Meanings of decision-making in probability and statistics: comparing Chilean and Danish upper secondary school curricula

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In this article I investigate the roles given to decision-making in probability and statistics upper secondary school curriculum, in a comparative study between Chile and Denmark. Drawing upon Fairclough's model for *Critical discourse analysis*, I analyse selected official curricular texts as examples of broader discursive practices. In particular, I focus on the positioning of social actors and legitimation strategies. Present discourses position students as active decision makers, though in the Chilean case this is only evident in more recent texts. The teaching and learning of probability and statistics are legitimised through the appeal to political, professional and educational authorities, and through narratives of necessity for rational, grounded, evidence-based decisions. In this aspect Danish texts are more open to complex social and political decision processes. The analysis illustrates a common search for linking mathematics education to democratic involvement in social and political decision-making, but failing to specify the relevance of probability and statistics beyond the individual psychological scope.

The Nordic countries have recognised a necessity for linking mathema-tics education with higher democratic values. In the pre ICME-10 document titled *Mathematics education – the Nordic way,* Dahl and Stedøy (2004) provide a shared story on the democratisation of the Nordic count-ries, in hand with a broadening access to mathematics education for all. At the same time, they recognise the challenge of balancing the planning of such universal access while allowing a more active role of educational communities involved. As Skovsmose and Valero (2002) put it, "the very process of planning, carefully and in detail, access to any kind of ideas obstructs the possibility of making this access democratic" (p. 399). In a similar way, Latin American countries started processes of democratisation

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during the 1990s after their military dictatorship regimes, which have also led them to reflect and propose ways of dealing with such paradoxes. For example, Valero (1999) draws on a teachers' development programme in Colombia to build on the notion of deliberative democracy, where collective and transformative action takes place as a result of a *coflective* (co-reflective) and deliberative process. I find it of most inte-rest that such distanced regions, both geographically and culturally, share a similar challenge of linking mathematics education and democracy.

Deliberative democracy (Skovsmose, 1998; Valero, 1999), in contrast to representative democracy, presupposes a direct sovereignty of the people by providing participants an active role in political discussions and decision-making. One possible link between this notion and mathematics education is what Skovsmose (1998) defines as *deliberative interaction* in the classroom as a micro-society. With an illustrative case, Skovsmose describes how a group of 16-year-old students participated in a project that took into account local inquiries that required mathematics not only to describe them, but to make decisions of common interest. The goal was to provide opportunities to tackle decisions that refer to mathematical arguments. But in a modern risk society, where factual calculations and subjective perceptions are entangled (Beck, 2000, pp. 214–215), decisions are made under uncertainty and risk, raising the necessity for new powerful mathematical ideas in school (Skovsmose & Valero, 2002), namely probability and statistics.

The historical development of probability and statistics as fields of scholarly knowledge has been driven, to a large extent, by the need for making decisions in situations when uncertain outcomes are to be evaluated. According to Hacking's (1975) historical and philosophical address, a pre mathematical notion of probability during the Middle Ages indicated "approval or acceptability [of opinions] by intelligent people" (p. 22). Later, since the seventeenth century, probability, induction and statistical inference have been developed to decide how to apportion a fair share when a gambling game is interrupted, whether an accused person is to be condemned, to decide the price of annuities and policy regarding pensions, and to take a stand on whether a scientific hypothesis is true or not. So risk and decision-making define the "logic" of probability (Borovcnik, 2015). Risk, as vague as this concept may be, is the way we evaluate decisions under uncertainty beyond the possible impacts of different choices, but also with some weight given to their likelihood. Probability has emerged as an attempt to quantify these levels of likelihood.

There is a vast collection of literature in the cognitive and behavioural psychology agenda addressing decision-making under uncertainty, and

characterising biases and heuristics that drive us away from a rational ideal. Well-known theories vary from a two-system mind according to the cognitive load involved (Kahneman, 2011), to a notion of ecological rationality (Gigerenzer, 2008), where heuristics actually work in the proper context. But a common element in this agenda is the focus on individual and economic-inclined types of decisions. Statistics education research has taken this agenda into consideration, but evidence shows that school statistics and specific training do not contribute to prevent the persistence of these biases and heuristics (Garfield & Ben-Zvi, 2007). Moreover, studies in laboratory conditions with the use of gambling games neglect the role of context in the complexity of real-life decisions (Pratt et al., 2011). In light of this, Pfannkuch (2018) has identified emerging curricular approaches to be addressed in future research. One proposition is to get *"more insight into fostering statistical argumentation* including learning how to make evidence-based claims in data-rich environments and critically evaluating data-based arguments in diverse media from a statistical literacy perspective" (Pfannkuch, 2018, p. 407, emphasis in original). This trend is grounded in research experiences focused on the complexity and scaffolding of decision-making tasks under uncertainty. Researching the "critical lens" of decision processes would require different approaches, such as action research, phenomenography, and critical discourse analysis (Petocz, Reid & Gal, 2018, p. 81).

There arises a general *problematique* of coherence between two big ideas: the interest of linking school mathematics with a critical democratic deliberation, and the inherent connection of probability and statistics to processes of decision-making under risk and uncertainty. It is of particular interest to explore whether mathematics curricula reflect both critical ambitions and decision-orientation in their inclusion of probability and statistics. Mentioning and connecting these notions potentially provide a thorough justification for the learning of data and chance in school for every future citizen.

In this article I attempt to join the justification concern brought up above by investigating the role of decision-making in current and upcoming probability and statistics official curricula, through a version of C*ritical discourse analysis* [CDA]. In order to narrow down and have a sense of what is the aim regarding students' future participation as citizens, I make an exemplary analysis that focuses on upper secondary school mathematics curriculum. As a Latin American researcher in a Nordic environment, I delineate the analysis as a comparative study between Chile and Denmark, my home and host countries, respectively.

The research questions addressed in this article are: (1) what is the role given to notions of decision-making in the Chilean and Danish upper

secondary school probability and statistics curricula?; and (2) which similitudes and differences can be drawn from these two cases? These enquiries contribute to the field of statistics education research by providing an exploration of how curricula reflect justifications that appeal to greater ideas of decision-making, democracy and citizenship, thus moving towards a more transparent link between school statistics and its declared goals.

Interpreting roles and spotting changes are broad intentions, so in the following section I address the conceptual framework and methods concerning a version of CDA, in order to understand the research questions in a more precise way.

Analytical strategy

Inspired by Fairclough (2010), I am describing the methodology as an altogether analytical strategy, since "we cannot so sharply separate theory and method" (p. 234) while constructing the object of research. Starting with theoretical generalities, I then provide the necessary conceptual definitions required by the methods for selection and analysis.

Key interrelated concepts are social structures, practices and events, where "social structures define what is possible, social events constitute what is actual, and the relationship between potential and actual is mediated by social practices" (Fairclough, 2003, p. 223). In this frame language is the social structure. Among its infinite possibilities, choices are made to produce texts as part of social events, mediated by discursive practices, where discourse is understood as semiosis, i.e. the process of meaning-making.

In line with this theoretical framework, my analytical strategy consists of navigating backwards the language-discourse-text hierarchy. I first select texts from upper secondary school Chilean and Danish official curricula that refer to decision-making in probability and statistics. I then extract elements of textual analysis, which finally illustrate broader discursive practices. The textual analysis describes what is in the texts, and the discursive aspect addresses how these elements give meaning. The upper level of social structures would involve a discussion on all there is possible to write and mean about decision-making in the curriculum, through a comprehensive global-scale study, which is beyond the realm of this article. Nevertheless, at the end of the article I offer some perspectives beyond the results of the analysis, connected to the broader *problematique* presented in the introduction.

Data collection

Chilean secondary education is defined from grades 7 to 12. These are called 7th-basic and 8th-basic, and from 1st-middle (9th-grade) to 4th-middle (12th-grade).[1] All of these grades have their own study programmes, but more general curricular frameworks are available for 7th-basic to 2nd-middle and the one for 3rd and 4th-middle is under construction. The text selection for Chile takes into account 3rd and 4th-middle curricula, and the aforementioned upcoming curricular framework.

For current versions of the curriculum, I will only take common plan programmes, i.e. those directed to every student in the school system. Excluded programmes are those differentiated mathematics studies defined for scientific education. Therefore, the texts are extracted from grades 3rd-middle and 4th-middle study programmes. These documents also contain didactical guidelines for teachers.

With regard to Denmark, upper secondary education (*gymnasium*) can be taken after ten years of mandatory basic education (*grundskole*). There exist four main gymnasium programmes, namely STX, HTX, HHF, and HF, and three possible levels of mathematics within them: A, B and C. In order to narrow down to one mainstream version of mathematics "for all", I select texts from study plans and guidelines for B-level mathematics. An additional document not authored but endorsed and published

Table 1. *Sources of selected texts*

Document	Label	Year	Curriculum	Author
Mathematics Study Programme, Third Middle	CL315	2015	2009	Curriculum and Assessment Unit, Chilean Ministry of Education (MINEDUC)
Mathematics Study Programme, Fourth Middle	CL415	2015	2009	
Curricular proposal for 3rd- and 4th-middle: Public consultation document	CL3417	2017	2019	
Mathematics B Study Plan, HTX	DKB17	2017	2017	Board of Education and Quality, Danish Ministry of Education (UVM)
Mathematics A/B – HTX: Guidelines	DKAB18	2018	2017	
Mathematics B/C – HF: Guidelines	DKBC18	2018	2017	
Mathematics A/B/C – STX: Guidelines	DKABC18	2018	2017	
Statistics in high school	DKSG18	2017	2017	Ekstrøm, C. T., Hansen, E. & Brockhoff, P. B.

by the Danish Ministry of Education is included as a particular set of guidelines with respect to statistics.

All sources of texts are summarised in table 1. For each source document are given: a label, year of publication or last update, curriculum it belongs to, and authors.

I look for sentences making references to "decision" and "choice"; respectively, *decisión* and *elección* in Spanish, and *beslutning* and *valg* in Danish, including conjugations of the verbs "to decide" and "to choose". These sentences compose the texts to be analysed.

Chilean study programmes are divided into units (numbers, algebra, geometry, and data and chance). I focus the scope of the search on the units of data and chance. The same selection is applied to the upcoming curricular proposal for these last two grades (CL3417), within the mathematics subject. It still does not have the form of study programmes, but it can be considered to be of the same genre, for it means to become the next version of such. It is a general proposal based on a process that involved national experts, international experiences and consultation with civil society participants. The document is published on MINEDUC's website.

Danish study plans and guidelines are not divided into grades, and probability and statistics appear among learning goals instead of main educational unit headlines, making the scope of the search in the full documents broader.

Not all appearances – close to 100 – are included in the selection, since my purpose is to investigate links between decision-making and subject matter probability and statistics. Therefore, mentions of pedagogical choices in content and methods, or students' decisions for a particular exam are discarded. At the same time, as seen in table 1, not all Danish study plans and guidelines are included as sources, where no texts fulfilling the criteria for selection were found.

Data analysis

The analysis is performed at two levels: textual and discursive. Among the several possibilities for identifying and categorising elements in the texts, I will focus on three: modality, collocation and intertextuality. Modality expresses a commitment the author makes to truth and necessity, respectively by epistemic and deontic modalities (Fairclough, 2003, p. 165). It can be evidenced – although non-exclusively – by the use of modal verbs, such as "may" (epistemic) and "must" (deontic). Collocation is the repeated co-occurrence of certain concepts, for example "hard-working" appears as a common binomial in political speeches. Intertextuality is the more or less explicit presence of elements of other texts and

their authors' voices. These elements can be dialogued with, assumed, rejected, and so on (pp. 41–42).

Beyond the mere identification of modality, collocation and intertextuality in texts, I will point out the way they are chosen to be expressed as indicators of discursive practices. In particular, I focus on legitimation strategies. Legitimation is the discursive practice of justifying what is made factual in the texts, through reference to authority, value systems or utility, or conveyed through narrative (Fairclough, 2003, p. 98).

Coming back to the research questions, I define the "role" of decision-making as choices for the use of legitimation strategies as discursive practices. Additionally, I shall identify changes from the current to the upcoming version of the curriculum. Characterising this shift is the final stage of the analysis.

Selected texts

As a way of organisation, Chilean study programmes are divided into units. For each unit are described: purpose, previous knowledge, key concepts, contents, abilities, attitudes, expected learning outcomes (and their respective assessment indicators), didactical orientations, and suggested activities for each expected learning outcome. The following are the selected texts, and above them are the contexts within the study programmes where they are found. All translations from Spanish are made by me as literally as possible.

In the 3rd-middle grade current curriculum (CL315), "decision" appears in the form of "decision tree representations", as a follow-up for the goal of understanding the concept of conditional probability. I will not take it as part of the analysis, since it actually refers to "probability tree representations". Then it forms part of the general didactical orientations for the unit.

3rd-middle. Unit 4: Data and chance

Purpose

1 Experimental problems are worked with *decision* tree representations, which enable a greater understanding of content and [are] a tool for probabilistic calculations. (CL315, p. 120, emphasis added)

Didactical orientations

2 In line with this, it is fundamental that the teacher promotes the development of random thinking, i.e. that students learn to make *decisions* with evidence in situations of uncertainty. (CL315, p. 123, emphasis added)

In the last grade's current study programme (CL415) there are actually two units about data and chance, with no mention of "decision" in "data and chance 2", which includes graphic notions about binomial and normal distributions. In "data and chance 1", "decision" is part of the didactical orientations as in the 3rd-middle grade. Later on, there is a reference to "decision" as a comment for teachers when engaging in activities for the learning goal to critically evaluate information.

4th-middle. Unit 3: Data and chance 1

Didactical orientations

3 In this unit, it is expected that students critically evaluate information published on the media and Internet, from the analysis, interpretation and synthesis of such information, with which they can obtain results about a population considering its size and the variable's distribution, infer conclusions from the mean, variance and standard deviation, and make *decisions* grounded in statistically significant information. (CL415, p. 86, emphasis added)

Suggested activities

4 Furthermore, it is important that the teacher promotes contextualised learning so students progressively develop statistical literacy, which gives them tools for making grounded *decisions*. (CL415, p. 88, emphasis added)

As for the public consultation document for the upcoming curricular framework (CL3417), decision first appears as part of the general purposes of the mathematics subject.

Mathematics: Formative purposes

5 In order to achieve the latter, students will work collaboratively in mathematical modelling of situations, to make grounded *decisions* in disciplinary problems, as well as in the interdisciplinary, social, environmental or economic scope. (CL3417, p. 49, emphasis added)

And then, utterances about decisions are part of mathematics learning goals in both grades.

Learning goals for 3rd-middle

6 It is expected that students will be capable of making *decisions* in situations of uncertainty, with information involving dispersion measures, double-entry tables and conditional probabilities. (CL3417, p. 52, emphasis added)

Learning goals for 4th-middle

7 It is expected that students will be capable of solving problems in contexts of uncertainty, through the application of the binomial distribution and calculation of probabilities, for *decision*-making and critical analysis of statistical information. (CL3417, p. 52, emphasis added)

Both learning goals have a parallel in the previous texts. In the current study programmes (CL315 and CL415), topics such as dispersion measures, conditional probabilities and the critical analysis of statistical nformation are covered.

Danish study plans are published as one whole mathematics unit and their content is organised into: identity and purpose; subject goals and content; organisation; and evaluation. Guideline documents have the same structure, and refer to the study plans in a more detailed and comprehensive manner. All texts were translated to English with Google translator, and revised by a Danish colleague.

Making decisions is first mentioned as part of the general purpose of mathematics in high school in a study plan (DKB17) and guideline document (DKAB18), in regard to what students are enabled to do.

> *Purpose*
> 8 The work on mathematical material leads the student to gain knowledge and skills within mathematics, and enables the individual to understand, analyse, evaluate and make *decisions* in social, professional and study contexts. (DKB17, p. 1, emphasis added)

> *Purpose*
> 9 In addition to gaining knowledge and skills within mathematics, the individual student is enabled to understand, evaluate and make *decisions* in everyday, professional and study contexts. (DKAB18, p. 2, emphasis added)

In a more specific reference to statistics, the relevance of data-driven decisions in policy is drawn in two guideline documents (DKBC18, DKABC18).

> *Subject goals and content – core material and minimum requirements – statistics*
> 10 With the development of computers, the Internet and means of communication, amounts of data are now stored in an order of magnitude that is growing explosively, and statistics based on data is increasingly being used in political *decisions*. (DKBC18, p. 8; DKABC18, p. 8; emphasis added)

The one utterance about choice is contained in the Danish statistics experts' document, in the context of explaining the importance of learning statistics.

> *Why is it important to learn statistics?*
> 11 This applies, for example, to the Monty Hall problem (Ellis, 1995), where a participant on a TV show has to *choose* between three doors behind one of which there is a car. (DKSG17, p. 2, emphasis added)

The above 11 sentences are the units of analysis for the following textual analysis and identification of discursive practices. Narrowing down to isolated sentences is a clear limitation of the proposed analytical strategy, and I address it by two means. First, sources and headlines (in italic font) are considered in the analysis as being part of the texts, as an explicit acknowledgement of the context they are written in. Second, textual and discursive elements of the analysis take other parts of the source documents when perceived necessary.

Textual analysis

I shall first identify and describe elements of textual analysis found in the excerpts so as to provide input to the following discussion. In other words, in this section I extract what is in the texts, before arguing their meaning. I focus on modality, collocation and intertextuality.

Modality

Starting with the Chilean texts, modal forms expressed as "it is expected that students" (3, 6 and 7) can be identified as epistemic modalities, i.e. as expressions of probability and truth, in this case, the expected and not certain to happen. It can be argued that, given the official character of curricula, these are actual expressions of the necessary, falling into the category of deontic modalities.

Explicit deontic modalities are evidenced as "it is fundamental that" (2) and "it is important that" (4), and they express the necessity for particular attention to be paid by the teachers in enabling students' decision-making skills.

Danish extracts express modalities in a more implicit way. Except for the one modal form "has to choose" (11), which refers to the rules of a game show, deontic modalities are exposed in the sections they are written. The mention of the Monty Hall problem exemplifies, in part, *why it is important to learn statistics* (11). Likewise, texts (8) and (9) are part of the subject's *purpose*, so it is desired for the student to "gain knowledge" and "be enabled" (8, 9).

Collocation

References to decisions do not appear alone. A habitual co-occurrence of the substantive "decision(s)" in Chilean texts comes with "grounded" as company, both as an adjective as in "grounded decision-making" (4) and "grounded decisions" (5), and as an adverbial form, as in "decisions grounded in ..." (3). A similar adverbial accompanying form is "decisions

with evidence" (2) and "decisions with information" (6). This collocation suggests a reference to a particular type of decision or decision-making processes, based on quantitative arguments, distinguishable from a mere act of making a choice.

A different collocation appears in Danish texts. Decisions are accompanied by their contexts, whether they are "in social, professional and study" (8), "everyday, professional and study" (9), or "political" (10). This is also the case for the verb "to choose" written together with its particular context of a TV show (11).

Intertextuality

Intertextuality is found within selected texts. In the Chilean case, texts 2 and 4 can be read in parallel as having the same structure.

In line with this, it is fundamental that the teacher promotes the development of random thinking, i.e. that students learn to make decisions with evidence in situations of uncertainty. (2)	Furthermore, it is important that the teacher promotes contextualized learning so students progressively develop statistical literacy, which gives them tools for making grounded decisions. (4)

These texts make references to two different concepts, namely "random thinking" (2) and "statistical literacy" (4) in a similar way: as notions to be promoted by the teacher, driving students to make grounded decisions.

From the Danish sources, texts 8 and 9 also share structure and content. As much as guideline documents refer to the corresponding study plans, text 9 is not an actual quote from text 8.

The work on mathematical material leads the student to gain knowledge and skills within mathematics, and enables the individual to understand, analyse, evaluate and make decisions in social, professional and study contexts. (8)	In addition to gain knowledge and skills within mathematics, the individual student is enabled to understand, evaluate, and make decisions in everyday, professional and study context. (9)

It is no surprise that a text from the guidelines refers to the corresponding study plan, but more interesting are the subtle differences. From the B mathematics programme (8) to the guidelines for A/B mathematics (9), the ability to "analyse" decisions is cut out. Moreover, decisions in "social" (8) contexts are exchanged for "everyday" (9) context.

There are also references to texts other than those selected. "Random thinking" (2) and "statistical literacy" (4) are not defined in the documents, but they are traces of another text published by the Chilean Statistics Society as experts' curricular recommendations (Araneda et al., 2011).

To the best of my knowledge, this is the one Chilean document containing both concepts together as key necessary goals of learning statistics in high school. The term "random thinking" is not found in the literature, rather I presume the intention is to refer to "statistical" (Garfield & Ben-Zvi, 2007) or "probabilistic thinking" (Chernoff & Sriraman, 2014). Both "statistical thinking" and "statistical literacy" are described as core answers to the section "Why teach statistics?" (pp. 11–17), referencing a paper published in the International Statistical Review (Garfield & Ben-Zvi, 2007). Overall, texts 2 and 4 are referring to authority, introducing notions without further explanation, which are developed within national and international statisticians' associations.

In Danish texts the one and explicit reference to an external text is a mention of "the Monty Hall problem" (11) by citing a website where it is explained. It is not an academic or official reference, and the website does not provide information on the author's credentials. The relevance of this remark lies in the problem itself being a classic example of how intuition and a probability-based rationality lead to opposite courses of action. This statement is the core of the research agenda of behavioural economics and rational choice (Kahneman, 2011), and it has been proposed in statistics education research as a resource to address probabilistic misconceptions (Batanero, Fernandes & Contreras, 2009; Elicer & Carrasco, 2017).

Discussion: discursive practices

Within Fairclough's (1992) three-dimensional model, texts are part of social events, and they are signs of discursive practices. Discourse mediates the relationships between texts and social structures. In the following, I point out and problematise representations of social actors and legitimation strategies evidenced by the preceding elements of the textual analysis.

Representations of social actors

As a whole, and given the genre, texts include three generic and classified social actors: students; teachers; and politicians. The first two are, of course, the case of upper secondary school students and teachers. In different texts referring to decisions, these actors are also excluded via suppression and backgrounding (Fairclough, 2003, p. 145).

Teachers are suppressed from the current (CL315, CL415) to the future Chilean curriculum (4M17). In 2 and 4, teachers' actions are the ones triggering students' decision processes in a certain way, whilst in 5

this is engaged through students' collaborative work. In hand with this suppression of teachers, students' presence changes from passive (as affected by teachers' actions) to active. Moreover, a key shift in formulation is evidenced by the fact that students' (or future citizens') decisions are present not only in declarations of purpose (1, 5), didactical orientations (2) and suggested activities (4), but as part of the expected learning outcomes. In particular, students are expected to be capable of "making decisions" (6) and "solving problems [...] for decision making".

The shift in Chilean texts is coherent with the Danish case. Students' ability to make decisions (8, 9) is a result of the "work on mathematical material" (8), leaving teachers' participation implicit.

The collocation of decisions to their contexts in Danish texts indicates a scope that Chilean texts do not hold, making students able to decide in "social" (8), "everyday" (9), "professional and study" (8, 9) contexts. It must be noted that these texts come from the generic mathematics programme, as opposed to specific "data and chance" educational units. In fact, Chilean texts 6 and 7 reflect an alteration that is coherent with another broader reference to "decision" in the diagnosis chapter of the same document: "They demand protagonism in decision making and aspire to contribute to solve problems in the world they live in, such as poverty eradication, climate change and sustainable development" (CL3417, p. 13).

The wish for future citizens' participation in decisions under uncertainty goes in hand with Beck's (2000) observation about the role of experts in the risk society. Experts point out risks and the people decide to take action in a complex political process. But "how one acts in this situation is no longer something that can be decided by experts" (p. 217). It can be argued that as much as both countries push for an increasingly active role of students in deliberation as part of the purposes of school mathematics, the specificity of probability and statistics has not yet been addressed.

Legitimation

Making decisions appears as a way of justifying the teaching and learning of statistics in the curricula. The texts fulfil the purpose of not only saying what and how to teach and learn, but also why and for what. Fairclough (2003, p. 219) claims that much of the legitimation of a social order –such as the inclusion of particular knowledge in the official curriculum – is textual. In particular, the texts show three of the legitimation strategies identified by Van Leeuwen (2007), namely through mythopoesis, rationalisation, and authorisation.

Mythopoesis or legitimation conveyed through narrative is evidenced by the collocation of "decision" with "grounded" (3, 4 and 5), and similar accompanying adverbial forms such as "with evidence" (2) and "with information" (6) in Chilean texts. This strategy is stable through the texts and makes the case for justifying probability and statistics, since not just any kind of decision is included, but only those rooted in data and mathematical rationality. It allows a steady association between such school subjects and rational choice. This is not the case for Danish texts, where decisions are instead collocated with a variety of contexts, "political decisions" being the only ones said to be made using "statistics based on data" (10).

Merely associating probability and statistics to rational decisions is not enough. It is still necessary to justify the teaching. Aforementioned deontic modalities are evidence of legitimation through rationalisation, which refers to utility. Within the current Chilean curriculum (CL315, CL415) teachers' promotions are "fundamental" (2) and "important" (4) for students to make grounded decisions. The upcoming proposal (CL3417) provides a similar rationalisation in the form of "in order to achieve the latter" and "to make grounded decisions" (5), but this time it is mediated by the students' action, namely their collaborative work in mathematical modelling situations (5). In a way, this change represents another shift in the agency from teachers to students in the teaching?learning process. In a similar but implicit manner, modalities in Danish texts reflect that part of the purpose of the "work on mathematics" (8, 9) is to lead the student to make decisions, and that "it is important to learn statistics" (11) to address choices obscured by intuition.

As already suggested by intertextual elements, traces of an authorisation strategy are found in texts from the current Chilean curriculum (CL315, CL415), through the inclusion, without further definition, of "random thinking" and "statistical literacy", and of the Monty Hall problem in the Danish case. I highlight the fact that the authorisation strategy is not found in the Chilean upcoming curricular framework (CL3417). This shift resonates with the bottom-up nature of the latter document, where civil society plays a bigger role in the justification of curriculum.

A difference worth noting is that, while Chilean curricula do not make explicit reference to statisticians' recommendations (Araneda et al., 2011), the Danish Ministry of Education actually endorses and publishes experts' impressions, and so they are taken as a source in this article (DKSG18). Moreover, the collocation of "political decisions" (10) as a case for the relevance of learning statistics defines a clear call for authority; if "statistics based on data is increasingly being used" (10) by

political authorities, students and future citizens ought to participate in statistical deliberation.

Concluding remarks

My overall intention is to investigate discourses of decision-making within official curricular documents regarding probability and statistics as school mathematics content. The goal is to exemplify possible links between mathematics education and deliberative democracy, a concern shared by the Nordic and Latin American communities.

Critical discourse analysis allows me to see texts as part of social structures mediated by discursive practices, which give meaning to what is in the texts, operationalising the search for meaning in a pragmatic and systematic way, in order to address two research questions.

The first research question intends to reveal a diversity of discourses attached to notions of decision-making as expressed in Chilean and Danish upper secondary school curriculum. It is possible to evidence strategies of positioning social actors and legitimation. Decisions position actors depending on who is making them and students are encouraged to participate in such processes either by teachers' actions or by their own work in school mathematics. Present legitimation strategies show that official study programmes not only describe what is to be taught, but also for what purposes. Decision-making appears as a key element of such legitimation, by establishing a narrative of probability and statistics linked to grounded, evidence-based and rational choice, and the teaching of those subjects as necessary and relevant. Legitimation appeals to political authorities, professional statisticians and statistics education researchers' voices.

The second research question refers to contrast, and though curricular texts selected from both countries reveal similar discursive practices, they perform them differently. The analysis illustrates signs of change towards a justification that relies less on professional statisticians and educational researchers as authorities in Chile, and more on students' own learning activities as decision-makers. This shift is part of the wish for a bigger role in decision processes that go beyond the disciplinary scope. In the Danish case, students' active role is already stressed in the curriculum.

Chilean curricular excerpts reveal a persistent legitimation of the learning of probability and statistics conveyed through narratives of rational decision-making. Instead, Danish texts tend to attach decisions to a diversity of contexts, from the general political and social, to explicit mentions of games and school subject matter. There is a common appeal

to statisticians and statistics education researchers within the psychological and economic theories of rational choice, the Danish case being more explicit than the Chilean. Despite the distinctions, a key finding in this respect is that both countries exemplify the purpose of mathematics education to enable students to participate in societal and political decision-making processes, but fail to specify the relevance of probability and statistics in such contexts, relegating their scope to gambling games and other individual rational choices.

As opposed to Skovsmose and Valero's (2002) notion of *powerful mathematical ideas*, probability and statistics are perceived as relevant for being *mathematical ideas of the powerful*. It is said that political authorities and certain professionals make use of this knowledge to make socially relevant decisions, but it is not clear if and how it empowers them. Moreover, given that most students will certainly not be in those positions of power, the empowerment of all citizens through probability and statistics is yet to be thoroughly discussed.

Perspectives

I would like to finish by offering some perspectives for future research in probability and statistics education, beyond the narrow scope of the analysis above. One of the limitations of this study is that I attempt to let the texts provide their meaning, drawing upon general discursive practices, namely the positioning of social actors and legitimation. Future curricular analysis can take these strategies as a point of departure to investigate whether curricular documents illustrate notions that are of specific interest.

Social actors are portrayed in different roles, and discourses of critical citizenship enlightened by probabilistic and statistical ideas can be explored through the degree of agency given to students. In fact, the distinction between consumers and producers of data is a key issue in the problematisation of the critical lens of statistical literacy (Weiland, 2017). In investigating resources used by student teachers, Monteiro and Ainley (2007) provide a framework defining reading settings (data consumers) as seen by Gal (2002), enquiry settings (data producers), and pedagogic settings as a third category. The encouragement of agency of future citizens would be incoherent if enquiry settings are left aside in the curriculum.

Legitimation strategies include the discourse of associating statistics education with rational decisions in a variety of contexts; study, professional, social and political. As stated in the introduction, the psychological research agenda has inspired research into individually relevant decisions under uncertainty. One can adjust the lens in investigating the

role of context in curricula, as it has been investigated in experimental settings with student teachers (e.g. Pratt et al., 2011), by broadening the scope of decisions' consequences and students' participation.

The source CL3417 was used for the analysis on this paper in its version up to May 2019. However, by the publication date of this article, it has been updated by the Chilean Ministry of Education, from a document for public consultation to an official curricular framework. Formulations of texts 6 and 7 have been altered, whereby students are not expected to "make" decisions anymore, but to "ground" or "justify" them. This shift is of utmost relevance for future analysis.

Finally, it would be interesting to develop further research into classroom settings where decisions are made and turned into actions. The project "Our community" (Skovsmose, 1998), in which students had the chance of discussing with their mayor, shows the potential of school mathematics as a means for actual democratic deliberation. If social and political decisions are among the justifications of probabilistic and statistical ideas in the curriculum, evidence of such empowerment is needed to make that connection more transparent.

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Notes

1 Not to be confused with middle school, as in between primary and secondary school. In Chile, *enseñanza media* (middle education) is the bridge between *enseñanza básica* (basic education) and *educación superior* (higher education).

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