

# Editorial

This issue of NOMAD is a special issue targeting practice-based research on mathematics teaching. When research is said to be *practice-based*, it often involves some kind of co-learning experience and design, where researchers explore from the outside while teachers explore from the inside (Jaworski, 2003), both aiming at improving practice. The term practice-based may be used in different ways, including a variety of research approaches. In order to understand what might be involved in a practice-based approach to research on mathematics teaching, we first need to elaborate on the core concepts of *practice* and *teaching*.

The word *practice* has been used in different ways, and Lampert (2010) identifies four common usages. First, practice may signify something that is practical and not theoretical. In this sense, practice refers to people's actions rather than their thinking or knowledge. Lampert links this dualism between theory and practice to the Cartesian split between body and mind. In a second usage, teaching is described as a collection of practices. This is common in the core practices literature (e.g. Grossman, 2018), but it has also been used in literature that emphasizes best teaching practices. Third, the word practice has also been used as a verb, to emphasize the need to practice in preparation for teaching. A recent example is the emphasis on rehearsals in practice-based teacher education (e.g. Lampert et al., 2013). A fourth way in which the word practice has been used is in the sense of professional practice. Lawyers or medical doctors have their practice, and teaching practice has been described in a similar sense with reference to everything that is involved in the teaching profession.

Whereas *practice* is used in at least four different ways, the research literature appears to consider *teaching* mainly in two different ways. First, and arguably most common, is to consider teaching as an activity or something teachers *do* to stimulate students' learning. An example is Gage's (1978, p. 14) definition of teaching as "any activity on the part of one person intended to facilitate learning on the part of another". A second way that teaching has been considered in the research literature relates to the fourth usage of the word practice, where teaching is considered as a professional practice, or as *work*. For instance, Ball and Forzani (2009) describe teaching as work that is constituted by several tasks of teaching that teachers are faced with. Similarly, Cohen (2011) describes the work of teaching by identifying predicaments that are involved, and Lampert (2001) unpacks the problems of teaching.

With these distinctions in mind, practice-based approaches to research on mathematics teaching tend to investigate the work that is to be done and the problems that are entailed in this work of teaching mathematics. However, attending to problems of teaching may not be enough to develop practice. It is also necessary to figure out how the knowledge gained from such studies can be shared among teachers outside the participating schools (cf. Ball et al., 2014; Enthoven & de Bruijn, 2010). Morris and Hiebert (2011) suggest that practice-based research can stimulate the development of a knowledge base for teachers – a kind of public knowledge that could also be useful for teachers who did not participate in a study. In this special issue, we have had the intention of bringing forward state-of-the-art practice-based studies of mathematics teaching and thereby contributing to the development of the professional knowledge base that Morris and Hiebert called for.

It has been argued that improving the impact of educational research on practice requires closer attention to the instructional problems that teachers want to solve (Cai et al., 2017; Cai et al., 2017a). Since Stigler and Hiebert (1999) introduced lesson study to the Western world, this form of practice-based collaborative research with teachers, carried out in classrooms, has developed into a research field and gained increasing attention throughout the world (Cai et al., 2017b). A reason to encourage collaboration between researchers and teachers is that without the involvement of practitioners in knowledge development and theory building, it is difficult to gain clear insight into potential curriculum implementation problems and to generate measures to reduce these problems. New interventions – regardless of how imaginative their design might be, or how theoretically sound they are – require continuous reflection on implementation issues to improve their fitness for survival in real-life contexts (Van den Akker, 2010). In addition, Jaworski (2003) suggests that collaborative inquiry bridging the teacher-researcher, or school-university, divide is enabling growth of both individual teachers and researchers, as well as the wider educational community.

## Nordic perspectives

Because of the variety of questions posed in this area of research, practice-based research on teaching involves a diversity of approaches. For instance, action research, lesson studies, design-based research, and other professional learning communities, are used to address the problems that teachers face in the classroom. It becomes evident, however, that practice-based research cannot be restricted to simply improving teaching as a practical activity; there are theoretical gains to achieve in this kind

of research as well, and such gains can be observed in the large body of research that is described as practice-based. In the Nordic countries, lesson studies and learning studies have been used to generate knowledge about the relation between teachers' instruction and students' learning, and action research is widely used to investigate and theorize teachers' questions that derive from their own classroom practice. When surveying articles published in NOMAD during a five-year period (2016–2020), we notice that research on mathematics teaching covers a broad spectrum of content, methodologies and study designs. We find textbook analyses, curriculum analyses, and descriptions of teachers', students' and student teachers' experiences of different educational phenomena. Studies are conducted both in close proximity to practice (e.g. classroom observations), and at a greater distance (reviews of literature and historical text analyses). Narrowing down the focus to what can be labeled as practice-based research on mathematics teaching, we find studies with an overall aim of developing practices for learning, either through developing methods or professional competence. These studies do not focus solely on developing teachers' knowledge, but implications for students' learning are also discussed.

Several of the Nordic studies foreground the collaboration between researchers and teachers in investigating various features of mathematics teaching. Collaboration is described as essential in contributing to the development of teaching methods and theoretical principles, and deepening the common knowledge of how to support students' learning of mathematics. Thus, in the context of NOMAD, practice-based research is not only research *on* or *for* teachers, but it is a collective work *with* teachers and researchers that impacts teaching practices. During the last five years, about 100 articles have been published in NOMAD. From reviewing the abstracts of these articles, only about a dozen of these articles appear to fall into the frames of practice-based research as outlined above. Clearly, several of these studies aim at implementing interventions and studying possible effects on students' learning of mathematical content (e.g. Lindenskov & Tonnesen, 2020; Svanteson Wester & Kullberg, 2020). A number of these studies are intervention studies where the conditions for students' learning are explored and revised (e.g. Eriksson et al., 2019; Fred, 2019). This indicates an interest in identifying theoretical underpinnings and principles for developing teaching practices – not only on the local level, but also for the general good and theory development (e.g. Eriksson, 2019). Some studies aim at enhancing pedagogical content knowledge in collaborative projects (e.g. Kilhamn & Røj-Lindberg, 2019), or involve teachers researching their own practice in order to better understand the processes and potential that are embedded in this practice (e.g. Eikset &

Meaney, 2018; Fauskanger, 2019; Sjöblom, 2018). Among the latter group of studies, we notice that the conducting of interventions becomes a shared object for both teaching and research (e.g. Gade, 2017). Despite the above-mentioned examples, relatively few practice-based studies have been published in NOMAD. Thus, the aspiration of this special issue is to highlight and discuss benefits, shortcomings and future endeavors within this area of research.

## The contributions of the special issue

This special issue targets research characterized by practice-based approaches to the study of mathematics teaching, that is, research that searches for direct and immediate knowledge from and for the practice phenomenon that is the object of inquiry. The eight articles in this special issue represent three themes: *development of teaching, what teachers and pre-service teachers learn*, and *studies on practice-based research*. Most of the articles are related to more than just one of these themes.

### *Development of teaching*

Two articles focus on the development of teaching through a cyclic process of planning and revising lessons in a lesson study or learning study. The first article focuses on the opportunities provided for student learning. How students can become engaged in "theoretical work" when working on algebraic tasks is shown in Eriksson et al.'s article *Tasks, tools, and mediated actions – promoting collective theoretical work on algebraic expression*. The teaching intervention was used to explore the teaching of tasks that aim at engaging students (Grade 7) in *theoretical work* in algebra. The teachers used ideas from *learning activity* to design and enact the activities in the classroom. Eriksson et al. show that tasks with built-in contradictions in the representations of algebraic expressions engage students in theoretical work that promotes understanding of the algebraic expressions. The second article, by Tyskerud, *Utvikling av matematikkundervisning – en kommognitiv analyse av rutiner i klasserommet*, shows how a group of teachers develop their teaching, when involved in three consecutive lesson studies, towards more student-centered teaching based on "explorative" tasks, in contrast to more traditional teaching with "ritual-based" tasks. In the article, a commognitive perspective, in which development of teaching is defined as a change in routines, is used for analysis. Changes in the teachers' routines during the course of the lesson studies in the desired direction are identified, but the author acknowledges that the change of routines may not be persistent.

### *What teachers and pre-service teachers learn*

Four articles concern teachers' or pre-service teachers' learning, in which co-planning is a central part of the intervention that the teachers are participating in. In the article by Fauskanger and Bjuland, *Opportunities to learn ambitious mathematics teaching from co-planning instruction*, "dialogue moves" during co-planning sessions are analyzed. They show how the teachers get the opportunity to develop skills in predicting student responses, representing student responses, and meeting the goal for the lesson when reasoning with colleagues in collaborative practice. Björklund and Ekdahl, in their article *Learning to teach mathematics in pre-school through a theory-driven intervention*, analyze the changes in one pre-school teacher's teaching after having been part of a theory-driven intervention study about teaching number and early arithmetic skills. Before the intervention, the teacher conducted activities with the children that did not develop or challenge their understanding of numbers. During the intervention, the teacher became more focused on what questions to ask to promote children's understanding, and how to handle situations that could help children develop their arithmetic skills and understanding of numbers, underpinned by theoretical principles. The study thereby sheds light on how a particular intervention may affect teachers' teaching practices and ability to act in relation to children's learning.

In the article by Krog Skott et al., *New mathematics teachers' learning when participating in induction on mathematics education. A case study of two lower secondary teachers*, beginning in-service teachers' development is in focus. Two newly qualified teachers participated in an induction program with mentors and researchers in which a problem from practice was addressed through a collaborative effort that resembled lesson study. Social practice theory and patterns of participation served as theoretical lenses in the analysis of the significance of the program, which showed two contrasting developments for the participating teachers. Similarly, pre-service teachers' learning from learning studies in a teacher education course is shown by Mårtensson and Ekdahl in their article *Variation theory and teaching experiences as tools to generate knowledge about teaching and learning mathematics – the case of pre-service teachers*. They show how pre-service teachers change the tasks they use with students after being involved in a learning study process using variation theory. They identify five different ways in which the pre-service teachers improve their tasks. The changes made are seen as being the result of knowledge that was generated during the learning study, where theory is connected to the practice of teaching.

### *Studies on practice-based research*

Two of the articles address issues of quality in practice-based research on teaching. Säfström et al., in their article *Initiating teacher-researcher collaboration to support students' mathematical problem-solving*, report on the interplay between symmetry and complementarity in research that involves collaboration between researchers and teachers. For instance, they show how initial asymmetric and non-complementary approaches can develop into symmetric and complementary approaches in the teacher-researcher collaboration, and how teachers and researchers enhanced their understanding of one another's practices over time. In their article, *Teachers' participation in practice-based research – a methodological retrospect*, Palmér and van Bommel discuss quality criteria in educational design research, presenting three different ways of collaborating with teachers in practice-based research about teaching problem-solving in the Swedish preschool class (6-year-old children). They highlight the tension between ensuring the external validity of a study and enabling internal validity and improvement of practice. When teachers get more involved in carrying out the research, certain quality criteria become more prominent; this is not the case when teachers are less involved. Their findings might inform research carried out in collaboration with teachers – in other words, all of the studies included in this special issue.

### Conclusions and ways forward

In her commentary on this special issue, Venkat identifies a developmental turn in the interpretation, characterization, theorization, and empirical orchestration of practice-based research – based on her own observations of the field over 20 years. Amidst this turn, practice-based studies have maintained a focus on understanding mathematics teaching as it is carried out in classrooms. Venkat observes how the articles in this special issue not only describe the dynamic interactions of classroom practice; they also discuss how these practices can be developed. Practice-based research on mathematics teaching might thus follow a natural trajectory of development, where understanding the nature of the field and its practices is a prerequisite for persistent change. Although contributing to change is a main motivation for practice-based research, change can only be successfully implemented if it is adapted to the fueling conditions of the target practice. In many of the studies in this issue, teachers are included in the process of formulating research questions and shaping the methodological approaches. In this sense, teachers are given agency. This agency might enable them to develop practices that are suitable for – and sensitive to – particular features of their own professional context,

as opposed to aiming for high fidelity in implementing pre-designed programs (Van den Akker, 2010). Ideally, then, questions that are relevant for teaching can be identified, and teachers can benefit from theoretical constructs and scientifically grounded principles that are developed in educational research. On the other hand, this approach to the study and development of practice introduces new demands on design and methodology. Collaboration between teachers and researchers can never be taken for granted; it is a complex process that involves navigating questions about the nature of educational research aimed at developing teaching practices – including questions about power and justice.

Careful consideration of the past and present can inform the future development of a field of research – like that of practice-based research on mathematics teaching. This special issue in *NOMAD* represents such an opportunity. Learning from past and present practice-based studies not only contributes to our understanding of mathematics teaching and what goes on in the mathematics classroom, but it can also contribute to the development of practice. Teachers' involvement is crucial to such development, not only for implementing designs and testing theoretical principles, but also to shape questions and designs in order to make them relevant and operational for practice. Here, Hoover and Ball direct attention to a critical issue: many questions that are raised in studies that claim to be practice-based are not questions that teachers themselves are likely to raise. We are thus faced with an inherent dilemma of maintaining the integrity of teaching as a practice, and at the same time conducting practice-based research that contributes to improving teaching. For instance, teaching is conceptualized in different ways across research articles, and these different conceptualizations inform both implementation and interpretation of successful mathematics teaching. This complexity becomes evident in the special issue, and is discussed in the commentary articles (Hoover & Ball, 2021; Venkat, 2021). In order to understand the phenomenon of "practice-based research", it is necessary to attend to the interactions between micro- and macro-perspectives, as well as to the contextual, historical, and individual aspects involved – although a single study may focus on one or several of these. Thus, we do not expect this special issue to lead to consensus about what practice-based research in mathematics teaching is, or how it can best inform teaching practice. We do, however, hope that the special issue draws attention to questions about what it means that a study is practice-based. We believe that some consensus is necessary for the field to be able to move forward and call attention to questions of what we mean by core terms like "practice-based", "teaching", and "research" – for instance by horizontal and vertical comparisons of core questions, aims, and approaches across

studies – which can provide a productive starting point for a constructive dialogue and a way forward.

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