further examinations and theoretical developments are needed.

Further, how should the framework of contextual resources and contexts be used by researchers interested in examining mathematical teaching and learning? As discussed above, to what extent are a priori analyses of contextual resources necessary? Are some contextual resources less interesting for mathematics education researchers? I propose two different ways of including mathematical aspects within the framework of contextual resources. First, it is possible to stress mathematical aspects of the contextual resources? For example and in relation to the transcript, when considering the contextual resource of communicative genre, the analyst could stress aspects that are of mathematical nature such as ways of handling rounded decimals within mathematical solutions or in the case of actors' knowledge and beliefs about communicative project (cf. contextual resource 4) aspects such as the prospective teachers' beliefs about mathematical problem solving. Second, the framework could be complemented by a permanent contextual resource that accounts for mathematical aspects of the interaction. Personally I prefer the first alternative since it seems that mathematical aspects are inherent aspects of many contextual resources, as implicitly shown in the introduction of the framework.

Independent of whether the first or the second alternative is used, I propose that some kinds of a priori analysis of contextual resources is necessary if the analyst should be able to say something substantiated about them. It is not possible to totally specify what is required in each and every study, but since Linell (1998) states that every theory of discourse needs a theory of contexts, I regard it as important for researchers using DA to specify some guidelines for how to handle the

examinations of contextual resources. One suggestion may be that researchers make explicit in which ways they handle each contextual resource. Thus, specifying the contextual resources of relevance for the study, how the researcher goes about examining them, and how the relevant contextual resources come into play in the analysis of the unit of analysis are important elements to be made explicit.

On a more general level and in relation to Niss' (2007a, 2007b) discussion of theories and theory use in mathematics education, it becomes interesting to discuss what it means to *use* DA. For instance, is it necessary to use the analytical constructs or methods of analysis from DA or could the analyst just refer to the three basic principles of DA? Personally, I regard it as important to use analytical constructs that are derived from the principles to analyze data since it helps the analyst to conceptualize interactions in line with the basic principles of DA and also avoiding common-sense analysis. This does not, however, imply that data are squeezed into pre-given categories. For instance, the construct of communicative project does not direct the researcher to specific kinds of projects that are co-constructed but instead direct attention to the fact that interactions should be seen as sequential, co-constructed, and act-activity interdependent.

Finally, if this article is seen as a turn in the research interaction I hope that other researchers contribute to the co-constructing the communicative project of making explicit the usefulness of DA for mathematics education researchers.

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Notes

1 The concepts of interaction and discourse are used interchangeably which is in harmony with Linell (1998).

2 The abbreviation DA is often used for denoting Discourse Analysis, a term used to refer to several approaches focused on talk and texts. The dialogical approach could certainly be included as such an approach but in this article DA is used as an abbreviation for the approach itself.

3 That is, the beginning and the end of a turn or a topical episode could relatively easy be detected in a transcript.

4 Communicative genres could therefore be seen as established ways of solving large communicative projects (cf. Linell, in press).

5 I know that Cobb and collaborators stress that their analytical approach could be used to analyze data and therefore should not solely be seen as a guide to design classroom practices. Nevertheless, I thought it was hard to use it on my data.

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Sammanfattning

Den dialogiska approachen har introducerats för att studera matematiska klassrumssamtal i ett växande antal publikationer genomförda och skrivna av nordiska forskare. Eftersom den dialogiska approachen är utvecklad för att studera mänskligt handlande, kommunikation och kognition i allmänhet är det viktigt att explicit diskutera hur den kan utvecklas och kompletteras för att uppfylla de ändamål som matematikdidaktisk forskning kräver. I denna artikel initierar jag en sådan diskussion med utgångspunkt i teoretiska analyser och empiriska exempel från min egen forskning. Genom att relatera den till ett ramverk för kvalité inom matematikdidaktisk forskning visas att den dialogiska approachen är ett verktyg som kan användas för att uppfylla många av dessa kriterier. Artikeln avslutas med förslag på aspekter som behöver diskuteras och utvecklas för att göra den dialogiska approachen ännu mer användbar för att förstå lärande och undervisning inom matematik.