Effective mathematical communication in play-based activities: a case study of a Norwegian preschool

BEATE NERGÅRD AND KJERSTI WÆGE

This article focuses on effective mathematical communication in preschool. Based on a qualitative case study of a Norwegian preschool, we explore how visual mediators, gestures and mathematical concepts support effective mathematical communication in play-based activities. The article shows how these modes, and the links between them, were crucial for establishing effective communication. Visual mediators, gestures and mathematical concepts functioned as means for making the focal projects and the contexts explicit in the conversations, and thus for communicating effectively.

The role of discussions and interactions in mathematical education has attracted much attention in mathematical education research (e.g. Cobb, 1994; Sfard, 2008). Research on mathematical communication in the context of preschool has focused on the teacher's role in the mathematical discourse, and how the teacher can support children's mathematical learning in adult-initiated activities (e.g. Carlsen et al., 2016; Fosse, 2016; Saebbe & Mosvold, 2016). Studies that explore children's communication in play have focused on how children use gestures and visual mediators when they communicate mathematically (e.g. Johansson et al., 2014; Sumpter & Hedefalk, 2015). The findings show that gestures and visual mediators are an important part of children's mathematical communication. Trawick-Smith et al. (2016) argue that more research is needed to explore communication, interactions and children's mathematical learning in play.

The literature points out the importance of mathematical communication for children's learning of mathematics (e.g. Sfard, 2001; Sfard & Lavie, 2005). However, simply communicating about and discussing

Beate Nergård, *Queen Maud University College*
Kjersti Wæge, *Norwegian University of Science and Technology*

Nergård, B. & Wæge, K. (2021). Effective mathematical communication in play-based activities: a case study of a Norwegian preschool. *Nordic Studies in Mathematics Education*, 26 (2), 47–66.

mathematical ideas is no guarantee of meaningful learning. Sfard and Kieran (Kieran, 2001; Sfard, 2001; Sfard & Kieran, 2001) argue that communication must be effective for children to learn mathematics. They developed a framework and explored effective mathematical communication in student (13 years old) group work. Their findings show that the students' communication was not always effective and thus did not support learning. Ryve, Nilsson and colleagues (Nilsson & Ryve, 2010; Ryve et al., 2013) developed an analytical framework and studied effective mathematical communication in student group work (12–13 years). Nilsson and Ryve (2010) showed how the constructs of focal projects and contextualisation helped in organising and analysing effective communication, and their findings revealed the importance of communicating with compatible focal projects. Ryve et al. (2013) found that the link between visual mediators and technical terms is crucial in students' attempts to communicate effectively. As more research is needed on effective communication (Ryve et al., 2013; Sfard, 2001), the aim of this study is to provide new insight into effective communication in preschool. The following research question is addressed: How can visual mediators, gestures and mathematical concepts support effective mathematical communication in play-based activities in preschool? This article draws on a qualitative case study of a Norwegian preschool. We answer the research question by analysing video recordings of play-based activities.

Research on mathematical communication in preschool

Some studies have investigated how mathematical conversations provide learning opportunities for children. Clements and Sarama (2007) explored mathematical conversations in play. They showed how conversations provided opportunities for children to use mathematical language and mathematise the content of their play. Björklund et al. (2018) examined the teacher's involvement in children's mathematising within play. They identified four lines of teacher actions that provided opportunities for children's learning: confirming direction of interest; providing strategies; situating known concepts; and challenging concept meaning. Other studies have examined how the teacher may engage the children in mathematical conversations and how conversations may support children's mathematical development (e.g. Carlsen, 2013; Doverborg & Samuelsson, 2000; Hundeland et al., 2014). Carlsen (2013) showed how the use of questioning and such tools as voice, facial expressions and concrete materials characterised the teacher's orchestrating of a mathematical activity. While most research has focused on the teacher's role in mathematical conversations, some research has studied the use of different modes,

such as gestures and concrete objects, in children's communication (Flottorp, 2010; Johansson et al., 2014; Sumpter & Hedefalk, 2015). Sumpter and Hedefalk (2015) found that the children used a variety of products, such as concrete materials, fingers and mathematical procedures in their mathematical argumentation. Similarly, a study by Johansson et al. (2014) illustrated how the relationship between verbal language, gestures and concrete objects could be viewed in relation to young children's explanations. Other studies show that children often use mathematical concepts to describe and explore ideas and mathematical relationships (Björklund, 2008; Ginsburg & Seo, 2004), for example those related to quantities, shapes and sizes. In this article we focus on how the use of visual mediators, gestures and mathematical concepts helps the teachers and the children to establish common FPs and contexts, and thus allow them to communicate effectively about mathematical content.

Communication and effective communication

Our study is informed by sociocultural views on communication as a cultural and historical activity, where interactions are characterised by the use of tools and especially by the use of language (Vygotsky, 1978). We consider communication in preschool to be a social and situated practice through which adults and children develop explanations and provide justifications to negotiate shared understanding on the same object.

Sfard (2001) describes communication as "an attempt to make other people act or feel according to one's intentions" (p. 38). In the ongoing communication, a sender tries to get a receiver to recognise a thought or an action. If the receiver's response is in accordance with the sender's expectations, the purpose of the communication will be fulfilled. The receiver is not passive, and from his or her point of view, communication means trying to make sense of and provide feedback on the speaker's message. Sfard and Kieran (2001) define communication as effective if "it fulfills its communicative purpose, that is, the different utterances of the interlocutors evoke responses that are in tune with the speakers' meta-discursive expectations" (p. 49). Meta-discursive expectations are understood as an indication that the response must contain expected content rather than specific content (Sfard & Kieran, 2001).

Contextualisation

We use Nilsson and Ryve's framework of contextualisation (2010) to conceptualise the meaning of meta-discursive expectations and account for effective communication in play. We explore the participants' individual

focal projects (FPs) and the way they are contextualised. An individual's FP refers to the problem or the project he or she engages in and interprets as his or her task to solve (Ryve et al., 2013). The participants' contextualisation of the FP is related to how they interpret the different contextual elements. We distinguish between a conceptual, situational and cultural context as part of the current activity (Halldèn, 1999; Nilsson & Ryve, 2010). The conceptual context refers to the children's personal constructions of, here, mathematical concepts and their mathematical understanding. The situational context refers to which interpretations the children have in the interaction with their environments, including their interpretations of figurative material, possible actions and directly transferable experiences. The cultural context refers to the children's interpretations of the norms in the discursive practice (Halldèn, 1999).

An FP can be handled in different ways, and how the participants deal with the FP depends on how they contextualise the project (Nilsson & Ryve, 2010). An example from the current study shows how the participants fail to communicate effectively as they develop different contextualisations of FPs in an activity involving weight while playing with a scale and concrete objects. The teacher engages in a FP about what is "heaviest" and locates this FP within the context of determining which of the two sides of the scale is heaviest. One girl (Siri) locates the FP to decide which is the heaviest in the context of the heaviest object (a box of yeast sachets). The teacher and Siri have established a common FP, but the contextualisation of the FP differs. This type of discrepancy makes it hard for them to respond within the frame of each other's meta-discursive expectations.

Nilsson and Ryve (2010) show how the participant's contextualisations produce FPs and how these are then contextualised into new FPs. These chains of contextualisations and FPs can be studied to examine how the teacher and the children relate to each other's meta-discursive expectations, thus examining whether the participants communicate effectively.

Visual mediators

Sfard (2008) considers the realisation of objects as visual mediators. Visual mediators, such as images and blocks, are considered to be an essential part of children's communication. Sfard (2008) argues that the children's interaction and communication with and about these objects can help them to identify the object they are talking about and coordinate their mathematical communication. Sfard and Lavie (2005) point out that children often use concrete objects to play with, as in this study with the weight and yeast box, and such play-material mediates colloquial

discourses between them. The physical manipulation of the visual mediators can be seen as an embodiment of the verbal response as well as a nonverbal response to mathematical problems (Sfard & Lavie, 2005).

Mathematical concepts

Lithner (2008) emphasises the importance of establishing how children's reasoning can be viewed as mathematical, pointing out that it must be anchored in relevant mathematical properties of the components one is talking about. These components are objects, transformations and concepts. Objects are the fundamental entities, "the thing that one is doing something with" (Lithner, 2008, p. 261), a transformation is the process that is done to the object and concepts are central mathematical ideas based on the objects, the transformations and their properties (Lithner, 2008). Drawing on Lithner (2008), we consider mathematical concepts as being anchored in relevant mathematical properties of the FP and the visual mediator that the children are reasoning about, for example concepts such as "weighs" and "heaviest".

Gestures

Sfard's (2008) definition of communication includes verbal talk, gestures and body language. Sfard (2009) also claims that gestures are invaluable means for ensuring that the participants in a mathematical conversation are talking about the same mathematical object. Using gestures to explain one's solution is an effective way to help the participants interpret mathematical problems in the same way. According to Radford (2002), children's use of gestures plays a significant role in mediating their mathematical thought. In the current study we use a multimodal approach to take actions into account, such as pointing and other gestures.

Play-based activities

Play is at the core of early childhood education (Singer, 2013), and has an important role in the Norwegian preschool curriculum. It is an inherent social activity that underpins mathematical thinking as children explain, explore and undertake activities that include mathematical experiences (Ginsburg, 2006). A play-based approach embodies a sociocultural view on play, and in play-based activities, children are free to choose from a range of play stations and materials (Walsh et al., 2006). The teacher's role is to follow the child's lead and guide without disturbing the play, more or less extending the mathematical content within the play in different

ways (Björklund, et al., 2018). The child may perform an action, word or sentence, understood as mathematical by the adult, unintentionally. However, according to van Oers (2010), the children's actions only gain mathematical meaning or experience when the adult reacts in a mathematical way. In this study we examine effective communication in the context of play-based activities.

Setting and participants

Our study has been conducted in a privately operated preschool in Norway. The preschool is special as it focuses on natural-science activities, and in the course of the year, five-year-olds from four different preschools visit it, one week at a time every five weeks. We studied one group of children from one of the four preschools over one year. The research was carried out when these children were present in the preschool, a period of seven weeks spread over one year. The participants in this study were 25 five-year-old children, and adult staff; three teachers and two teaching assistants.

The Norwegian preschool context follows a sociocultural tradition (OECD, 2006) where mathematics normally is taught through everyday activities, adult-initiated activities and play situations. In the preschool we studied, play-based activities consisted of an adult-structured play environment where the children had control over what and how they wanted to play with the available play material, and the teacher interacted, as appropriate, by letting the children explore. The adults arranged such play environments every day, where the children had the opportunity to explore mathematics within the play.

Method and analysis

The aim of this study is to explore effective communication in play-based activities in one preschool. It can be characterised as a qualitative case study as these studies are defined as being bound by time and place (Creswell & Poth, 2016), such as in this case, where we studied a preschool for a period of seven weeks spread over one year, focusing on mathematical communication and language. The empirical material used in this study, a subset of our data involving all our video recordings of play-based activities, comprises 13 video recordings of play-based activities, each lasting between three and 21 minutes. The data collection was conducted by the first author. A Q4 Handy Video Recorder was used to record the play-based activities. It was held close to the activity and the participants

so that use of the play-material and the participants' facial expressions and body language could be read clearly.

Nvivo data analysis software was used to analyse the video recordings. We explored the interactions between the participants and examined their utterances, gestures and use of visual mediators in their ongoing communication. It was impossible to have direct access to the participants' thinking. However, the video recording of the activity enabled us to observe their behaviour, gestures, articulated expressions and use of visual mediators.

We drew on Ryve and Nilsson's (2010, 2013) analytical framework to analyse the conversations. We were interested in the participants' FPs and how they contextualised them, and examined whether their FPs were compatible, that is, we analysed which problems the participants were engaged in and tried to solve, and which personal or mental context the individual was operating in when dealing with the problem. Moreover, we used a multimodal approach that covered a broader range of modes than spoken language, such as manipulation of visual mediators and use of gestures. We were interested in how the use of visual mediators, gestures and mathematical concepts affected the participants' communication. We wrote analytical memos to analyse and connect the data and then selected representative examples of the understanding that we as researchers had developed to present our findings.

The first exchange, lasting 13 minutes, is quite long. Siri (S) and Anna (A) are sitting at a table and playing with various objects. A scale is standing on the table. The teacher (T) is sitting next to Siri. The activity is spontaneous and initiated by Siri, and in their collaboration, both the girls and the teacher are active, with abundant use of visual mediators and variations of mathematical talk during the activity. This is a typical play-based activity in this particular preschool. The presentation will follow a chronological order to highlight sequences according to the nature of the interactions and particularly to changes in effectiveness in the participants' communication.

Findings

Siri has placed two large plastic teddy bears and a small teddy bear on the left side of the scale, and on the right, she has placed a small tin and a box of raisins. Anna, sitting next to Siri, is building a tower on the table using various gram weights. Anna follows the conversation and activity between Siri and the teacher, and often stops to look at what they are doing (excerpt 1).

Excerpt 1

Who	Said	Done	Scale's position
1. S:	Oh! This big one here weighs a lot!	Puts a box (of yeast sachets) on the left side of the scale	
2. T:	How much does it weigh then, hmm? Do they weigh the same?		
3. S:		Removes objects on the right side, looks at T and removes all the objects except the box	
4. T:	Hm!		
5. S:		Smiles, looks at both sides, places the small sandwich spread tin on the left, but moves it to the right and puts everything (small sandwich spread tin, raisin box, three teddy bears) on the right except a single sachet of yeast which is lying on the left side together with the box	
6. T:	Which one weighs the most now, then?		
7. S:		Points from side to side and smiles at the teacher	
8. T:	Which one is the heaviest?		
9. S:		Picks up the box and shakes it, and puts it back on the left side	

In [1] Siri has observed a big dip on the scale when she puts the box on it. She says "Weighs a lot" to explain her observation. T's [2] question is: "How much does it weigh then? Do they weigh the same?" This functions as a starting point for establishing an FP to determine how much the box weighs in the context of the scale, the objects and balancing the scale. She uses the same concepts as Siri does, "weighs" and "much", when she formulates the question, exploiting the opportunity to invite Siri into a mathematical discourse about weight. The choice of mathematical concepts is important for establishing a mathematical conversation where the participants have a common FP by referencing the same visual mediators. The use of the same mathematical concepts and the common establishment of an FP in the same context prove to be important for the effective development of the conversation.

Siri establishes a compatible FP in [3]–[5] to determine how much the box weighs in the context of the scale, the objects and weighing the same. This context leads to a new local FP about finding what she

has to place on the right side of the scale to weigh the same as the box (and the one sachet) on the left side of the scale. When Siri handles the objects, they help her to explore the FP. Siri's physical manipulation of the visual mediators, the scale and the objects is a nonverbal response to the mathematical FP. This example shows how the combination of the visual mediators, the scale and the objects, and the use of mathematical concepts ("weighs", "much", "the same") establish a common FP and a common context between Siri and T. The communication between them is effective in this sequence of the conversation.

When Siri [5] stops after placing objects on the right side of the scale, it is still unbalanced. The left side (with the box) is still heavier. In [6] there is a shift in the communication. T asks Siri "Which one weighs the most now, then?" T introduces a new FP about what weighs most in the context of determining which of the two sides is heaviest. Siri [7] may be uncertain because of T's shift in the FP, and she responds by pointing from the scale's left to right side. This may be interpreted as Siri giving a response to the FP she is personally interested in, which is to determine how much the box weights in the context of the scale, the objects and weighing the same. Siri has established that the small tin, the box of raisins and the three teddy bears weigh almost as much as the box. She uses gestures (pointing) to explain what she has found, and the scale functions as a visual mediator supporting her response. T [8] asks her question again by switching mathematical concepts from "weighs the most" to "heaviest". The change of concepts causes Siri [9] to change her response. She gives a nonverbal answer to the question by lifting the box from the scale. She changes her FP [9] to decide which is heaviest, and she localises the FP in the context of the heaviest object. T and Siri have now established a common FP, related to determining what is heaviest, but they localise it in different contexts. T is not clear when it comes to how she localises the FP and which context she refers to when she uses the word "which". T's change in the use of concepts from "weighs most" to "heaviest" and her reference to the context by using "which" contribute to Siri being unable to follow T's thoughts. None of the objects, the context nor T's use of concepts support the communication between them.

When T changes her question in [6] – [9], she changes her FP, and when she is also unclear about the FP context, it adversely affects the communication; it goes from being effective to being ineffective. Siri wants to talk about her strategy by using gestures, but T does not act on this nonverbal response so the communication between them is not effective.

A few moments later the second instance occurs (excerpt 2), where Siri has a dialogue with herself and we can see how her mathematical concepts support her thinking.

Excerpt 2

Who	Said	Done	Scale's position
18. T:	But it was quite heavy, wasn't it? Since there was room for –		
19. S:	It's really that –	Looks at the scale	
20. S:		Moves everything (small sandwich spread tin, raisin box, three teddy bears) from the right side of the scale to the left side. The box is not on the scale	
21. S:	What do these things here weigh, then?		
22. S:		Puts the box on the right side of the scale	
23. S:	Look here! This weighs the most.	Points to the box	
24. T:	Yes, that weighs the most. But that's a bit strange. Because there are – How many are there? There's only one thing, and there, there are one, two, three, four, five things, and they weigh less than the one thing. That's really strange.	Points to the box before pointing at and counting the objects on the left	
25. S:		Looks back and forth from one side of the scale to the other, and moves a teddy bear to the right and back again. Then she places the box on the left side and the small sandwich spread tin, raisin box and three teddy bears on the right	
26. S:	What's happening?	Laughs	
27. T:	Yes, what's happening?		

T invites Siri back into the mathematical discourse by saying [18] "But it was quite heavy, wasn't it?" using mathematical concepts dealing with the earlier shared FP. When T refers to the box and how heavy it is, she leads the communication back to an FP about weight in the context of the scale, the objects and heaviness, aiming her question at Siri's local FP (the heaviness of the box) from earlier in the activity. T's use of concepts, localisation of the FP and the context establishes a more effective communication.

In [19] – [23] Siri has a conversation with herself, and the objects are used as a nonverbal response to the mathematical problem she is trying to solve. After the observation in [19], she starts to handle the objects in [20]. Her utterance [21] indicates that she has established an individual FP about the weight of the objects. When T does not respond to Siri's question [21], she uses the box in her further exploration by placing it on the right side [22]. This action is a physical solution to her individual FP. When she then in [22] puts the box on the right side, saying in [23] "Look here! This weighs the most", she is again back in the FP about the heaviness of the box, but the context has changed to being about the objects on each side and the scale's position. When Siri in [23] points to the box and says, "weighs the most", she shows that she localises her FP explicitly on the box and its heaviness.

In [24] T supports Siri's solution, responding by repeating what Siri found out about the weight of the box and using the same concept, "it weighs the most". However, in the same utterance she introduces a new FP about how several objects can weigh less than one object, "... five things, and they weigh less than the one thing". T points to the concrete objects while using numerals to indicate the contextualisation of her FP. The combination of pointing to the objects while counting them helps T to make the localisation of the FP explicit. Even though T uses gestures to support the verbal language in her question and is specific about which context she is operating in, Siri appears uncertain. She reacts by moving the objects back as they were earlier, the box on the left side and the other objects on the right side. This may be interpreted as her attempting to follow T's thinking, but when she observes that after switching sides the position of the scale also changes, she is puzzled. The sequence reveals the importance of gestures and pointing in establishing a common FP, hence the importance for establishing effective communication.

When Anna enters in the next sequence (excerpt 3), her visual mediators combined with verbal language create effective communication between her and T. In [28] T introduces an FP about weighing the same, which is localised in the context of what can be done so the two sides of the scale will weigh the same. Her question establishes this as a shared context with Siri to determine when something weighs the same, and the scale and the objects function as visual mediators to achieve this. T points to both sides of the scale, using such concepts as "weigh the same" to indicate the contextualisation. The combination of using visual mediators, mathematical concepts and gestures helps T to make the localisation of the FP explicit. Siri looks at both sides of the scale and displays uncertainty by biting her finger. Anna enters the conversation by answering T's question. Anna's answer [30] indicates that she is focusing on the same

Excerpt 3

Who	Said	Done	Scale's position
28. T:	Really - But where do you need to have...? If they are to weigh the same, what do you need to do then?	Points to both sides	
29. S:	Hm	Bites her finger and looks back and forth between both sides of the scale	
30. A:	Put on more things.	Looks at T	
31. T:	Yes! But where do we need to put those things?		
32. S:		Puts a plastic syringe on the right side	
33. T:	Yes, that was a syringe. Now let's look.		
34. A:		Puts a tower of gram weights on the right side	
35. S:	No, Anna, then it will be the most.	Removes Anna's tower, watches the scale dipping	
36. T:	OK! Now it's beginning to stabilise itself		The scale is dipping
37. A:	But this weighs only one.	Holds up a weight of one gram	
38. T:	Does it weigh that? Does it say so? One. Yes, it weighs one gram. Right.		
39. A:	It weighs only one		
40. S:		Watching the scale	
41. S:	Come on!		The scale is dipping
42. T:	OK; one gram. What about that green one there, then, how much does it weigh?	Points to the weight indicating 10 g	
43. S:		Puts a stethoscope on the scale which spans both sides	
44. S:	But what does this long thing here mean, then?	Pointing to the stethoscope	Left
45. T:	Yes, stethoscope on.		
46. S:	Ha! This one won here! It won!	Looks at the scale with wondering eyes, puts her hand on the box on the left side and presses the scale down	
47. T:	Did it win?		

FP and context as T, and her response leads T to introduce a new, local FP to determine where the objects should be placed on the scale. Siri's and Anna's responses in [32] and [34] indicate that they are engaged in this local FP. Siri's actions [32] and Anna's actions [34] are both nonverbal responses to their common FP, where they both put more objects on the right side so they will weigh the same. T's, Siri's and Anna's FPs and contexts harmonise, and in this case, the communication is effective. This example shows how the communication is effective because T is clear in the contextualisation by pointing. When the other participants also focus on the same FP and give nonverbal responses, which coincide with T's explicit FP, a more effective communication is established.

When Siri in [35] removes Anna's objects from the scale, a marked change takes place in the communication and collaboration between the girls. Anna changes the focus to an FP about how much one of the gram weights weighs. She holds a weight up to indicate the contextualisation of her new FP. This weight functions as a visual mediator that helps Anna to make the localisation of the context explicit. T engages in this FP, and she localises it in the context of how much the weights weigh in grams (a number indicating measure). The dialogue between Anna and T from [37] to [42] is effective between them, as they are both in the same FP and they localise the FP within the same visual mediators (the gram weights). They are also clear in their verbal language by using the same mathematical concept, "weighs", and they use gestures clearly. The clarity of the utterances and gestures enables Anna and T to establish an effective communication.

While T and Anna have established an effective communication, Siri has a dialogue with herself. She continues to focus on the FP about weighing the same, watching while the scale stabilises. When the scale is almost in balance, Siri [41] exclaims "Come on!", and her action in [43] where she puts the stethoscope on both sides of the scale might suggest that she sees that it is almost in balance and tries to distribute the weight of the stethoscope to create balance. Siri's verbal and nonverbal communication in [40] and [42] focuses more on herself than the others. She "talks" to the scale and uses the objects in a dialogue with herself. When Siri's actions [43] do not solve her FP, she asks [44] "But what does this long thing here mean?", and uses her fingers to point to the stethoscope to indicate the contextualisation of the FP which is about what this object means for weighing the same. Siri's handling of the objects supports her exploration of the mathematical problem.

T's answer [45], "Yes, stethoscope on", may indicate that she responds to what the name of the objects is, but not to Siri's FP. T has been busy communicating with Anna about their shared FP, and the communication

with Siri is not effective. The sequence shows that T's response affects Siri's dialogue with herself in the sense that T does not notice the strategy Siri uses by adding new objects to solve her FP about weighing the same. Siri then puts her hand on the box saying [46] "Ha! This one won here! It won!" This indicates that she has given up on her attempt to find

Excerpt 4

Who	Said	Done	Plastic bears
1. B:	I just want yellow small bears.	Points at the row he has made	Four small, two on each side of a big one in the middle
2. B:	One, two – one, two Two on each	Points at one and one bear from the right, hops over the big one in the middle	
3. T:	Yeah, two on each side		
4. B:	One more on each	Puts a little bear on each side	
5. T:	How many do you have then?		
6. B:	One, two, three	Points to one and one bear on the left side of the row and looks at the adult	
7. T:	Yeah, three on each side. How many do you have all together then?		
8. B:	Three?	Looks at the adult	
9. T:	Together, in the whole row?		
10. B:		Smiles, looks at the row. Points at the three outermost bears on each side	
11. B:	Three?	Looks at the teacher questionably	
12. T:	Yeah, you've got three on each side. But what if we want to find out how many you have in a whole row? Shall we count together?	Draws hand back and forth over the row of bears	
13. B:	You can count there and I can count here	Points to the side that he wants the adult to count, the right side	
14. B og T:		Point at and count the bears on each their side. They both hop over the big one in the middle	
15. T:	One, two, three, four, five, six	Counts out loud, pointing	
16. B:	Yeah, I counted six too, and one in the middle	Looks at the teacher and smiles	

an answer to the FP in terms of weighing the same, and she again has her focus on her original FP, that the box is the heaviest of the objects.

In the second exchange, a boy and a teacher are sitting at a table playing with plastic bears in multiple colors and two sizes. The boy has made a row of five yellow bears (excerpt 4). In [1] and [2] Brian says that he wants two yellow bears on each side. T responds by repeating the boy's claim [3]. This functions as a starting point for establishing an FP to determine the number of bears on each side. In [7] there is a shift in the communication. T introduces a new FP about the total number of bears. Brian [8] may be uncertain because of T's shift in the FP, and responds by giving the answer to the original FP, about the number on each side [8, 11]. He uses pointing to support his answer [10]. T [12] asks her question again and uses gestures to support it. This causes Brian [13] to localise his FP [13] in the context of the total number of bears. Brian starts counting by pointing to bears [14], and in [15] they count aloud together pointing to the bears. T and Brian have now established a common FP and context. The sequence illustrates the importance of gestures and pointing in establishing a common FP and context.

Discussion

In this article, we have explored how visual mediators, gestures and mathematical concepts supported the participants in establishing compatible FPs and contexts, thereby allowing them to communicate effectively. As in other studies of communication in preschool (e.g. Sumpter & Hedefalk, 2015; Carlsen, 2013), our findings show that the teacher and children used a broad range of modes, such as speech, pointing and moving concrete objects, in their communication. Moreover, we found that these modes were crucial for establishing effective communication (Sfard, 2009; Ryve et al., 2013). Visual mediators, gestures and mathematical concepts functioned as a means for making the FPs and contexts explicit in the conversations (Sfard, 2008).

Visual mediators

Similar to the studies by Johansson et al. (2014) and Sumpter & Hedefalk (2015), our study shows that visual mediators are an important part of the children's communication. Our findings reveal that the material or objects the children played with, and speech and gestures, were important for making the FPs and contexts explicit, and thus for establishing effective communication (Ryve et al., 2013). Moving, lifting or pointing to concrete objects, such as the yeast box, helped the participants to

establish a common FP and context, both when communicating verbally and nonverbally.

Mathematical conversations in preschool are not just about establishing effective communication but also about engaging in productive communication where the children engage in multiple forms of mathematical representations and reasoning (e.g. van Oers, 2010). Similar to Ryve et al.'s study (2013) of student group work, we found that there were instances in which the children focused on specific visual mediators, such as the yeast box, and were not interested in engaging in other representations suggested by the other participants. According to Ryve et al. (2013), there might be a conflict between establishing effective communication by means of a specific visual mediator and the children's motivation to engage in other kinds of representations. Doverborg and Samuelsson (2000) suggest that teachers should use the children's interest, engagement and curiosity as a starting point for mathematical conversations.

Gestures

Carlsen (2013) emphasises the importance of combining speech, gesture and demonstrative actions to establish intersubjectivity and develop shared meanings. A study by Johansson et al. (2014) shows how children use gestures, such as pointing, and actions with concrete objects in their mathematical explanations. Our study shows that gestures, in combination with visual mediators, were important for establishing effective communication, and reveals that both the teachers and the children used pointing, in combination with speech and visual mediators, in their questions and explanations to make explicit the location of the FPs and contexts. Moreover, the children used pointing as a nonverbal response to specify the FP and how it was contextualised. The participants pointed to the concrete objects to explain their thinking, making the localisation of the FP and context explicit, where the play material functioned as visual mediators supporting their response and thus the establishment of effective communication.

Mathematical concepts

Similar to the studies by Björklund (2008) and Ginsburg (2006), our study shows that the children often used mathematical concepts to describe and explore ideas and mathematical relationships. The participants' use of mathematical concepts, in combination with visual mediators and pointing, was crucial for establishing and making explicit the FPs and contexts (Corneille, 1997).

According to Björklund et al. (2018), the teacher's role is to expand the children's encounters with mathematics, and to try to introduce mathematical concepts or operations to the children's play activities (van Oers, 2010). We found that the teacher introduced new FPs by asking questions or by introducing new mathematical concepts, often in combination with visual mediators and pointing. The questions and mathematical concepts functioned as starting points for establishing common FPs and contexts. However, we found that small changes in the teacher's use of mathematical concepts, for example from "weighs most" to "heaviest", might influence whether the teacher and the children manage to establish effective communication.

Conclusion

The focus of this article has been on how specific aspects of interaction influence effective communication. We have examined how visual mediators, gestures and mathematical concepts supported the participants in establishing compatible FPs and contexts. The findings show that using mathematical concepts, moving, lifting or pointing to concrete objects – and the links between them – were crucial for making explicit the location of FPs and context, and thus for establishing effective communication. The findings might help teachers to identify the children's FPs and contexts, which may in turn inform their communication and teaching. Several studies (Björklund et al., 2018; van Oers, 2010) discuss important aspects for creating productive communication. Our findings show how the analytical approach of Ryve et al. (2013) could be adapted to and implemented in the preschool context to explore effective communication. One limitation is that we only explored effective communication in play-based activities. It has also been beyond the scope of the study to investigate the relation between effective communication and children's learning. While this study provides insight into effective communication in play-based activities, more research is needed to explore the relationship between effective communication, productive communication and opportunities for learning.

References

Björklund, C. (2008). Toddlers' opportunities to learn mathematics. *International Journal of Early Childhood*, 40 (1), 81–95.

Björklund, C., Magnusson, M. & Palmér, H. (2018). Teachers' involvement in children's mathematizing – beyond dichotomization between play and teaching. *European Early Childhood Education Research Journal*, 26 (4), 469–480. doi: 10.1080/1350293X.2018.1487162

Carlsen, M. (2013). Engaging with mathematics in the kindergarten. Orchestrating a fairy tale through questioning and use of tools. *European Early Childhood Education Research Journal*, 21 (4), 502–513. doi: 10.1080/1350293X.2013.845439

Carlsen, M., Erfjord, I., Hundeland, P. S. & Monaghan, J. D. (2016). Kindergarten teachers' orchestration of mathematical activities afforded by technology: agency and mediation. *Educational Studies in Mathematics*, 93 (1), 1–17.

Clements, D. H. & Sarama, J. (2007). Effects of a preschool mathematics curriculum: summative research on the Building blocks project. *Journal for Research in Mathematics Education*, 38 (2), 136–163.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23 (7), 13–20. doi: 10.2307/1176934

Corneille, B. (1997). Early childhood corner: recognizing the mathematics. *Teaching Children Mathematics*, 4 (2), 112–115.

Creswell, J. W. & Poth, C. N. (2016). *Qualitative inquiry and research design: choosing among five approaches*. SAGE.

Doverborg, E. & Samuelsson, I. P. (2000). To develop young children's conception of numbers. *Early Child Development and Care*, 162 (1), 81–107. doi: 10.1080/0300443001620107

Flottorp, V. (2010). Hvordan kommer matematisk meningsskaping til syne i barns lek? En casestudie. *Nordisk Barnehageforskning*, 3 (3), 95–104.

Fosse, T. (2016). What characterises mathematical conversation in a Norwegian kindergarten? *Nordic Studies in Mathematics Education*, 21 (4), 135–153.

Ginsburg, H. & Seo, K. H. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D. H. Clements & J. Sarama (Eds.), *Engaging young children in mathematics: standards for early childhood mathematics education* (pp. 91–104). Routledge.

Ginsburg, H. P. (2006). Mathematical play and playful mathematics: a guide for early education. In R. M. Golinkoff, D. G. Singer & K. Hirsh-Pasek (Eds.), *Play = learning: how play motivates and enhances children's cognitive and social-emotional growth* (pp. 145–165). Oxford University Press.

Halldèn, O. (1999). Conceptual change and contextualization In W. Schnotz, M. Carretero & S. Vosniadou (Eds.), *New perspectives on conceptual change* (pp. 55–65). Elsevier.

Hundeland, P. S., Carlsen, M. & Erfjord, I. (2014). Children's engagement with mathematics in kindergarten mediated by the use of digital tools. In U. Kortenkamp, B. Brandt, C. Benz, G. Krummheuer, S. Ladel & R. Vogel (Eds.), *Early mathematics learning* (pp. 207–221). Springer.

Johansson, M., Lange, T., Meaney, T., Riesbeck, E. & Wernberg, A. (2014). Young children's multimodal mathematical explanations. *ZDM*, 46 (6), 895–909. doi: 10.1007/s11858-014-0614-y

Kieran, C. (2001). The mathematical discourse of 13-year-old partnered problem solving and its relation to the mathematics that emerges. *Educational Studies in Mathematics*, 46 (1), 187–228. doi: 10.1023/a:1014040725558

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, 67 (3), 255–276. doi: 10.1007/s10649-007-9104-2

Nilsson, P. & Ryve, A. (2010). Focal event, contextualization, and effective communication in the mathematics classroom. *Educational Studies in Mathematics*, 74, 241–258. doi: 10.1007/s10649-010-9236-7

OECD (2006). *Starting strong II: early childhood education and care*. OECD.

Oers, B. van (2010). Emergent mathematical thinking in the context of play. *Educational Studies in Mathematics*, 74 (1), 23–37.

Radford, L. (2002). The seen, the spoken and the written: a semiotic approach to the problem of objectification of mathematical knowledge. *For the Learning of Mathematics*, 22 (2), 14–23.

Ryve, A., Nilsson, P. & Pettersson, K. (2013). Analyzing effective communication in mathematics group work: the role of visual mediators and technical terms. *Educational Studies in Mathematics*, 82 (3), 497–514. doi: 10.1007/s10649-012-9442-6

Saebbe, P.-E. & Mosvold, R. (2016). Initiating a conceptualization of the professional work of teaching mathematics in kindergarten in terms of discourse. *Nordic Studies in Mathematics Education*, 21 (4), 79–93.

Sfard, A. (2001). There is more to discourse than meets the ears: looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics*, 46, 13–57. doi: 10.1023/A:1014097416157

Sfard, A. (2008). *Thinking as communicating. Human development, the growth of discourses and mathematizing*. Cambridge University Press.

Sfard, A. (2009). What's all the fuss about gestures? A commentary. *Educational Studies in Mathematics*, 70 (2), 191–200. doi: 10.1007/s10649-008-9161-1

Sfard, A. & Kieran, C. (2001). Cognition as communication: rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity*, 8 (1), 42–76.

Sfard, A. & Lavie, I. (2005). Why cannot children see as the same what grown-ups cannot see as different? – early numerical thinking revisited. *Cognition and Instruction*, 23 (2), 237–309. doi: 10.1207/s1532690xci2302_3

Singer, E. (2013). Play and playfulness, basic features of early childhood education. *European Early Childhood Education Research Journal*, 21 (2), 172–184. doi: 10.1080/1350293X.2013.789198

Sumpter, L. & Hedefalk, M. (2015). Preschool children's collective mathematical reasoning during free outdoor play. *The Journal of Mathematical Behavior*, 39, 1–10.

Trawick-Smith, J., Swaminathan, S. & Liu, X. (2016). The relationship of teacher–child play interactions to mathematics learning in preschool. *Early Child Development and Care*, 186 (5), 716–733. doi: 10.1080/03004430.2015.1054818

Vygotsky, L. S. (1978). *Mind in society : the development of higher psychological processes*. Harvard University Press.

Walsh, G., Sproule, L., McGuinness, C., Trew, K., Rafferty, H. & Sheehy, N. (2006). An appropriate curriculum for 4–5-year-old children in Northern Ireland: comparing play-based and formal approaches. *Early Years*, 26 (2), 201–221. doi: 10.1080/09575140600760003

Beate Nergård

Beate Nergård is a PhD candidate at Department of Teacher Education, Faculty of Social and Educational Sciences, NTNU. She is employee at Queen Maud University Collage of Early Childhood Education. Her research interest is mathematics in kindergarten, with a main interest in how children encounter mathematics through language and activities.

bne@dmmh.no

Kjersti Wæge

Kjersti Wæge is the director of the Norwegian Centre for Mathematics Education at the Norwegian University of Science and Technology. Her research interest is related to student' motivation in mathematics, ambitious mathematics teaching, and professional development of mathematics teachers.

kjersti.wage@ntnu.no