

Editorial

Mathematics textbooks and other learning resources have come into focus again in times when we see an explosion of easily attainable material for teachers and pupils on the internet. The quality and the possible learning potential of such resources are frequently debated. In this thematic issue of NOMAD we present papers written by members in the *Network for research on mathematics textbooks in the Nordic and Baltic countries*. The network was presented in NOMAD four years ago (Grevholm, 2011) and has been alive since 2006, when it was born in a workshop organized by the *Nordic graduate school in mathematics education* (Grevholm, 2006). Thus, the mathematics textbook, understood in the widest possible sense, is the focus of all the papers. It has been underlined by TIMSS and other international comparative studies that in the Nordic and Baltic countries, the use of textbooks seems to be even more intense than in other parts of the world. Over the years, several of the members in the network have contributed with papers on textbooks to NOMAD (Bjarnadóttir, Lepik & Christiansen, 2013; Jakobsson-Åhl, 2008; Johansson, 2006; Kongelf, 2011; Randahl & Grevholm, 2010; Österholm, 2008). In this thematic issue, we are able to present several papers based on research studies that have been carried out over the years 2011–2015, partly as a consequence of financial support from NordForsk to the network. The papers reflect the fact that members of the network are persons working on all possible research levels; master students, doctoral students, senior researchers and groups of researchers. The five Nordic and three Baltic countries have all been involved in the network. Additionally, the network has since the beginning created international links and collaborated with international scholars. This is reflected in two of the papers by Dutch and German contributors, respectively. Guest editor for this thematic issue is professor emerita Barbro Grevholm, University of Agder and Kristianstad University.

In this issue

The thematic issue starts with two historical papers that form a background to the papers on current books, followed by three papers concerning different aspects of the content of mathematics textbooks. After that, we find four papers, dealing with pupils' and teachers' views on and use of the books and resources more generally. One of the following

papers is on students at tertiary level and their view of and use of books in linear algebra and finally an overview paper discusses issues on methodology.

The first historical paper is written by Bjarnadóttir and deals with a mathematics textbook that was introduced in Iceland in the 1960s, and has the title *Tölur og mengi – Numbers and sets. A New Math textbook in Iceland in the 1960s*. Bjarnadóttir carried out her doctoral study on historical developments of mathematics education in Iceland and has herself published textbooks, so she is well familiar with the area. She points out that only few mathematics textbooks were developed in Iceland during 1930–1966. Iceland, as most other countries in Europe, was reached by an international school-mathematics reform movement in the mid 1960s – the New Math. The learning resources introduced in the beginning of the New Math movement and lasting until the 1970s, were mainly imported from neighbouring countries. There was one exception. The author of the textbook *Tölur og mengi* [Numbers and sets] was the main active person in the introduction of the New Math. Bjarnadóttir describes how this author and his collaborator introduced the new ideas and the methods they used. Among their tools, Bjarnadóttir places the textbook, and indicates how it influenced other mathematics textbooks for adolescents.

Christiansen's contribution is an analysis of two 19th century Norwegian geometry books, and he discusses the reactions they caused. Holmboe, who was professor in mathematics, wrote several textbooks on mathematics. His presentation of geometry can be seen as traditional and is in line with Euclidean ideas. Hansteen, who was professor in applied mathematics, wrote a textbook on geometry, where he wanted to challenge the Euclidean geometry. Christiansen analyses the different approaches to some basic definitions in the two geometry textbooks mentioned, and the debate that followed. The main focus is on the presentation of basic concepts in geometry. In the contemporary society we rarely experience mathematics textbooks mentioned in the public debate and thus this case is rather unique.

In the next section, the focus is on the content of the textbooks, e.g. decimal numbers and introduction of algebra. These articles illustrate the numerous choices of content areas that are possible when studying textbooks. The section starts with a paper exploring Dutch mathematics textbooks for the early school years.

The Realistic Mathematics Education (RME) reform from around 1970 in the Netherlands is well known internationally. According to the authors of the Dutch paper, van Zanten and van den Heuvel-Panhuizen, most contemporary textbook series for primary school mathematics are written in the spirit of this reform. Van Zanten and van den Heuvel-Panhuizen

investigate what the reform leads to when it comes to the approach to decimal numbers. They analyse how this specific content area is presented in a contemporary RME-oriented textbook series and in two pre-RME textbook series. They find that most of the RME characteristics included in the framework they use for the analysis are present in the contemporary RME-oriented textbook. Additionally they find that several early traces of RME characteristics were already present in the two older textbooks that date from before the RME reform started. Thus, the reform was obviously well grounded in RME-ideas long before it became visible.

In the paper by Kongelf, the object of study is the introduction of algebra in mathematics textbooks for lower secondary school in Norway. In his paper, the findings from an analysis of the introduction chapter of algebra in six different textbook-series are presented. The introduction to letters as symbols for variable phenomena varies depending on age group, amount and context. He characterizes critical aspects of the chapters using an inductive, qualitative content analysis. The main findings are that the concept of variable is not presented clearly enough and the authors hardly use the opportunities to build further on arithmetic. Additionally he points to erroneous formulations, illustrations and mathematical reasoning, which can create foundations for misconceptions. He also illustrates how the authors could have changed an introduction of variables to meet the needs of pupils in better ways. In doing so he exemplifies opportunities to improve textbooks.

In her study, Halldórsdóttir compares characteristics of three Icelandic mathematics textbooks for grade 8, which have been used for more than 25 years. She focuses on aspects as structure, content, perspectives and mathematical competences of the textbooks. The official curricula enlighten her comparisons. Her aim is to explore the implementation of policy through the focal point of textbooks. The textbooks differ considerably with respect to the characteristics studied. She shows that the objectives of mathematics formulated in the national curriculum are reflected in the most recent textbook, but in another widely used textbook with foreign origin they differ in important ways from the curriculum. One important conclusion is that the role of the mathematics textbooks, as an interpretation of policy and the influence of them on classroom instruction, demands that teachers select their textbooks wisely. For doing so, teachers need good knowledge in evaluation of textbooks.

The next three papers form a section of comparative studies on teachers' and pupils' views upon and use of textbooks. The first comparative paper by Lepik, Grevholm and Viholainen builds on a survey of more than 400 teachers in Estonia, Finland and Norway. Mathematics

teachers' self-reported practices of textbook use were investigated. The questions asked were for example about approaches to the use of textbooks and to what extent teachers rely on textbooks in planning and preparing their lessons. A search was done for what kinds of patterns of use exist in teachers' practice when using textbooks in mathematics lessons. The findings indicate that in Estonia and Finland teachers have similar attitudes towards textbooks. The textbook has a strong effect on their didactical choices. The Norwegian teachers, however, say that they are less dependent on the textbook. In Finland the textbook is the main source for exercises while in Estonia and Norway teachers use other resources more often. Almost every second teacher uses the textbook only as an exercise book. Thus the authors claim that the full potential of the textbook is not used. Pupils rarely read theory in the book. As a result the pupils do not get the opportunity to fully exploit the book as a multifaceted learning resource.

Viholainen, Partanen, Piironen, Asikainen and Hirvonen have studied both teachers' and pupils' use of textbooks. Textbooks are tools for students, but they can also be seen as mediators between the intended curriculum and teachers' practices. The question is how students and teachers at Finnish upper secondary schools perceive and describe their use of the theory, examples, and exercises in the mathematics textbooks. Data was collected from 71 students in a short survey. Interviews were carried out with six of the students and three of their teachers. The results indicate that the theory, examples, and exercises in the textbooks have a strong influence on teachers' work, but for the students the textbooks primarily represent a source of exercises. Again, as in the study by Lepik et al. mentioned above, it is found that students don't take full advantage of textbooks.

Comparing teacher guides in Iceland and Sweden is the aim of the paper by Ahl, Koljonen, Gunnarsdóttir and Pálsdóttir. They notice that curriculum resources have a potential to support teachers' design of lessons according to research findings. Thus the aim is to investigate how Icelandic and Swedish teachers interact with and use teacher guides as a tool for planning. The authors interviewed five teachers, teaching lower grade levels, in each country about how they use teacher guides. The interviews focused on the different features of the guides and revealed what the teachers were searching for in the guides and for what purpose. An analytical tool was used to investigate the teacher guides.. The results show that in the case of the studied teachers, the ones using educative teacher guides were more likely to use a more varied range of lesson design reflections compared to teachers using more traditional teacher guides.

Grave and Pepin present outcomes from the master thesis of Grave from 2013. Four primary school teachers were studied with focus on how they use resources in their mathematics teaching. Five different usage categories are introduced and exemplified from the teachers' lessons. The categories presented and discussed are 1) resources to manage the teaching objectives; 2) resources to "inspire" teaching; 3) resources for student work; 4) resources to adapt the teaching to individual students' needs (differentiation); and 5) resources to organize the teaching. Although two of the teachers use the same textbooks it becomes obvious that they use it differently. Especially teachers act differently in order to differentiate learning opportunities and meet students' individual needs.

One study deals with textbooks at tertiary level. Rensaa and Grevholm write about engineering students' views on a textbook in linear algebra. One part of their paper reports on a content analysis of parts of a specific linear algebra book used by the students in the study. A theoretical model building on many factors influencing textbooks, factors in or properties of the textbook as such and factors influenced by the textbook is basis for the analysis. The results indicate that characteristics like motivating examples and visual design of text and pictures appeal to the engineering students. Another part of the paper is to present an analysis of engineering students' views on the specific textbook. The textbook is appreciated by the students and they point to the examples in the book as the most valued part. Characteristic for the textbook is to present theory in small portions, often in examples using specific values to illustrate theoretical arguments. Such a design seems to be a success factor.

The final paper, by Rezat and Sträßler, gives an overview of methodological issues on textbook research. The authors try to classify earlier research mainly from the Nordic and Baltic countries according to what they call the socio-didactical tetrahedron. They use some of their earlier papers on the didactical tetrahedron, and detail the discussion more than they have done in earlier versions. The authors start from the assumption that research on mathematics textbooks can be divided into three areas, namely research that focuses on a) the influences on textbooks, b) the mathematics textbook itself or c) the use of mathematics textbooks and its impact. By exemplifying with research done in the Nordic and Baltic countries, the paper presents an overview of methods used in all three areas. The findings from an analysis of 24 papers are that research related to area a) and b) usually builds on surveys or content analysis of textbooks while research in area c) has to struggle with the methodological challenges of how to gather valid data and of being able to generalize. They discuss possible ways to handle such challenges.

If we try to classify the papers in this volume according to the three areas above we find two papers dealing with the influences on textbooks (Bjarnadóttir, Christiansen), four papers on the textbook itself (van Zanten & van den Heuvel-Panhuizen, Kongelf, Halldorsdottir, partly Rensaa & Grevholm), and five papers on the use of mathematics textbooks and their impact (Lepik, Grevholm & Viholainen, Viholainen et al., Ahl et al., Grave & Pepin, partly Rensaa & Grevholm). The final paper falls outside all of the three areas.

What is waiting for us in the future?

Given the topics in the papers in this volume it is possible to see that there is a need for many more studies on textbooks. In order not to end up in too fragmented findings, when seen together, it would have been valuable to form a stricter research agenda and to systematize the studies and results. In the summer of 2014, an international conference on mathematics textbooks research and development was organized at the University of Southampton (Jones, Bokhove, Howson & Fan, 2014). No less than 11 members of the Nordic Baltic network took part and many presented their studies. A second conference of the same kind will take place in Brazil in 2017 and a hope could be that one task for the conference would be to map research results so far and create an agenda for future research in the area of textbooks. A tentative research agenda for studies on textbooks was presented in a discussion group at PME in Kiel in 2013. In short it was expressed like this (Grevholm, Rezat & Fan, 2015).

Research agenda:

- The development of e-textbooks and research on these in mutual relations of all kinds.
- Development of e-textbooks that really take advantage of the possibilities of ICT and studies of their effect.
- Continued comparative studies and studies based on content analysis from different aspects.
- More research attention to the role of textbook developers, policy makers, curriculum specialist, teacher educators and evaluators in the process of textbook development, adoption and use, and how they interact with each other. Development should be based on research but the development processes should also be researched.
- Future research on mathematics textbooks for a more integrated view on reading and learning from textbooks.

There has been discussions about student teachers' lack of opportunities to learn about textbook analysis and about frameworks or criteria for evaluating textbooks. Teacher education and development need to emphasize work on all aspects of textbooks. The fourth point in the suggested agenda above points to urgent need to find ways for different kinds of competencies to collaborate in the creation and design of textbooks and additional resources. A rather large on-going project led by Susanne Prediger in Germany is following that idea.

Let us imagine all the pupils using mathematics textbooks. And imagine just a small improvement of these books to make them become a more valuable tool for the learning of mathematics. A small improvement multiplied with the number of all users would make a huge change in the learning situation. This thought indicates how important research and development of textbooks are and we have to be aware of the large resources spent on textbooks and other learning material, not least in developing countries. All actions that can raise the quality are important. In the current situation where many schools in Europe and elsewhere face the need to offer equity in learning opportunities in spite of a multicultural classroom the need for high quality textbooks is growing. This thematic issue of *NOMAD* can perhaps inspire to some future important studies on mathematics textbooks and complementary resources.

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The Editors

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