

Editorial

Digital resources in mathematics education

We are pleased to introduce the 2023 thematic issue of *Nordic Studies in Mathematics Education (NOMAD)* on *Digital resources in mathematics education*. Digital resources seem to play an ever-increasing role in mathematics education and are now part of the teaching and learning of most mathematical topics. While resources that are no longer new, such as the calculator, are still being used frequently, new and more sophisticated resources seem to emerge. Thus, the landscape of available digital resources in mathematics education includes textbooks, infrastructure, and topic specific software/hardware, making it complex to navigate for both teachers and learners. In the Nordic countries, the complexity is further accelerated by ongoing or imminent discussions and initiatives of revising the mathematics curriculum to include programming, computational thinking, and digital competence. This landscape adds implications for practice, policy and for what is needed of our theoretical frameworks to study mathematics education in such a context adequately. This special issue includes contributions from the Nordic countries that investigate a broad spectrum of how practitioners, researchers and decision-makers navigate digital resources in mathematics education from pre-school to higher education.

The relevance of the topic is further evident in the interest drawn to the call for papers (see appendix). As a result, this is a double issue containing ten articles. All articles were subject to an abstract proposal screening and a double-blind peer review process with three reviewers per contribution. In principle, each manuscript was reviewed by another author participating in the call for papers, and researchers in and out of the Nordic context with ad hoc expertise in the respective contributions.

Summary of contributions in this issue

Digital technology in mathematics education

The first three papers focus on purposes and approaches to include digital technology in mathematics education.

Jankvist, Misfeldt, Geraniou, Aguilar and Baccaglioni-Frank explore the concept of technocritical mathematics education, which stems from critical mathematics education and addresses the use of digital technologies in mathematics teaching. It emphasizes the growing presence of hidden mathematics in today's technology-driven society, both inside and outside the classroom. The paper argues that a technocritical mathematics education should empower students to uncover and apply hidden mathematics as essential skills for modern citizenship. Furthermore, it raises questions about which mathematics scenarios can help students develop competence in these processes and what defines such scenarios.

Johansen conducted a literature review to investigate potentials and challenges of digital technologies in mathematics education in relation to students' *Allgemeinbildung*. Her findings take the form of dilemmas related to perceiving and understanding a data-drenched world, knowing and comprehending mathematical objects and facts, and the acquisition of capabilities for problem-solving freed from tidy calculations.

Granone's and Reikerås' study explores how preschool teachers can boost kids' mathematics skills using a coding robot toy. By analyzing video recordings of a teacher interacting with four 3- to 5-year-old students during their monthly three-hour robot play sessions, they apply the theoretical concept of semiotic mediation in mathematics education. Their findings emphasize that the coding toy serves as a valuable tool for achieving mathematics learning objectives with teacher guidance, including problem-solving, counting, and measurement.

Pre- and in-service teachers

A sub-set of the articles in this special issue is dedicated to studies how the roles of and challenges faced by both pre-and in-service teachers.

The study by Vineran, Liljekvist and Brandl begins by addressing the challenge faced by prospective teachers in connecting school mathematics with university-level content. This issue arises during transitions from high school to university and back to teaching careers. To address this, the study explores the effectiveness of *Interactive mathematical maps* in a geometry course for future teachers. These maps illustrate how mathematical concepts evolve from initial problems. The study aims to understand how these maps can bridge continuity from secondary to tertiary mathematics education and foster positive beliefs about mathematics. The results of this study show that participants find the maps useful and easy to use, promoting a process-oriented and open-minded approach to mathematics.

The paper by Meaney, Huru and Kvivesen studies how Norwegian pre- and in-service teachers consider different digital tools would support differentiated teaching, how they relate to progress in students' mathematical learning and how they apply to multilingual classes. The digital tools were diverse and included non-mathematics specific tools such as social media, Minecraft, Kahoot, Power Point and Interactive whiteboards but also included tools with a closer relation to mathematics such as Excel and Geogebra. The findings of the study reveal similarities in pre- and in-service teachers' evaluation of the digital tools with regards to differentiated mathematics teaching and mathematics teaching in multilingual contexts. The study however found indicators of uncertainty in terms of how the digital tools could support specific groups of students.

Task design in digital environments

Bach's paper focuses on a design research project regarding the design principles for tasks, which focus on "exercising" or "putting into practice" students' mathematical communication competency when using a *Dynamic geometry environment* (DGE). Relying on perspectives from the instrumental approach, semiotic registers and mathematical language, Bach analyzed two 9th grade (15–16 years old) students' interactions with GeoGebra and was able to refine humble task design heuristics into design principles. Among other things, the task design heuristics emphasize the need of ensuring progression of content complexity across situations and context, of addressing two semiotic registers, and of allowing individual and collective processes by including individual tasks and tasks that focus on communication.

In his paper, Heinesen explores a task design in GeoGebra and the role of Dynamic geometry environments in relation to grade 8 (14–15 years old) students' mathematical reasoning and proving activities. The study draws on the Theory of semiotic mediation and design-based research methodology pursuing the question of how students' conjecturing activities in DGE can be combined with theoretical validation with the purpose of making theoretical validation a meaningful activity for the students. The task design address specific pieces of knowledge within geometry. The activities consist of a worksheet prepared with a shape, a number of sub-questions and finally asking students for a conjecture regarding the shape and its properties followed by an explanation why the conjecture may hold true. For the latter students are invited to draw on the toolbox designed for them containing theorems from geometry and other pieces of knowledge. Højsted argues that the "toolbox puzzle" from his study may bridge conjecturing activities in DGE.

Pedersen conducts a review of the use of DGE and Computer algebra systems in relation to the mathematical thinking competency as defined in the Danish competency framework. Pedersen's analysis finds that activities related to the mathematical thinking competency may be delicate situations related to reasoning, problem-solving and problem-posing. She also finds that DGE and Computer algebra systems has the potential to act as tools for activities to enhance the mathematical thinking competency, especially in relation to question-answer, generalization-abstractation and scope of concept aspects, but that there is a need for an explicit connection between mathematical thinking competency and DGE and Computer algebra systems.

Programming and computational thinking

The two last papers focus on programming and computational thinking and their interplay with mathematics. Borg and Fahlgren explore the mathematical value of programming by analyzing a case of students working with problem solving through a programming language. They find that some programming concepts and techniques are important in the problem-solving activities, and that some of these concepts and techniques are of clear relevance to mathematics, while others are more oriented towards programming and computer science knowledge.

Elicer and Tamborg investigate the relations between programming and computational thinking and mathematics as depicted in the Danish technology comprehension curricular pilot reform that ran between 2018–2021. Their study is based on Anthropological theory of didactic situation and find notable misalignments between intentions depicted in the curriculum and the resources developed to support curriculum implementation. While the curriculum juxtaposes PCT and mathematical competences, teaching resources are predominantly integrated, and scholarly knowledge positions PCT as subordinated to mathematical problem solving and modelling which the former do not particularly engage with.

A Nordic flavor to digital resources in mathematics education

Nordic countries are rich in comparison to many other parts of the world, and access to digital technologies can be taken for granted. This is reflected in the fact that the papers in this special issue primarily deal with how mathematics education can gain the maximum benefit from the access to such resources, e.g. through how mathematics teachers are

trained, and how we design tasks for mathematical instruction. The collection of papers also illustrate that digital resources in mathematics education not only represents a world of opportunities, but requires both teachers and students to develop new kinds of mathematically based critical mindsets.

Researchers have examined the impact of didactical resources, including digital tools, to understand the process of teachers' resource adaptation and implementation in classrooms (Gueudet et al., 2012; Remillard & Heck, 2014). Recently, there has been a focus on how the availability of different resources affects teachers' professional development (Giti-rana et al., 2018). In Nordic countries, teachers are generally considered autonomous (Blossing et al., 2014; Fauskanger et al., 2022), which suggests that Nordic teacher education should support teachers' decision-making rather than prescribing their actions. Education in the Nordic countries is often described as "the Nordic model" with key characteristics that also apply mathematics education. These characteristics include that all Nordic countries have a national mathematics curriculum, and a commitment to student centered mathematics education. To some extent, the Nordic model also applies to the areas of digitalization and technology in mathematics education, in that Nordic countries have acknowledged that digital competencies related to technology and programming should or already have become part of educational policies. While the inclusion of programming and digital skills is a global trend, the Nordic countries' approaches have specific characteristics. Despite the diversity of rationales, fostering problem-solving and logical thinking skills are transversal to all of them (Bocconi et al., 2018). This explains, in part, why aspects of programming and digital competencies are consistently being integrated into the mathematics curricula. Consequently, in the Nordic countries, the discussion of programming and computational thinking becoming part of schooling needs to be in the mathematics education research agenda, which is reflected in several of the papers of this special issue. This strongly implies that an outlet such as *NOMAD* constitutes an important forum, in which academic discussions and insights on challenges related to this specific Nordic approach can be disseminated and discussed. The fact that the number of accepted papers of this special issue on digital resources in a Nordic context have led to a double issue strongly indicates both a high level of interest and research activity on this matter, which in all likelihood will maintain to be important in our part of the world.

It is our hope and strong belief that the papers within this double-issue of *NOMAD* in sum contribute to showcase state-of-the art in both

research and practice and can bring forward our knowledge and further research endeavors into the important questions related to how, why and with what reservations digital resources should be part of mathematics education now and in the future in a Nordic context.

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Appendix

Old text from the call for papers

Authors' contributions can relate to multiple aspects of the overarching theme. These can report both on empirical and theoretical studies and refer (but not be limited) to the following topics:

How have digital tools and aids been included in the mathematics curriculum? What explicit justifications and hidden reasons can be identified? Are there notable similarities and differences between countries?

What do experiences and results say about the mediating role of digital technologies in students' mathematics learning?

How have digital tools facilitated or eventually obscured the assessment of mathematics learning?

What are the roles of digital resources in bridging mathematical competencies with other non-mathematical fields of practice (e.g., design) and theoretical constructs (e.g., computational thinking)?

Which opportunities and challenges are there in the use of online platforms for teaching and learning mathematics, including online synchronous and asynchronous teaching and learning?

How can digital resources and their role be defined and classified in the context of mathematics education? How do different theoretical perspectives frame computing hardware and other tangibles, mathematics-oriented software, digital textbooks, programming languages, online teaching platforms and learning communities?

Which challenges and dispositions do teachers find in adopting and using digital resources before and during their practice? To what extent do technologies respond to their necessities? Are there gaps associated with, for example, age?

Which socio-political aspects can be highlighted with the irruption of digital resources in mathematics education? Which political agendas and corporate motives are interwoven in the mainstreaming of digital resources?

