Learning to teach mathematics in preschool through theory-driven interventions

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In this paper we aim to highlight how teachers' ways of experiencing mathematics teaching for preschoolers is reflected in their teaching acts. The specific research question is how one teacher's way of experiencing the task of teaching numbers to preschool children change when participating in a professional development project informed by Variation theory principles. We analyze the teaching of one teacher participating in a professional development project during one preschool year with particular interest in how principles for teaching numbers in preschool are implemented in the teacher's teaching. Observations from authentic teaching situations and an interview with the teacher are data for analysis. From a variation theoretical analysis, we draw conclusions of how the teacher's ways of experiencing mathematics teaching to preschoolers is reflected in her teaching acts and what challenges there are to implement theory-driven mathematics education in preschool.

According to the *Education act* in Sweden (SFS 2010:800), preschool is to offer education that meets every child's potential and needs to develop their knowledge and skills in social as well as academic knowledge areas, mathematics being one of these areas. This education is to be conducted based on research, which indicates that the teaching conducted in preschool should have a theoretical foundation but also be based in good practice experiences. This is also in line with the definition of a profession (Korthagen, 2010). Professions legitimate their work by reference to research and theories and change their practices not only because the conditions of the practice change (such as curricula, practical circumstances, or policies) but due to the process of knowledge growth, critical examination, and academic development that leads to new understandings, new perspectives, or new ways of interpreting the surrounding

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world. The curriculum for preschool (National Agency for Education, 2018) points out broad areas that preschool children are to get acquainted with and develop their basic knowledge of, but how the teaching is to be implemented is up to every teacher to decide. Thus, the professional preschool teacher needs knowledge based in research to conduct his/her pedagogical profession.

There are two critical issues raised in evaluations of the success of developmental programs that aim to strengthen teachers' work: Does professional development actually have impact on pedagogical practice, and more importantly, on children's learning outcomes? In order to impact learning outcomes of preschool number teaching we introduced a theory-driven intervention in which preschool teachers were involved in developing, conducting and revising their teaching (see Björklund et al., 2018) in accordance with Variation theory principles (Marton, 2015). However, as Guskey (2002) raised some critical concerns about necessary conditions for implementing research that changes teacher practice in a true sense, we find it important to raise awareness of teacher knowledge and in particular how teacher knowledge affects what young children are afforded to learn. The aim of this particular paper is to highlight how teachers' ways of experiencing the teaching of mathematics to preschoolers is reflected in their teaching acts. We do this by asking the question: How does one teacher's way of experiencing the task of teaching numbers to preschool children change when participating in a practice-based professional development project? To answer this question, we analyze the teaching of one teacher participating in the theory-driven professional development project during one preschool year. In particular, we analyze how principles for teaching numbers in preschool, informed by Variation theory (Marton, 2015), are implemented in the teacher's teaching acts during her participating in the project, and how her way of experiencing her teaching is reflected in her teaching acts. This is done by thorough analysis of documentations of authentic teaching acts, here illustrated by a selection of three video-recorded observations at different times and a follow-up interview with the teacher.

Background

The project is situated in Swedish preschool context, where preschool teachers have a broad education (bachelor level) and should be prepared to teach all subjects and all preschool ages (which in Sweden may range between 1-year-olds and 6-year-olds, sometimes all ages in the same child group). While this broad education and exam is one of the strengths of the preschool profession, it may also reduce the possibilities to bring out

the most from the programs and materials that are available. In order to know what to teach and how to use a certain artefact to facilitate the best possible learning opportunities, the teacher needs sufficient knowledge of for example the number concept, how numbers are constituted, how children learn about numbers and how to teach about numbers in ways that are appropriate for the specific learners (also referred to as *Pedagogical content knowledge* and *Mathematical knowledge for teaching* by Ball, Thames & Phelps, 2008). Pedagogical content knowledge also includes knowing how to make a content comprehensible for another person and knowing why some content is difficult for some students to learn (Shulman, 1986). This requires extensive knowledge of both content and pedagogy.

Professional development is complex in any area of expertise, but perhaps in particular when it comes to teachers where it is not about performing an act to result in a product, but a profession characterized by dynamic interaction between learners and teacher, and an orientation towards a learning object. Some have tried to describe the developing teacher in terms of novice and expert: Katz (1972) claimed that a teacher proceeds through stages of knowledge and skills. Korthagen (2010) described teachers' professional development in three stages, from intuitive actions, through more strategic choices of actions, towards a deeper more generalized understanding that informs actions. Thus, teachers' ways of teaching are related to concept structure and schemata, around which meaning is organized at increasingly abstract levels of cognition. Transition to higher levels of understanding is supposed to happen in social interaction and through a bottom-up process; that is, based on their own experienced concerns in real contexts that are systematically reflected upon.

There are, however, doubts about the stage developmental perspectives of teachers' professional development. A more nuanced way of describing teacher professionality, emphasizes not the progression but the core of the professional enactments and is framed as "knowledgein-action" (Schön, 1983). This means an implicit choice of goal-oriented actions based on empirical and theoretical knowledge. Cheng (2008) suggested that meta-learning is an essential component for the professional development of teachers, particularly in changing contexts. This view on teacher knowledge raises an important issue, regarding if professional development should be about learning about one's own learning, that is, teachers become meta-learners in order to take charge of their own development and learning. Furthermore, professional development should according to Timperley (2008), facilitate teachers' understanding of the impact that a change in their pedagogical practice may have on the children's learning, since this change in seeing their own practice and professional needs results in greater teacher responsibility. According to Guskey (2002), if the intention is to change practice, professional development will work best if it provides solutions to problems that the teachers have encountered in their teaching, thus helping teachers to better understand both what they teach and how students learn specific content and skills.

While taking a Variation theory perspective on learning (see Marton & Booth, 1997; Marton, 2015), whether it concerns the learning of children or teachers, learning is defined as a person's way of experiencing a phenomenon that is changed due to more and new aspects being discerned. This enables the enactment of new and more advanced acts and approaches. That is, the teacher's teaching reflects his or her way of understanding the teaching task, which necessarily includes awareness of several aspects of the teaching practice. Consequently, teacher development could be seen, in this perspective, as a change in the teacher's way of experiencing his or her teaching with implications for children's possible learning outcomes.

The study

Our inquiry is based on empirical work in collaboration between nine preschool teachers and researchers. A fundamental part of the project¹ was the teachers' implementation of not only activities and use of artefacts in preschool practice, but rather their implementation of *principles* to facilitate conceptual understanding of numbers among the five-yearold children they were working with. These principles are based in Variation theory of learning (Marton, 2015): learning is seen as a change in ways of experiencing phenomena and how a person experiences a phenomenon is considered to be a function of those aspects of that phenomenon that the person is able to discern in a certain situation. More aspects discerned changes the way the person sees the phenomenon and consequently how the person acts and reasons in relation to the same phenomenon. Teaching is in line with this theoretical framework an act of offering the learner opportunities to discern such aspects, and in such ways, that will develop his or her way of seeing the phenomenon in question. A key feature is that in order to discern a new aspect, features of that aspect have to vary as dimensions of variation. In a teaching act, this is done through a carefully designed and enacted pattern of variation and invariance where the aspect that is to be discerned is varied and other aspects are kept invariant. The intervention that was to be conducted by the teachers thereby aimed at emphasizing and enabling critical aspects of numbers to be discerned by the preschool children through designed activities and artefacts. In particular, numbers as composite sets, representations of numbers, and part-whole relationship of numbers were made objects of learning. These aspects were brought fore to be explored in the teaching acts for example by contrasting examples and connecting between representations, in accordance with Variation theory principles (see Björklund et al., 2021; Ekdahl, 2020).

The FASETT project is highly theory-driven in that the theoretical principles were to be implemented both through the designed activities and through the empirical principles of collective learning (described below). What we aimed for in the project in terms of professional development was an advanced understanding of how an artefact or activity would be used in order to facilitate conceptual development and understanding of numbers and additive relations among the children.

Earlier studies have shown (e.g. Guskey, 2002) that research and scientific knowledge does not rub off to practice if the teachers do not feel it is a gain for their own work. Thus, we wanted teachers in our project to *own the theory* we sought to implement in the project, by involving teachers in developing the artefacts and acts in an iterative process. The purpose was thereby not to give a frame for teachers to apply to their practice as a script to follow, we intended for them to change their way of seeing their teaching such that they would identify a need for the theory to develop their practice.

Project design and principles

The project design has elements of an educational design research approach, characterized by being; interventionistic, iterative, involving practitioners, process-oriented, utility-oriented and theory-oriented (Akker, 1999; 2006). The project is interventionistic, since it addresses questions of learning related to the preschool practice. The teachers and the researchers worked in close collaboration during the whole process; planning, developing and revising activities and resources based on empirical findings and prior research on early number learning and principles from Variation theory. The activities were implemented by the teachers in their preschool groups and some of these teaching occasions were video recorded. The meetings with researchers and teachers (twelve meetings in total during an eight-month long period) took as their point of departure reflections on the video recorded activities conducted by the teachers. which included the children's responses to the enacted teaching. In preparation for each meeting the research group analyzed the recordings together and selected key episodes of importance for the aim of the project and to be of particular interest for further discussion with the teachers. In the collective meetings the researchers and teachers together watched the key episodes, analyzed and reflected on them. The discussions were *theory-oriented*, framed by underlying variation theoretical principles that for instance learning is relational and aspects are made discernable through patterns of variation. Following this, it was of importance to take the learners' way of experiencing numbers and a given task into consideration when trying to support them to learn the intended. In this way, the teachers were introduced to the theory and successively became familiar with the theoretical principles while integrating them in the pedagogical work and collective discussions.

Both the teachers and the researchers contributed with their reflections in the meetings and came up with ideas to improve the activities. The joint discussions were based on the researchers' view of theoretical assumptions underlying the study, but equally on the teachers' experiences and knowledge of their practice and the children. After having analyzed and commonly reflected on the teaching and learning opportunities that were offered in the conducted activities, these meetings often ended up with a collective decision to refine the activity and conduct the revised teaching with the children again. After having taught the activities repeatedly, new activities and resources were introduced and reflected on in a similar *iterative process*. Thus, the participating teachers were involved through the whole intervention process.

Data collection and data analysis

At the end of the second semester of the project, the participating teachers were interviewed. The aim was to study in what way they had appropriated the theoretical concepts of the project.

One of the nine preschool teachers participating in the project, here called Anne², was selected for specific inquiry, since she from the start of the project had continuously video recorded teaching episodes where the commonly planned activities as well as own planned number activities were taught. Anne had 27 years of working experience in Swedish preschool and had not been attending additional mathematics education courses. Three documentations of Anne's teaching as well as the interview (40 minutes) were selected for a thorough analysis in this paper. The documentations of her teaching are sampled as they are selected by the teacher herself as examples of her teaching:

A A planned card game conducted before the intervention program started, where the teacher distributed one card with written

numerals to each child. The children are asked to match their card to cards with corresponding number of items on the floor.

- B An activity where the teacher was conducting one of the collectively planned activities. The teacher presented short number stories of different types within the number range 1–10. The children were encouraged to model the problems on their fingers.
- C A self-initiated teaching activity with "jumping frogs" conducted during the intervention (as an addition to the collectively planned activities). The children were asked to throw ten paper clips ("frogs") in a bowl. The teacher and the children discussed the number of paper clips found inside and outside the bowl.

The video-recorded teaching episodes as well as the interview were transcribed verbatim. To examine how the mathematical content was handled, we analyzed how principles for teaching in the preschool groups were implemented in the teaching acts during Anne's participation in the project. We then compared the principles that become visible in the different teaching acts and analyzed Anne's reflections on her teaching in the interview. Following the variation theoretical assumption (Marton, 2015) that a person's (the teacher Anne in our case) way of experiencing a phenomenon (mathematics teaching for preschoolers) is reflected in her teaching act, we were able to describe changes in Anne's way of experiencing the task of teaching preschoolers about numbers and additive relations.

Results

Three documentations are selected to illustrate occasions where the teacher's way of experiencing the teaching of numbers and additive relations are reflected in her teaching. Each episode will be discussed in terms of the teacher's way of experiencing her teaching through the analysis of what is taught and made possible for the children to learn as well as her own reflections of her ways of teaching.

Teaching the already known

In the beginning of the project the teachers were documenting a regular activity where they were teaching numbers in their preschool settings. Our target teacher chose to share her documentations that were both routine situations that are common for preschool practice in Sweden, such as setting the table for a meal, and structured activities shaped as mathematical tasks. The following excerpt is one of the latter examples.

Five children are attending circle time. The teacher gives them one card each (up-side-down). On the floor in front of them are six fish-shaped cards with different number of items (2–8) printed on them.

- Anne: Let's see, can you take the same number. You have a number on your fish. Pick the right number of hats, suns, butterflies, or what it is, that belongs to the number you have. Harry, can you start, turn your card around and see what number you got.
- Harry: [turns his card with number 5 printed on it] five.
- Anne: A five. Can you pick, where can you find a five?
- Harry: Five [points at a card with five butterflies].
- Anne: You think it's that one. What's on the picture?
- Harry: Butterflies.
- Anne: How many butterflies is it then?
- Harry: Five.
- Anne: How did you see that?
- Harry: I counted.
- Anne: You counted them. Then you can take the butterfly fish, they make a pair [Harry takes the card with butterflies and the number card and takes his seat again].

The episode continues in similar way until all children have had their turn and matched their number card with one fish card on the floor. The children are invited to experience numerals matched with a number of (identical) items. The number range is 2-8, which allows for some of the children to subitize (instantly identify small amounts as exact numbers). For a larger number, the children need to determine by for example counting. These children master their given tasks without any hesitation. The teacher's actions and decisions in the teaching act, which are of interest in our study, frames the situation as isolated tasks directed at individual children. The goal for each task seems to be experienced as to find the pair, that is, to solve the task. The teacher takes some initiatives to ask how a child found the right number of items to pair with the numeral but accepts the child's answer without further exploration. The tasks become solitary since no connections between the given tasks are made, either by the teacher or the children. When analyzing these kind of teaching activities, we find that the children master the tasks as they already have the skills to pair a cardinal number with a group of items. The teaching thus becomes confirmatory, not deliberately extending the children's existing knowledge of numbers.

How then, is this teacher experiencing her teaching? In the interview she says:

I had no idea how one could teach mathematics whatsoever, not in that way, anyhow [referring to the intervention program's designed teaching activities]. You think that maths in preschool, everybody does it, they count in circle time and saying the counting rhyme because you don't know if they've understood or not, before.

The teacher expresses an uncertainty of her teaching and of teaching in general in preschool. Even though she identifies several instances where mathematics and in particular counting is used, she experiences these situations as acts that will not reveal if the children possess those skills or not. Through the developmental project where the teacher participated in discussions about children's ways of understanding additive relations tasks, she has been exposed to a different approach for working with numbers in preschool that is based on the children's actual knowledge. In reflecting on this way of developing her teaching approach she says:

I think I've got more of a sense now for how to ask questions and such. Then again, they are quite difficult questions which one has not asked before, because one did not know if they knew the answer. And had no clue how to teach either to get them to answer the questions. So, for natural reasons, such questions have not been asked before.

What stands out in her reflection is that her earlier approach, which was primarily confirmatory, was based on her insecurity to encounter a situation where the child *did not* master the tasks. This insecurity, she claims, is based on her not knowing how to respond and facilitate the child's further learning of what he or she does not yet master. In other words, she has not discerned what knowledge of numbers constitute and thereby is not able to offer experiences that would enable the children to broaden their knowledge of numbers.

Teaching according to a script

The previous episode lacks in reflecting theoretical underpinnings to the choice of teaching content and acts. In the developmental project we introduced theoretical principles for teaching and learning by collectively analyzing children's responses to numerical tasks. The motor in these discussions was always the questions "how does the child understand the task at hand?" and "how can we help the child see the task in a more powerful way?" This kind of reasoning about task development and teaching acts were to most of the participating teachers unfamiliar and even though efforts were made to use contrasting examples to illustrate how small differences in enactment could have significant effects for the learning opportunities offered, it turned out to be a demanding task to import the theoretical principles into their practice. The target teacher in this study expressed her need for a script in order to enact her teaching. She says:

I try, based on how I have understood or learnt, to ask questions, but then again, in the end I had to write down exactly how to ask, otherwise I mixed it all up. There were quite many questions to ask. Ten-seven, ten-six, ten-five. And then some discussion which I could not remember. I was so focused on the discussion. If I was to ask ten-six or was it ten-three, I had to write it all down, maybe my memory is bad.

The teacher is eager to apply the new learnt principles in her teaching practice but in the forefront of her way of experiencing the teaching act is differentiated tasks that are to be conducted in a certain order. The principles based on Variation theory inform a systematic variation of how examples or tasks are ordered in that critical aspects are to become discernable to the children. Since the teacher does not yet experience the *connections* between the examples or tasks it becomes a difficult act to coordinate the tasks in that the principles will be adhered. Still, her efforts to implement the principles are coming through in the following documented teaching session.

Anne: Let's say we have three baby elephants and they meet five others.

Kate and Marc show first three fingers on one hand then the other hand with all fingers unfolded.

- Anne: How many are they then together?
- Marc: Eight buddies.
- Anne: How did you know?
- Kate: Three plus five [shows three on one hand, five on the other and moves them together] is eight.
- Anne: Three together with five is eight. Now then, if we have five baby elephants taking a walk and they meet another three, how many are they together then?
- Kate: [shows the same finger pattern immediately] Eight.
- Anne: How did you know?
- Marc: You did the same question.
- Anne: Did I do the same thing?

Marc:	Yes.
Anne:	Exactly the same question?
Kate:	Not the same question, but you did the same numbers.
Anne:	It was the same numbers? Not the same question? What did I do then with the numbers?
Kate:	You took three with five [hits the finger pattern three and then five on the table].
Marc:	Before you took three with five.
Anne:	What did I do this time then??
Kate:	Five with three [hits the finger pattern five followed by the pattern three on the table].
Anne:	I took five with three, aha! [imitates Kate's finger patterns] What happens then, is it the opposite? Five and three.

This episode shows a distinctly different teaching approach than the previous one. The teacher has in both activities planned the activity, chosen what numbers to work with and they are framed in ways that are familiar to the context of preschool. The episode presented above is however designed particularly to emphasize number relations and the examples carefully chosen to highlight number (de)composition and commutativity. The teacher follows up on the children's suggestions and keeps attention to how the different examples relate and connect to one another. Nevertheless, the teacher's expressed experience of this designed teaching act was, based on the interview data, rather as a prepared script to follow as not to forget the order of the tasks that the theoretical principles prescribed. However, what she did not need a script for was the approach to follow up children's acts and suggestions. In this way she opened up other necessary aspects that allowed the children to take notice of different ways of answering the given tasks, and thereby also allowed her to get to know their way of experiencing the number tasks they were exploring.

In sum, compared with the pre-intervention documentations the teacher has developed her ways of learning to know the children's knowing, which had been a critical feature for her developing her teaching. Yet, difficulties still remained concerning how to facilitate the children in their further development, that is, to teach for developing understanding of numbers and additive relations.

Making use of theoretical principles

Preschool is an empirical practice where education and care are conducted in many different kind of activities, spontaneous situations as well as play and designed tasks and games. To teach in this dynamic pedagogical setting requires the teacher to let loose of her script and enact theoretical teaching principles that she has made her own. This comes through in the teacher's reflections on her way of offering problem solving situations to the children.

We count for instance plates when we are setting the table. Cutlery and plates and try to, how many are missing on the table? Never before did I say: Do you remember there were five children, now you have two glasses, how many more glasses do you need? I never said that before. Instead, I said let's count how many more we need, and I think there is a big difference. How you express yourself to make children think. Earlier I gave them easy answers or easy solutions. That is, my goal was to get the table set so everyone gets a glass.

The teacher expresses her change in way of experiencing the teaching task, here exemplified by setting the table, as she was earlier aiming for the problem to be solved in an empirical sense. The table needs to be set so every child has a plate and glass. Her developed way of experiencing these situations in terms of teaching is characterized by the mathematical content in focus. Changes in ways of posing questions may seem as a simple development from a theoretically driven developmental project. but it contains a heightened awareness of what a question directs attention to may contribute to challenge the child's way of understanding the content in a task. It may however be difficult to unpack theoretical principles verbally, in that the enactment of such principles has to be lived through the teacher's actions, not relying on a script. Particularly in a practice where teaching most often is conducted in unforeseen settings and spontaneous questions raised. The following observation is an activity conducted on the teacher's own initiative in the mid part of the developmental project.

Five children and Anne sit on the floor playing "jumping frogs" with ten clips and a bowl.

- Paul: One [throws a clip, misses the bowl]
- Anne: One, you have thrown, yes.
- Paul: [throws a clip] two, [throws eight more clips one at a time] three, four, five, six, seven, eight, nine, ten. It slipped away! [the last clip hits the edge of the bowl]
- Anne: Great. Was it all? Paul, how many did hit the bowl? [points at the bowl] How many clips are there?
- Paul: Three.
- Anne: Three, this many [shows a finger pattern with her thumb, index and long finger on her left hand]. How many are outside the bowl then,

	that missed? [shows both of her hands to Paul with the three unfolded fingers and the other fingers folded]. Look at your hands.
Paul:	[looks at the clips on the floor] five.
Anne:	Three and five, is that ten?
Paul:	No.
Anne:	How many did you have from the beginning?
Paul:	Three there [points at the bowl] and five there [points outside the bowl].

This, and similar games that are common in preschool, easily trigger a counting procedure to determine a number of items that are visible. However, the empirical tasks rarely help the child move beyond counting if a particular mathematical perspective is not emphasized (in this case the part-whole relation of numbers). In the above excerpt Paul counts the visible items in the bowl (3) and those items he can see on the floor (5). The teacher initiates a mathematical perspective to the task when asking how many items there were from the beginning, that is, a whole, to which the parts should relate, even if some items are out of sight. This situation, and the teacher's mathematical direction in her teaching approach highlights particularly what the teacher had been afraid of before participating in the project, encountering a child who is not mastering the task at hand. Thus, she has to enact some way of supporting the child in discerning what is critical for understanding the numerical aspect of the task, not just solving the empirically appearing problems.

Anne:	What if you have three there [points at the clips in the bowl], those we can see Take your fingers up front [shows three fingers again], three.
Paul:	[unfolds index, long and ring finger, holding his hand on the floor in front of him].
Anne:	How many did you throw from the beginning? Do you remember, you counted them [makes a gesture of throwing] when you threw them? Only three hit the bowl, but how many did you have from the beginning?
Paul:	[holds his hands in the knee] I don't know.
Anne:	Look at your hands, how many fingers do you have? [holds up her ten fingers unfolded].
Paul:	five [holds both his hands with fingers unfolded in front of him on the floor].
Anne:	On both hands?
Paul:	Ten.
Anne:	Ten fingers [holding all ten fingers unfolded in front of them] How many clips did you throw?
Paul:	Ten.

Anne: You remember that, then you know Paul. You threw ten [showing ten fingers], if you hit three [folds all but three fingers]. Look at your fingers. There are three clips [puts the three clips from the bowl in front of Paul's three unfolded fingers one-to-one]. Look, three hit the bowl and you had ten from the beginning, how many did not hit the bowl then?

Two other children show different finger patterns.

Paul:	Seven.
Anne:	Yes, there are seven outside. How did you figure it out?
Paul:	I don't know.
Anne:	How did you figure it out?
Ivy:	[looks at her fingers, counting one by one] He's right!
Anne:	Yes, he's right, it's seven, but I wonder, how did you figure it out. How can you do it? [turning towards the other children]
Karen:	You can hold down three [shows three fingers]. You can count five. And six, seven.

To see the mathematics in the task is a difficult undertaking for many preschool children, such as Paul. The teacher offers him a representation in shape of finger patterns to help him identify a relation between the whole ten and the parts of which one was known (3). In this she also includes the physical items (3) and connect them to the finger pattern (3) while also relating the three-part to the whole ten. Seeing the three in the ten is a prerequisite for finding the missing part (7) and thus simultaneously experience how the three numbers relate to each other. In the teacher's enactment we see that these aspects are foregrounded, and she is making use of theoretical principles (representations and connection in particular) for making it possible for the child to discern the necessary mathematical relationship. What Anne is doing, is in fact a way of implementing the theoretical principles in her teaching acts, to facilitate the child seeing mathematical structures that will help him solve the task and develop his understanding of numbers and additive relations.

Discussion

We have in this paper directed attention to what it means to teach numbers in preschool on a scientific grounding. We describe the teacher's way of experiencing the task of teaching based on observations of her self-initiated teaching, co-planned teaching and verbal reflections of her intentions. With support from Variation theory (Marton, 2015), we interpret her learning as changes in ways of experiencing the task of teaching a certain content which is reflected in the opportunities for learning that the teacher offers the children. We can draw these conclusions since it is the same teacher who during her participating in the developmental project has changed her way of experiencing the task at hand, not a personality difference or group difference that could influence differences found in ways of enacting the teaching. Furthermore, our focus on how the learning object is handled, may contribute to the discussion about content knowledge and need for professional development among preschool teachers.

The iterative design of the project focusing on principles of teaching numbers conducted in only a few activities, over a longer period, seems to have contributed to the teacher developing a deeper understanding of the mathematical ideas possible to bring fore in teaching younger children about numbers. This was a demanding change, according to her own reflections. The design focusing on children's understanding and responses to the enacted teaching facilitated her developing the knowledge she identifies as to be critical for her professional development concerning mathematics teaching. The foci in the project, that is both on number concepts and children's learning, met her identified needs. We thereby suggest, in line with Guskey (2002) that the theory and research content helped her see the benefits for her preschoolers' learning outcomes and thus useful for her teaching practice. In the project, we actually dealt with a complex of phenomena to be developed: the children's learning of numbers and additive relations, and the teacher's teaching particularly directed towards young children's learning of numbers and additive relations. The latter has been in focus for this particular inquiry, but the teacher's professional development is in fact including both her developed way of seeing children's numerical learning and her way of seeing the act of teaching mathematics in a more general sense in preschool settings.

The project centered around implementing theory into pedagogical practice. However, theory achieves its power through simplification and narrowing of a field of study (Korthagen, 2010) and thus deals with the world in general, for the most part treating observations of variations as error and randomness as noise. Yet, the pedagogical practice is often "messy" and implementing theoretical principles is a delicate task when simultaneously adhering the dynamic flow of initiatives and alternative actions and responses in the teaching act. In other words, the responsibility of the implementing teacher is not simply to apply what he or she has learned to practice but to transform, adapt, merge, criticize, and invent in order to move from the theoretical and research-based knowledge of the researchers to the kind of practical pedagogical knowledge needed to engage in professional teaching.

We suggest, with support from Korthagen (2010), that the collective development of the project activities constitute a kind of community of practice, that hinders the developing teacher to be left alone with her experiences and instead thrive in the constant encounter with others' both empirical and theoretical experiences. This is also in line with the Variation theory principles of learning (see Marton, 2015), by which contrasting features of the same experienced phenomenon are necessary to motivate and trigger a change in teachers' ways of experiencing their mathematics teaching. So, it could be concluded that the teachers learned about the theory by testing it in the activities with the children and having it as a critical lens when analyzing the interaction and as guiding principles when designing activities together with other participating teachers and researchers.

Our study is small-scale but may nevertheless inform future research projects in that we contribute with an investigation of some aspect of development that seems important for bridging between the teaching practice and the research practice. That is, how participating teachers make theoretical principles their own, but these have to bear relevance for their practice, which in turn can give insights important for theory development as well.

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Notes

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- 2 All names of participants are fictional to secure their confidentiality. Ethical clearance has been given by the Regional ethical review board and written consent from research participants and representatives to generate and use data for research purposes.

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