Mediating activities in students' collaborative work on self-explanation prompts

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This study concerns mediating activities in student discussions during collaborative work with self-explanation prompts (SEPs). While the aim of most other tasks, from the students' perspective, can be perceived as finding the correct answer, discussions supported by SEPs require a different approach, because students must engage in mathematical discussions, and explain their insights into the mathematics at hand. In this study, we explore activities that are fostered by SEPs. The analysis of the activities taking place, reveal five mediating activities to promote in teaching, but also potential hinders for the intended discussion to occur.

In relation to the development of conceptual understanding [1], procedural skills, and multimodal reading, tasks with *self-explanation prompts* (SEPs) have been described as useful tools (e.g. Roelle et al., 2014; McEldoon et al., 2013; Kwon et al., 2011). SEPs are requests asking a student to explain some crucial aspect to him- or herself in order to make sense of new content. Although SEPs focus on explanations for *oneself*, they might also be useful in collaborative work, as they foster mathematical discussions (Dyrvold & Bergvall, 2019). SEPs can be of several different types. An example of a task comprising an SEP (italicized) is shown in figure 1 (other types of SEPs used in this study are introduced in the section Data and method).

When working collaboratively with SEPs, students are encouraged to discuss for example the meaning of a mathematical concept, a representation, or part of a method. In such a discussion between students, utterances in the dialogue can function as a mediating activity (defined in Theory) and potentially lead to valuable meaning making. In this study, we seek to gain understanding of mediating activities SEPs can foster in

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Bergvall, I. & Dyrvold, A. (2023). Mediating activities in students' collaborative work on self-explanation prompts. *Nordic Studies in Mathematics Education*, 28 (1-2), 31–58.

Kenny takes a photo of his puppy. The sides of the photo are 6 and 8 centimeters long. Kenny's mother prints an enlarged photo of the puppy. The shorter side of the printout is 9 centimeters long.

If the lengths of all sides are increased by 3 centimeters, these new measures are somewhat wrong. Discuss why.

- <– fold

How long are the longer sides of the correctly enlarged printout?

Figure 1. *Task "The puppy" (SEP type 2)*

students' joint discussions. Such an understanding can, by highlighting characteristics of activities valuable to promote, be useful in teachers' support of students' development when using SEPs in teaching. The aim of the study was to contribute understanding of mediating activities in students' discussions and meaning making process in mathematics during collaborative work with SEPs. Two research questions are posed.

> Which mediating activities are formed when students work collaboratively with self-explanation prompts?
>
> What challenges are there for students' collaborative work with self-explanation prompts?

The point of departure for the study is sociocultural theory where the relationship between language and learning is emphasized. Participating in a mathematical discussion is thus regarded as central to the students' mathematical ability (see Theory).

The next sections of the paper are organized by first presenting an overview of research relevant to the purpose of the study. First, research on SEPs is described, followed by research related to learning through discussions. Challenges linked to learning through discussions are also addressed. The theoretical stances underlying the analysis are based in sociocultural theory, which is described in relation to learning through discussions fostered by SEPs. Then follows a section that describes the study's data collection and how the tasks with SEPs used in the study have been developed. These sections are followed by a description of the analysis process, followed by a description of the result. Finally, the results are discussed in relation to the previous research presented.

Background

Self-explanation prompts to support meaning making

The use of self-explanation prompts to support learning has been described and tested in several previous studies (e.g. see Dyrvold & Bergvall, 2019). There are convincing results supporting the usefulness of self-explanation prompts for enhancing meaning making; especially regarding conceptual understanding and procedural skills (e.g. Roelle et al., 2014; Booth et al., 2013). By directing students' attention, SEPs can induce focused processing of central principles, for example mathematical concepts.

Most previous studies on SEPs, however, investigate students' individual work, while the process of working collaboratively with SEPs is left unexplored. The present study contributes to filling this gap by studying the students' dialogue in depth. In collaborative work, the joint process can foster mediating activities. SEPs are, accordingly useful means of supporting the students' development by inducing focused processing of relevant aspects and nurturing individual development in the context of discussions.

SEPs have, for example been shown to encourage deep-oriented processing of learning content and, mediated by inferences, to contribute to the development of conceptual understanding (Roelle et al., 2014). Making inferences tends to put demands on students, demands that differ from student to student, depending on their prior knowledge (Roelle & Berthold, 2013). This does not need to be negative in collaborative work, since if SEPs are used in group work, each participant's contribution has the potential to function as mediation and thereby to aid the development of inferences.

However, there are also studies that do not reveal any positive learning gains after work with SEPs, something that, for example, may be related to the quality of the explanations (Hsu & Tsai, 2013). For SEPs to be effective, they need to be used in a thoughtful way. Based on a research review, Rittle-Johnsson et al. (2017) identified four evidence-based guidelines for effectively promoting self-explanation; scaffolding high quality explanations via training or structured responses, designing prompts to carefully balance attention to all of the important content, prompting learners to explain correct information, or to explain why incorrect information is incorrect when appropriate. To improve the possibilities of working with the first guideline, a deepened understanding of mediating activities in students' discussions during work with SEPs can be valuable. By analysing students' discussions as they work collaboratively with SEPs, this study can contribute such an understanding of mediating activities.

Rittle-Johnsson et al. (2017) also demand additional research to expand and refine guidelines how to formulate and use SEPs. In this study, we investigate mediating activities in students' discussions and meaning making process during collaborative work with SEPs. This understanding can potentially aid use of SEPs in teaching.

Learning mathematics through discussions

One way to encourage elaborated explanations is by collaborative work, something that can be done with SEPs. Studies that explore the connection between learning and language in collaborative work are often built on a sociocultural tradition (e.g. Webb et al., 2021; Genlott & Grönlund, 2016; Dawes et al., 2003; Mercer & Littleton, 2007). Webb et al. (2021) conducted an in-depth analysis of students' explanation of their own thinking and engagement with others' ideas. The study showed that students followed up with detailed questions if they did not follow the explanations. If students were having difficulty making their own ideas accepted, they gave more detailed justifications and explanations. In this way, the students created new connections between mathematical ideas and representations, and developed their problem-solving strategies considerably by participating in the discussions. The benefits of collaborative discussions are also investigated by Mercer and Sams (2006) who investigated how teachers can support students talk and reasoning during collaborative work. The important role of the teacher in modelling how to use language as a tool for collaborative work in mathematics, so called exploratory talk is emphasized. If students are taking part of this kind of modelling, they become prepared to use language themselves to discuss mathematics with peers, according to Mercer and Sams. The modelling of exploratory talk is investigated based on the extent to which teachers for example used ”why questions”, ”reasoning words”, offered reasons of their own, or sought agreement amongst the class at the end of a discussion. The results also revealed that language use in problem solving, and talk-based group activities can promote the development of individuals' mathematical reasoning, understanding and problem solving.

The advantage of discussions to develop mathematical thinking is also made explicit in a study by Kotsopoulos and Lee (2012), where students share and defend their mathematical thinking. The opportunity to present one's mathematical thinking is clearly beneficial, but the importance of collaboration with other students and the opportunity to ask follow-up-questions of a teacher is also emphasized. In line with what Mercer and Sams (2006) reveal, it is concluded by the researchers that the ”other” whether the teacher or a peer, may be necessary for some students to develop their conceptual understanding. Significant learning gains can be observed when discussion is promoted in mathematics learning (O'Connor et al., 2015) also for those who do not say much (O'Connor et al., 2017) but collaborative talk also increases the number of factors that influence the meaning making. In an analysis of types of utterances in collaborative problem solving Smith and Mancy (2018) distinguished between students' cognitive and metacognitive utterances. The study revealed that to contribute to collaborative activity, metacognitive utterances were the most efficient. Collaborative talk, that is reasoning that relates to reasoning of someone else, were however very rare; only 12 per cent of the utterances were collaborative. Accordingly, there may be challenges to achieve a rich collaborative learning process in group work.

Reasoning is often central in studies about collaboration and discussions, but there is no consensus of the meaning of mathematical reasoning. Jeannotte and Kieran (2017) addresses this issue and suggest a definition adopted in the current study. Mathematical reasoning is defined as "a process of communication with others or with oneself that allows for inferring mathematical utterances from other mathematical utterances" (Jeannotte and Kieran, 2017, p. 7). The current study analyses discussions, but in accordance with this definition, mathematical reasoning can also be constituted of several utterances made by one individual.

Challenges in relation to students' learning through discussions

Quality in discussions between participants in collaborative work is essential for learning, and knowledge about potential challenges for collaboration are therefore valuable. In Langer-Osunas (2016) analysis of group discussions in collaborative problem solving, such a potential challenge was perceptions of authority among the students because it play a crucial role in how the group work proceeds. To be able to harness the full potential of group discussions, Cross (2009) emphasises the teacher's role and the importance of giving the students sufficient time to establish classroom social norms around discourse. Other factors such as cognitive or social disagreements, misunderstandings, utterances of doubt, and students' persistence can also either inhibit or support the cognitive functioning of the group (Watson & Chick, 2001) but also the mere choice to not engage in the discussion at all (Cross, 2009). Such factors may be part of the explanation why the combination of writing and group discussion result in larger learning gains than solely either one of the two (Cross, 2009). The participants in group work are an asset, but these studies shows that collaboration is multifaceted and there are many factors that can affect whether high-quality discussions develop.

In a strive for high-quality discussions, an important factor is that a discussion that develops smoothly is not always the best circumstance for learning. Analyses of how students' understanding evolves through discussions and argumentation revealed a successive development of understanding where misunderstandings and disagreements plays an important role in the intersubjective process (Zack & Graves, 2001). Development does not necessarily depend on influence from the "more knowledgeable other" (in line with sociocultural theory), but can rather come from incorrect proposals, through questioning and responding to proposals made by others (Zack & Graves, 2001). Resistance in the form of insecurity and conflicting arguments is also important for the development of a shared meaning of concepts (Carlsen, 2010). In light of these challenges, it is apparent that students need guidance on how to achieve learning through discussion, and that disagreement are not necessarily negative and something that should be avoided.

In the present study, we take our point of departure in sociocultural theory to analyse the activities actually taking place in the students' discussions. In the next section, we discuss how students' discussions during collaborative work with SEPs can be understood from a sociocultural perspective.

Sociocultural theory

In the work with an SEP, the students receive support in two ways; on the one hand, the SEP itself may support the students by pointing out and highlighting central aspects. In addition, the discussion between the students has potential to support meaning making, because a discussion requires students to listen to other participants explanations and to use language themselves to express their thoughts.

In accordance with sociocultural theory (Vygotsky, 1986), language and thinking are closely interlinked. When thoughts are put into words, they are structured and clarified for the individual and verbalization thus supports thinking. In this study, we take the perspective that the discussion between students when working with SEPs, have potential to support students' meaning making and appropriation of mathematical tools and concepts and thus function as mediation. In accordance with Wathne and Carlsen (2022) learning is considered as a process of appropriating, when students, through joint discussions, become familiar with the mathematics and when and how it should be used in problem solving. In this study we investigate the mediating activities taking place in the analysed discussions, and thus laying the foundation for students' appropriation of mathematical tools and concepts.

Mediation is a core concept in Vygotsky's theory, and describes the process by which tools and signs act as mediators for mental processes. To grasp the notion of mediation, it is necessary to understand that the concept involves giving attention to the *participants* in the process as well as to the *process* itself. The mediating action is carried out by "individual(s)-operating-with-mediational-means" (Wertsch, 1993, p. 96). Vygotsky extended concept of mediation to include not only physical tools but also sign systems such as oral language, writing and number systems (Vygotsky, 1978). These sign systems, often referred to as psychological tools, are important as they are internally oriented and transform human abilities and skills into higher mental functions, in contrast to natural mental functions such as memory, attention, and perception (Vygotsky, 1986).

In the present study, the discussions fostered by the SEPs are considered as mediational means and plays the role of mediating actions, that in this context are called *mediating activities*. Understanding the function of these means and actions are essential to gaining a deep understanding of the students' meaning making. In fact, a central theme in Vygotsky's theory is that mental processes can only be understood if we understand the tools and signs that mediate them (e.g. Wertsch, 1985). The psychological tools that students are able to use have previously been internalized in joint activity with more knowledgeable others (e.g. Vygotsky, 1986), but students also have the possibility to develop new tools in the mediating activity together with their peers. For example, when a child is appropriating a mathematical concept, the concept is available as a tool for the student in future mental processes.

In essence, a sociocultural perspective provided the foundation for the design of the study, the method, and the analytical instrument used.

Data and method

The study was conducted as part of a larger one-year research project in collaboration between researchers and a teacher at a Swedish elementary school. In this paper we present parts of the project.

The students' discussions were observed as they jointly worked on tasks that included different types of SEPs. Data was collected on three different occasions during the year. Altogether 38 grade-four students in three classes participated in the project. The teacher, who taught all participating students mathematics during the course of the year, made groups with three students in each. In previous years, the 38 students had participated in three different mathematics classes with different teachers and thus, the students were used to somewhat different styles of teaching. The students were not extensively experienced in working collaboratively with mathematics tasks, nor were they familiar with SEPs. Before the session, each group was given oral instructions about the role of the SEPs in the tasks. Two groups worked simultaneously in the same classroom, separated by a screen wall. Each group worked on two different tasks on each occasion. If the students had not solved the task within 15 minutes, the researcher gently interrupted the work.

All participating students and their parents agreed to participate in the study after being informed about what their participation entailed and our confidentiality procedures and data management.

Designing tasks with SEPs

In total, six main variants of tasks containing three types of SEPs were designed and used in the study (in total 14 unique tasks, see table 1). The design process was a collaboration between the researchers and the teacher who participated in the research project. The teacher's knowledge was an important resource in the design process, for example contributing understanding of appropriate language and mathematical level for students in the grade. All tasks had one of three different types of SEPs previously identified in a literature review (Dyrvold & Bergvall, 2019). The first type of SEP used in the study is not part of the task itself (e.g. see figure 4). These SEPs are presented as standalone requests to explain a concept or an aspect of a concept. In relation to the four guidelines presented by Rittle-Johnsson et al. (2017), this type of SEP is intended to direct attention to important aspects of the content. The second type of SEP was designed as solved examples (see Große & Renkl, 2007) which students are asked to evaluate in terms of whether they are properly executed. In this variant of SEP, the solution process is either clearly stated and possible to follow, or containing a statement to be discussed (see figure 1 and figure 3). The idea of this type of SEP is to offer the students models for how to solve a problem or to reveal possible misconceptions. This type of SEP connects to the last two of Rittle-Johnson et al.'s (2017) two guidelines for how to promote self-explanations. The SEP can either foster the students to explain a correct solution process or support an explanation of why a solution is incorrect. The third type consisted of SEPs that served as subtasks in a major mathematics task. If the subtasks prompt the student to explain the general meaning of a concept, or part of a concept, this kind of subtask can function as an SEP. Similar to the first type of SEP, the third variant directs attention to important aspects of the content by highlighting different aspects, step-by-step (Rittle-Johnson & Alibali, 1999). In a task about proportionality, a subtask can constitute one such step by asking the student to explain the word "per" in the task.

In designing the tasks, emphasis was placed on ensuring that the SEPs had the potential to foster students' discussions of the concept of proportionality. The intention was not to evaluate or compare different types of SEPs, but rather to investigate mediating activities fostered by SEPs. All task in the first occasions contained the first type of SEP, which seemed to differ most from standard tasks in Swedish mathematics textbooks and was judged to have the greatest potential to foster discussions and to contribute something new to teaching. For the second occasion, the tasks were developed with SEPs of the second type, or SEPs of the first type refined based on lessons learned from the first occasion. For example, after seeing students skip the SEP and instead rush to find a solution to the final task, the final task printed at the bottom of the paper was folded backwards (figure 1) to force the students to actively engage with the SEP before trying to solve the tasks.

For the third occasions four completely new tasks were needed because all students had already encountered "The sunflower" (appendix 1) and "Candy" (figure 3) on the two previous occasions. For this occasion, SEPs of type three was introduced and new SEPs of type one and two were developed.

Table 1. *Tasks with different SEPs used on three occasions*

Occasion 1	Occasion 2	Occasion 3
Sunflower A[1] (SEP type 1) (appendix 1)	Sunflower C (SEP type 1) (appendix 3)	Juice (SEP type 3) (appendix 10)
Sunflower B (SEP type1) (appendix 2)	Sunflower D (SEP type 2) (appendix 4)	The puppy (SEP type 2) (figure 1 and appendix 11)
Candy A (SEP type 1) (figure 4 and appendix 5)	Candy C (SEP type 2) (figure 3 and appendix 7)	Matches (SEP type 3) (appendix 12)
Candy B (SEP type 1) (appendix 6)	Candy D (SEP type 2) (appendix 8)	Goal statistics A (SEP type 1) (appendix 13)
	Candy E (SEP type 1) (appendix 9)	Goal statistics B (SEP type 2) (appendix 14)

1. A–D denotes that different versions of tasks are used.

The students' discussions during all 14 collaboration sessions were filmed and transcribed verbatim and in its entirety. The analysis and results presented in the current article are based on all 14 sessions, and examples from two sessions are presented. These sessions were selected because they show dialogues that are relatively short and coherent and illustrate how mediating activities lead to some form of joint meaning-making without being interspersed side tracks.

Analysis process

The transcripts were analysed in three steps to investigate students' joint meaning making in the work with collaborative tasks. The analysis was conducted to all 14 transcripts from the students' work with SEPs on the three occasions.

In the first step (figure 2), we analysed the students' dialogue to track phases where the students are engaging with the SEPs. The phase was initiated when the SEP was first refereed to and left when the students ended the discussion on the SEP.

Figure 2. *Step 1–3 of the analysis – identifying mediating activities*

Each utterance made in the identified phases was then analyzed in step two. In this analysis the function of the utterance was described, in terms of its contribution to the discussion and in relation to the preceding and following utterances. Examples are, "links to a familiar everyday context" or "confirms the previous statement". Utterances that were incorrect or irrelevant to the SEP were also described.

In the third and last step of the analysis, the utterances were compiled in themes based on their function. Each identified function judged to have potential to support the discussion on the SEP, and thus contribute to the students meaning making, was identified as a *mediating activity*. Utterances that were of track and did not contribute to any progress in the discussion on the SEP, were analysed separately to capture challenges.

The analysis was based on the discussions, and the results accordingly provide insights about characteristics of collaborative meaning making in this particular setting. Video and sound recordings were used in this process to capture as accurately as possible intentions in utterances that are not easily interpreted with only transcripts. Unfinished utterances, for example, can signal insecurity, but also deep thinking or interruption by another student.

In the analytical process, each of the two participating researchers independently analysed the data in each of the three analytical steps. The results were then evaluated together, and possible disparities were assessed and solved. Findings on the two research questions are presented in next section.

Results

Mediating activities

The analysis in relation to the first research question revealed five types of mediating activities, and these are given names to reflect kinds of activities that can be fostered in collaborative work with SEPs. Reasoning, that is central in the last mediating activity, is defined in the section about learning through discussions.

Clarifications –utterances clarifying aspects of the task or highlighting important aspects of a concept/central aspect.

Concretizations –utterances that aid the meaning making by either referring to a context or by providing concretization of a concept/central aspect.

Triggers – utterances that expose premature or incorrect ideas, or statements with the potential to provoke reactions from others that lead the meaning-making process forward.

Consolidation – a summary of the preceding reasoning, laying the foundation for the continued work.

Stepwise reasoning –stepwise chain of reasoning given by one student or a dialogue where several students create a logical chain by filling in each other's statements.

Clarification is an activity where students, clarify interpretations or insights crucial for the mathematics at hand. For instance, a student might repeat the reading of the text, or might verbally emphasize a specific aspect of the task or the question itself. Clarifications can resolve doubts that arise in the discussion about, for example, a mathematical relation or a calculation. In our data clarifications have often been of importance at the very beginning of the sessions. In this way, the clarifications give conditions for the discussion to proceed. Another type of situation where clarifications are often used is when the discussion is stuck. In such cases, a clarification can lead the discussion back on track, allowing the students to continue working in the right direction. An example (example 1) of a clarification is given in the work with the task ”Candy D” 2 (figure 3). The task in figure 3 contains a type of SEP with a solved example, which the students are encouraged to evaluate.

> Carl buys two hectograms of candy at the gas station. The candy costs 16 SEK. Lisa buys three hectograms of candy at the supermarket. She pays 21 SEK. Johanna says that the candy is cheaper at the gas station because 16 SEK is less than 21 SEK.
>
> *Discuss whether or not Johanna is right and why.*

Figure 3. *Task "Candy D" (SEP type 2)*

In example 1, Beatrice and Annie first read the task. In line three, Beatrice attempts to discuss the task, but her voice is hesitant and she trails off. In line four, Annie makes a clarification by emphasizing that Carl and Lisa buy different quantities. This fact is central to the task, and is of crucial importance to gain insight into proportions, which is the central purpose of the SEP. Annie's clarification of the content in line four, early in the discussion, is therefore of importance by creating opportunities useful in a preceding discussion.

Example 1

1	Beatrice	Carl buys two hectos of candy at the gas station. The candy costs 16 crowns. Lisa buys three hectos of candy at the supermarket. She pays 21 crowns.
2	Annie	Johanna says that the candy is cheaper at the gas station because 16 crowns is less than 21. Discuss whether or not Johanna is right and why.
3	Beatrice	But she pays ... [with a hesitant tone, trailing off]
4	Annie	**But she buys three. And he buys two.**

This clarification is an example of a mediating activity that makes an important contribution for the discussion to proceed. However, in this particular example, the students do not immediately take advantage of this clarification, which is instead followed by comments off topic. The discussion on proportionality continues in line 39–41, as shown in example 2 when Annie once more emphasizes and clarifies that a crucial factor for the task is that the children buy *different amounts* of candy (bold in example 2). This shows that clarifications are not only given initially, but also later in the sessions. Some clarifications are also based on the previous discussion.

Example 2

39	Annie	Actually, they shouldn't have tried it this way, how much it costs at the gas station and how much it costs at both stores. They should have checked it. Then we would only have bought one hecto, or

both would have bought three, so that it would have been just the same. To see which is cheaper. I don't even understand why they try it like this. **They buy different amounts. She buys three, he buys two, and then they try to compare it**.

40	Beatrice	But he may not know how much he bought.
41	Annie	Exactly! But they may not know how much they each bought. **But in the future, in any case, they should ask what they have bought. And buy the same amount next time.**

Annie's utterance in line 41 (example 2), is also an example of another mediating activity identified in our study, *concretization*. Annie describes an appropriate approach to use the constant of proportionality in an imaginary situation where two people compare the price of candy at two different places of purchase. Annie's utterance is not explicitly about the SEP at hand, but rather a means to gain insight into proportionality. In other words, she elaborates on the constant of proportionality, which is a central aspect of the mathematical concept proportionality, in an everyday context, thereby illustrating its meaning by making a concretization. Because Annie imagines how the problem could be addressed in real life, the mathematical concept is concretized and the students' process of appropriation can be further developed in the dialogue. Concretization is achieved when a student, either by referring to an everyday context or by providing a concretization of the concept, gives an interpretation that might be easier to appropriate, and this often leads to further discussions and conclusions. Clarifications in contrast, are rather about pointing out or outlining important information more or less explicitly expressed in the task.

Another example of how the students use concretizations to develop their insight into proportionality is given in example 3. Ali, Chris and Ben are working on version A of the task "Candy" (figure 4).

Carl buys two hectograms of candy at the gas station. The candy costs 16 SEK. Lisa buys three hectograms of candy at the supermarket. She pays 21 SEK.

Discuss why it is good to know how much one hg costs in the various stores.

- <– fold

Task: Johanna always buys her candy in the store where it is cheapest. Where does she buy her candy?

Figure 4. *Task "Candy A" (SEP type 1)*

Example 3[3]

56	Ali	But we don't even know what this is. Let's discuss it first.
57	Chris	Then if, if you want to know. If you for example ... want two hectos you know what it costs ... If we take four if one-, then it becomes eight, yeah.
58	Ali	Yes.
59	Chris	And then ... you can take ... If you then have this, you can find out what you know ... how much it ...
60	Ben	Yes and then maybe a gram costs half as much, which is four, crowns. I mean two grams cost eight and then it will be a hecto. Yeah? Yes. Yes ... Uhmmmmm, ...
61	Ali	Yes ... Noo-noo ... yeeess ... No-yess, or, I don't know. ... Ehhhhehe
62	Ben	**It often says "how much", often "how much", like this: "one kilo costs yada yada yada"**. But...
63	Chris	**Per kilo** ...
64	Ben	Yes, exactly. It may not always say what one hecto costs. But then ...
65	Ali	How many?
66	Ben	Then it should be that you divide
67	Ali	Yes, **but how many hectos are one kilo?**
68	Ben	Ehhhhh ... I ... think ten.
69	Ali	It is ... So then if one kilo costs 20 crowns ...
70	Chris	Then you divide it.
71	Ali	Yes. By ten. And that makes two crowns.
72	Ben	Exactly. **Two crowns make one hecto**.
73	Chris	Yes.
74	Ali	Though that may not be what it costs in reality.
75	Chris	No.
76	Ali	No, not twenty crowns. Yes.
77	Ben	Yes, but then you know it. **Then you can know how much, so it doesn't have to be ... It might not say how much a hecto costs.**
78	Chris	No. Then you have to calculate that by yourself.

In line 62, Ben uses concretization by introducing an everyday context. Ben says ”It often says ’how much', often ’how much', like this: ’one kilo costs yada yada yada”, a reference to proportionality in a situation that even these young students have experienced. It is reasonable that it is the mediating function provided by this context that evokes Chris' previous experiences of the word ”per” as an aspect of proportionality. Recall that the SEP used in this group task does not include the word ”per”, and thus Chris' utterance ”Per kilo...” is important. Before Ben's introduction of this everyday context (line 59), Chris referred to proportionality using ”this”. Getting acquainted with the expression ”per” is an essential step toward appropriating the concept of proportionality, since it is a strict mathematical concept that is often used in everyday context.

Triggers make up the third type of mediating activity discerned in the analysis. Examples of triggers are students' expressions of contradictory thoughts or uncertainty. At first glance, this type of utterance may not seem to move the discussion forward, but in the analysis, triggers emerge as crucial for the progress of a discussion. A typical case is when a student expresses doubt about what has been said earlier, stating that he or she does not agree with previous statements. Such an example is given in the quote ”But we don't even know what this is. Let's discuss it first”, from line 56 in example 3. The statement directs the students' attention to an ambiguity that they can jointly work on and sort out. This statement of uncertainty clearly serves a mediating function, as it points out a gap in the understanding of the task, a gap that needs to be bridged before the discussion can proceed. Without this claim, the discussion may have gone completely wrong or led to a dead end. The analysis of the transcripts has revealed that it is important for the discussion that students dare to question statements that other students make because that can trigger further discussions. If they do not, incorrect assertions and conclusions might be accepted, obstructing the further process.

Another type of trigger consists of statements of uncertainty made by students expressing that they do not grasp some previous reasoning. At first, these statements may give the impression of inhibiting the discussion, but thorough analyses reveal that these statements can prevent a discussion from leading to wrong solutions. Such statements of uncertainty can steer the discussion in the right direction. These statements may also reveal that a student have not yet understood the concept. An example of a statement of uncertainty can be found in line 67 in example 3. Ali puts his uncertainty into words and lays the foundation for a dialogue in which the students help each other to clarify the uncertainty. The question in line 67 draws attention to the relation between one kilogram and one hectogram, triggering the subsequent discussion. This discussion concerns the commonly used comparison price, which is often stated as crowns per kilogram. Together, the students' addition of new information step-by-step, function as a mediating activity. The statement in line 67 does not explicitly concern the concept of proportionality, but functions as the first step in a process leading to the utilization of proportional thinking.

The fourth type of mediating activity identified in the analysis is *consolidation*. Consolidating utterances contribute by compiling the discussion, or ensuring the adequacy of a conclusion. In these types of statements, a summary of the previous discussion is given after the students have reached a tentative conclusion. This activity functions mediating because it ties together loose threads, enabling a conclusion to be drawn about the central issue. By reformulating what they have come up with, the students confirm the reasoning for each other and make well-informed decisions about how to continue. The quote above (example 3) illustrates consolidation in lines 72 and 77, where Ben confirms the previous reasoning given by Ali.

Consolidation can also be done by providing a new contextual example. In this case the students are trying to make the same reasoning in another example. In line 77, Ben summarizes the conclusions from the previous discussion by making a connection to an everyday context.

The last type of mediating activity identified is *stepwise reasoning*. Stepwise reasoning is often a joint activity, where several students reason together step by step, creating a logical chain that leads to a common understanding. The very goal of working with SEPs, is to get students to discuss and reason about, for example, an aspect of a concept or a solution method. Stepwise reasoning can take the role as a core activity in relation to that goal, and thereby contribute to the common meaning making. In the analysis of the transcripts, however, we have seen examples of different types of reasoning, more or less fruitful for developing the discussion.

A scenario that exemplifies the mediating activity stepwise reasoning is when a student produces a stepwise line of reasoning on his or her own, where the arguments are clear and explicit. In example 4, Annie, Carrie and Beatrice work with the task ”Candy D” (figure 3). In line 49, Annie reasons in a stepwise fashion to herself, putting her thoughts into words. Until line 49, she had been convinced that Lisa's candy is more expensive than Carl's because Lisa buys three hectograms and Carl only two.

Example 4

47	Carrie	But I think it will be very difficult to figure it out when there is one hecto more for Lisa.
48	Beatrice	Yes.
49	Annie	Yes. So, she has paid about seven crowns per ... no ... seven times three. Then she has paid seven crowns for one hectogram. And he has paid eight for one hectogram. Then this is cheaper!
50	Carrie	Yes.
51	Annie	Then Johanna is completely wrong!
52	Carrie	But Johanna says ...
53	Annie	Then Johanna is completely wrong! See, if he has eight. And then two hectos. If this is eight. So, she pays one hecto for ...

54 Carrie Then two hectos here cannot be 16.

55 Annie No it's not 16. 7 + 7.

This stepwise reasoning in line 49 clarifies the proportional relation in the task and leads to the new conclusion that Lisa's candy was cheaper. Annie's reasoning is convincing to her, but line 54 reveals that it has also made the relationship clear to Carrie.

The five mediating activities described above are examples of activities that have turned out to be beneficial for leading the discussion forward.

Challenges for collaboration

The analysis in relation to the second research question reveal actions with the potential to impede the discussion, and thus also impede students' construction of the concept. We have observed two main types of challenges to the collaboration. Firstly the division of labour is important. The collaboration suffers when one student is too dominant, and for example reasons with himself, but without explaining and substantiating their statements. The analyses also reveal cases, where a student expresses a knowledge of the concept which the SEP is about, early in the discussion. For some reason, this particular knowledge does not come into play in the group later on. We interpret this as meaning that some students are excluded from the discussion for social reasons. Either they are not allowed to take a seat in the discussion or they choose to tone down their ideas for the benefit of other students.

Secondly, there are problems connected to limitations in students' preunderstanding. This can either be limitations in the understanding of the context or limitations in the mathematical ability. For example a student did not know how to mix juice by taking one part of concentrated juice and four parts of water. The insufficient contextual familiarity complicated the discussion, which became side-tracked and the group lost focus on the SEP and the concept itself. Deficiencies in understanding the SEP can also apply to lack of prior mathematical knowledge. Several cases in the data reveal groups of students who get stuck on simple calculations as 42 divided by 7. The problem that arose in connection with this calculation meant that all focus on the SEP was lost.

Discussion

The current study explores and highlights mediating activities that take place during collaborative work with SEPs. We highlight and describe five types of mediating activities that appear to be successful for student discussion to progress, activities that the teacher can encourage their students to engage in, but the results also reveal challenges for fruitful mediating activities to come about in students' collaborative work with SEPs.

There are convincing results supporting the usefulness of self-explanation prompts; especially regarding conceptual understanding and procedural skills (e.g. Roelle et al., 2014; Booth et al., 2013). The current study introduces *collaborative work* with SEPs because discussions and the use of mathematical language is regarded as essential parts of the learning process (e.g. Webb et al., 2021; Dawes et al., 2003; Mercer & Sams, 2006; Mercer & Littleton, 2007; Genlott & Grönlund, 2016). Discussion as a tool for learning are thus well established in mathematics education. For example, Mercer and Sams (2006) pointed out the importance of the teacher modelling how to use language to support student to use so called exploratory talk. Exploratory talk relates to the current study because SEPs contain questions and prompts that correspond to some of the questions used to promote exploratory talk. For example, the "how" question and the use of "reasoning words" can be linked to prompts for discussion found in SEPs. In this way, the use of SEPs can fulfill a similar function as the teacher's modeling of exploratory talk as described by Mercer and Sams.

The focus of the present study is however which mediating activities are formed when students work collaboratively with SEPs. The mediating activity consolidation, can also be related to the study of Mercer and Sams. A teacher seeking agreement amongst the class at the end of a discussion aiming at modelling exploratory talk, can be compared to the mediated activity consolidation. This activity compiles the preceding reasoning, lays the foundation for the continued work.

As mentioned previously, there are challenges for fruitful mediating activities to come about in students' collaborative work with self-explanation prompts. According to Smith and Mancy (2018) only 12 per cent of student utterances during a group mathematical problem solving were collaborative. Smith and Mancy described that the most efficient utterances to promote collaboration was metacognitive utterances. A main idea of SEPs is to foster discussion for example concerning a concept or a solution method, which can be understood as metacognitive discussions as described by Smith and Mancy. Apparently, it is demanding to achieve real collaboration in student discussions. A criteria for the five mediating activities identified in this study, is that they contribute in making the discussion progress, thus this criteria signifies that collaboration is taking place. Therefore, the mediating activities can be used in teaching to promote students to engage in collaborative work. There are however also other challenges in group discussions, for example due to lack of prior knowledge. In line with the findings of Langer-Osuna (2016), our study also reveals situations where the discussion is hampered due to social tensions in the group, for example resulting in dominance by one student. A reason why this type of monologue arises might be a one-sided focus on individual performance or an excessive focus on the product – the solution of the task – instead of focusing on gaining further insights into mathematics and on the discussion itself. Possible ways around these problems could be to create the groups in a very deliberate way, where social tensions can be minimized.

Besides social tensions and lack of prior knowledge, students' doubts, or persistence can either support or inhibit the functioning in the group (Watson & Chick, 2001) and fruitful collaboration is not guaranteed. All challenges to collaboration cannot be eradicated but one way to boost development can be to put attention on what works because, apparently, development is aided by knowledge about the kind of skill or tool to develop. The opportunities for learning can be a question of how the group perceive the task to discuss, and regarding that issue we see a potential for support by promotion of the set of five mediating activates. With a shared understanding of for example *triggers* and *consolidation*, utterances that might by peers be perceived as useless contributions in collaboration in relation to SEP, may instead evoke further questions or demands for clarification that leads the discussion forward.

The activities *clarification*, *concretization* and *consolidation* can be suggested by the teacher, when helping students' who have been stuck, and students can be encouraged to continue a reasoning that can be developed *stepwise*, even if the student does not know the entire chain of reasoning from the beginning. A bit less intuitive is probably that, not only the seemingly problem-free discussion is worth striving for. *Triggers*, when students are questioning tentative results or sharing incorrect conclusions, were shown to be valuable despite at first glance seeming trivial or even detrimental for the collaborative meaning making. This is in line with the findings of Zack and Graves (2001), who suggest that incorrect proposals can lead to a more developed understanding because they trigger discussion. When students reveal doubts, or share thoughts that are premature or even obviously wrong to the other participants, this disrupts the discussion and it becomes possible to choose a new direction in the meaning making. Disruptions can entail a need to sharpen the arguments and further develop the process of appropriating by an elaborated reasoning.

Discussions where students share and defend their mathematical thinking is worthwhile (e.g. Kotsopoulos & Lee, 2012; Webb et al., 2021; Genlott & Grönlund, 2016; Dawes et al., 2003; Mercer & Littleton, 2007), but such reasoning must be learnt (Mercer & Sams, 2006; Rittle-Johnsson et al., 2017). A guideline for effectively promoting SEPs highlighted by Rittle-Johnson et al. (2017), is that students need scaffolding and training to develop high-quality explanations in relation to SEPs for example by modelling the use of strategies. When SEPs are used in groups, we argue that the five mediating activities can contribute such scaffolding. Modelling by a teacher has an evident role as guidance, but promotion of the five mediating activities contribute support has in the current study shown to be valuable throughout the collaborative work.

The five mediating activities that we have identified are based on the results from a few sessions, and further examples of activities are likely to emerge in other settings during work with SEPs. Future studies could contribute additional understanding of mediating activities during collaborative work, by examining the outcome of the promotion of these five mediating activities in other contexts.

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Notes

1 Earlier research uses the term "conceptual knowledge", but in order to emphasize that the development of this ability is an ongoing and continuous process, we have chosen to use "conceptual understanding".

2 In Sweden, children are accustomed to buying candy in a pick 'n' mix bag, paying a bulk price, similar to fruit.

3 In Sweden, it is common to use the abbreviation "hecto" (i.e. hectogram) in everyday contexts.

4 Due to copyright, another picture of the whole sunflower and lines in paper are offered.

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Appendix 1. *Task "Sunflower A" (SEP type 1)*

Kim grows sunflowers during the summer holidays. Summer holidays are 7 weeks long. The sunflowers sprout just as the summer holidays begin and then grow the same length per week.

First, discuss what it means for somethig to increase equally per week.

Task: One of Kim's sunflowers is 42 centimeters after the summer holidays. How tall was the sunflower two weeks after school ended if it grew the same length per week?

Appendix 2. *Task "Sunflower B" (SEP type 1)*

Kim grows sunflowers during the summer holidays. Summer holidays are 7 weeks long. The sunflowers sprout just as the summer holidays begin and then grow the same length per week.

First, discuss how to think if one should mark with lines how much the sunflower in the pricture has grown each week. Why do you put the lines exactly on the chosen spots? [4]

Task: One of Kim's sunflowers is 42 centimeters after the summer holidays. How tall was the sunflower two weeks after school ended if it grew the same length per week?

Appendix 3. *Task "Sunflower C" (SEP type 1)*

Kim grows sunflowers during the summer holidays. Summer holidays are 7 weeks long. The sunflowers sprout just as the summer holidays begin and then grow the same length per week.

First, discuss how much the sunflower grows each week.

- -<– fold

Task: One of Kim's sunflowers is 42 centimeters after the summer holidays. How tall was the sunflower two weeks after school ended if it grew the same length per week?

Appendix 4. *Task "Sunflower D" (SEP type 2)*

Kim grows sunflowers during the summer holidays. Summer holidays are 7 weeks long. The sunflowers sprout just as the summer holidays begin and then grow the same length per week.

One of Kim's sunflowers is 42 centimeters after the summer holidays. Kim's mum says, "When we went on vacation two weeks after graduation, your sunflower must have been four inches tall".

- <– fold

<u>Task</u>: Is Kim's mother right? Explain why or why not.

Appendix 5. *Task "Candy A" (SEP type 1)*

Carl buys two hectograms of candy at the gas station. The candy costs 16 SEK. Lisa buys three hectograms of candy at the supermarket. She pays 21 SEK.

Discuss why it is good to know how much one hg costs in the various stores.

- <– fold

<u>Task</u>: Johanna always buys her candy in the store where it is cheapest. Where does she buy her candy?

Appendix 6. *Task "Candy B" (SEP type 1)*

Carl buys two hectograms of candy at the gas station. The candy costs 16 SEK. Lisa buys three hectograms of candy at the supermarket. She pays 21 SEK.

First, discuss what it means for something to cost differently per hecto.

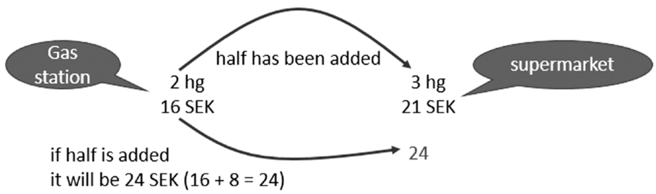
- <– fold

<u>Task</u>: Johanna always buys her candy in the store where it is cheapest. Where does she buy her candy?

Appendix 7. *Task "Candy C" (SEP type 2)*

Carl buys 2 hg of candy at the gas station. The candy costs 16 SEK. Lisa buys 3 hg candy at the supermarket. She pays 21 SEK. Is it cheaper to buy candy at the gas station or at the supermarket?

Here is Bea's solution. Discuss why you cannot solve the task in this way.



Answer: It is more expensive at the supermarket because SEK 24 is more than SEK 21.

Appendix 8. *Task "Candy D" (SEP type 2)*

Carl buys two hectograms of candy at the gas station. The candy costs 16 SEK. Lisa buys three hectograms of candy at the supermarket. She pays 21 SEK. Johanna says that the candy is cheaper at the gas station because 16 SEK is less than 21 SEK.

Discuss whether or not Johanna is right and why.

Appendix 9. *Task "Candy E" (SEP type 1)*

Carl buys two hectograms of candy at the gas station. The candy costs 16 SEK. Lisa buys three hectograms of candy at the supermarket. She pays 21 SEK.

First, discuss what it means for something to cost differently per hecto.

- <– fold

<u>Task</u>: Johanna buys 5 hg of sweets for 35 SEK. Has she shopped at the gas station or the supermarket?

Appendix 10. *Task "Juice" (SEP type 3)*

Moa mixes juice. She takes one part concentrated juice and four parts of water.

a. *How many parts are there in total in the juice mixture?*

b. *Discuss whether all the "parts" in a mixture have the same size.*

c. *Explain what is meant by "part".*

d. *How many deciliters of concentrated juice does Moa need if she is to mix 1.5 liters of juice? Show how you arrive at your answer.*

Appendix 11. *Task "The puppy" (SEP type 2)*

Kenny takes a photo of his puppy. The sides of the photo are 6 and 8 centimeters long. Kenny's mother prints an enlarged photo of the puppy. The shorter side of the printout is 9 centimeters long.

If the lengths of all sides are increased by 3 centimeters, these new measures are somewhat wrong. Discuss why.

- <– fold

How long are the longer sides of the correctly enlarged printout?

Appendix 12. *Task "Goal statistics A" (SEP type 1)*

Before a penalty shot, a player must be designated to take the penalty. Below you can see the number of goals per shot for some of thes corers.

– *Tonya Holloway: 5 goals out of 10 shots*
– *Nina Arlbrandt: 10 goals out of 20 shots*
– *Jonna Lindström: 5 goals out of 15 shots*

Explain what "goals per shots" means.

- <– fold

Are all players equally good scorers?

Appendix 13. *Task "Goal statistics B" (SEP type 2)*

Lisa solves the following task:

Before a penalty shot, a player must be designated too take the penalty. Below you can see the number of goals per shot for some of thes corers.

– *Tonya Holloway: 5 goals out of 10 shots*
– *Nina Arlbrandt: 10 goals out of 20 shots*
– *Jonna Lindström: 5 goals out of 15 shots*

Are all players equally good scorers?

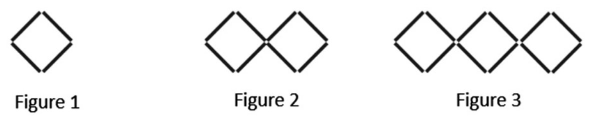
Lisa calculates like this:

$$\frac{5}{10} = \frac{1}{2}$$
$$\frac{10}{20} = \frac{1}{2}$$
$$\frac{5}{15} = \frac{1}{3}$$

Answer: Tonya and Nina are the best scorers

Explain if Lisa is right and why/why not.

Appendix 14. *Task "Matches" (SEP type 3)*



a. *Build the three figures above with matches.* (matches are offered)

b. *Build the fourth figure as the pattern continues to grow.*

c. *Also build the fifth figure.*

d. *Explain in words how the pattern grows with each new figure.*

e. *Complete the table below.*

Figure number	1	2	3	4	5	10	20	50
Number of matches								