The didactic divide and the education of teachers of mathematics in Sweden

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On a background of tradition versus renewal, a discussion on recent teacher education reforms in Sweden is given. A balance of different aspects of content and a need for research are seen as critical for the formation of mathematics teacher education. A conception of a 'didactic divide' between disciplinary and pedagogical knowledge is used as an analytic tool to describe the rationale behind the design of the reforms. Empirical data from student teachers enrolled in the new teacher education programme highlight how the structure and content of the programme contribute to overcoming the divide.

There has been during the past half century an increasing societal interest and concern for the teaching of science and mathematics in school (e.g. Comiti & Ball, 1996). In debates about schooling, and especially mathematics teaching, it is often claimed that nothing changes. The traditions are strong and teachers do as their own teachers did. Evident facts disprove such claims. In many countries curriculum reforms have followed the "trends" in psychological and educational research, from behavioural to cognitive, constructivist, sociocultural and communicational approaches (Sierpinska, 1996; Sierpinska & Lerman, 1996). The view of the connection between subject matter content and pedagogy has, during this evolution, changed considerably. We can observe a tension between a long term educational change and a short term resistance to change of teaching practice.

These developments put new questions in focus for the preparation of teachers. That ideas of a new kind of integration between disciplinary

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and pedagogical knowledge have emerged in the debate, is witnessed by the increased use of terms such as "teacher knowledge" and "didactical knowledge". The traditional divide between the preparation of mathematics teachers for primary and secondary school is beginning to be bridged in some recent teacher education programmes, however creating a sometimes hostile debate between different actors. In line with the development in education there is an inherent conflict deeply embedded in the formation of new teachers: tradition versus renewal.

In this paper we put forward the thesis that the development of the formal education of teachers in Sweden has been strongly influenced by deliberate attempts to handle the problem of a "didactic divide" between disciplinary and pedagogical knowledge. Our focus will be on mathematics, which along with Swedish and English, is one of the core subjects within general education in Sweden. Even though the development has been towards a more uniform conception of what kind of competencies future teachers will need, efforts to bridge the divide cannot be isolated from struggles to find appropriate forms and organisations of the practice of teacher education, nor from problems of recruitment of students. To support our claims, our discussion will lean on a description of the development of teacher education in Sweden, research literature, experience from local practice, and new empirical data from student teachers enrolled in a reformed programme.

Teacher knowledge in mathematics

Traditionally different kinds of knowledge needed to function successfully as a teacher of mathematics have been identified, i.e. subject matter knowledge, knowledge of the learners and of learning theory, knowledge of teaching strategies, and knowledge of the social context of schooling (Mewborn, 2000). However, subject matter knowledge covers several subcategories such as substantive knowledge (facts, procedures, concepts, ideas and relationships; see Brown, 1992), knowledge of mathematics as a discipline, and "pedagogical content knowledge" (Shulman, 1986) or "Stoffdidaktik" (see vom Hofe, 1998, p. 329, note 4). We include the first two subcategories in the term *disciplinary knowledge*. The third subcategory encompasses for example "ways of representing and formulating the subject that make it comprehensible to others" and "understanding of what makes the learning of topics easy or difficult" (Shulman, 1986, p. 9).

Studies have shown that a teacher's level of substantive knowledge has no direct correlation to results of teaching measured by student achievement, but seem to matter in relation to student understanding (Mewborn, 2000; Kilpatrick, Swafford & Findell, 2001; recent large scale classroom

studies [1] may bring new light to these complex issues). These facts point to the important issue of what kind and level of knowledge is needed and how this knowledge is fostered in the education of teachers. Boero, Dapueto and Parenti (1996) describe three "extreme orientations" in views of mathematics teacher education: if you know mathematics you know how to teach; a good mathematics teacher must master mathematics and be acquainted with the art of teaching; teaching is a professional competence grounded in different scientific domains (mathematics, education, didactics). Different interest groups still stress the importance of these orientations differently, though research, as well as new teacher education programmes, moves in direction to favour the development of the professional competence conception.

In the Danish KOM project (Niss & Højgaard Jensen, 2002), eight such professional competencies are structured in the two categories of language (e.g. competence to reason mathematically) and tools (e.g. competence to use the mathematical formalism). In the USA, the Mathematics Learning Study Committee in a similar vein uses the term *proficiency*, and identifies five interrelated components of mathematical teaching proficiency:

- *conceptual understanding* of the core knowledge required in the practice of teaching;
- *fluency* in carrying out basic instructional routines;
- *strategic competence* in planning effective instruction and solving problems that arise during instruction;
- *adaptive reasoning* in justifying and explaining one's instructional practices and in reflecting on those practices to improve them; and a
- *productive disposition* toward mathematics, teaching, learning, and the improvement of practice.

(Kilpatrick et al., 2001, p. 380) [2]

To develop the first of these components, teacher education needs to provide opportunities for student teachers to connect the different kinds of knowledge required, something that, however, often meets organisational problems of course integration (ibid. p. 381). When such connection is lacking we say that there is a *didactic divide* between the different kinds of knowledge. Such a divide may appear at a cognitive level as well as at an organisational level in for example a teacher education practice. According to Ball & Bass (2000), "teacher education across the 20th century has

consistently been severed by a persistent divide between subject matter knowledge and pedagogy", a gap that "fragments teacher education by fragmenting teaching" (p. 85).

Different teacher education programmes can be seen as different solutions of how to balance and link the different aspects of teacher knowledge. However, also issues of organisation, finance, and competence, are some of the constraints that strongly influence the chosen solutions (e.g. Malmros, 1986). In the next section we will focus on the relation between disciplinary and pedagogical knowledge of mathematics in the education of teachers in Sweden over the last 35 years. In the term "pedagogical knowledge" used here we include Shulman's (1986) categories pedagogical content knowledge and curriculum knowledge [3], as well as knowledge of general issues in education such as learning, developmental psychology, socialisation, etc.

Teacher education in Sweden

Since mathematics as practiced in school constitutes the future work field of a mathematics student teacher, we will first give a short description of school mathematics in Sweden, before discussing the reforms of teacher education programmes [4].

School mathematics

After the first national curriculum with a common compulsory school of 9 years was introduced in 1962, reformed national curricula have appeared in 1969, 1980, and 1994 (revised in 2000), all mirroring the international movements in mathematical education (see Skolverket, 1997). The curriculum for mathematics from 2000 is goal directed with specific goals in general terms of mathematical content that pupils should have attained after school years 5 and 9, respectively. Additional goals to strive for are described in terms of competencies of reasoning and communicating with mathematics. On the affective side, the desire to learn and the importance of developing a confidence in your own capabilities are also stressed. For the upper secondary level, enrolling almost all students leaving compulsory school, the subject-based national curriculum from 1970 was replaced by a course-based curriculum in 1994 (revised in 2000), where the mathematics study starts with a common course which is compulsory for students across all programmes.

Steps to a reformed teacher education

In line with the regulatory framework for teacher education in Sweden from 1967 (Prop. 1967: 4), prospective teachers for school years 1–3 and 4–6 were from 1968 educated at special teacher training colleges in separate class teacher programmes. Courses of mathematical content were here focused directly on what was going to be taught at school. Most teacher educators at these colleges were themselves experienced teachers without a research degree in their subjects, thus opening up for a didactic divide by putting an emphasis on experience-based pedago-gical knowledge in mathematics. There was one common education programme for teachers for lower secondary and upper secondary school levels, based on disciplinary studies at university, completed by one year of practical pedagogy. Here a didactic divide emerged from an emphasis on substantial knowledge in mathematics, since the major part of the education was on subject matter content, taught by academic teachers without a professional contact with or education for mathematics as an educational task in school settings (see e.g. Malmros, 1986).

The special teacher training colleges for primary school level became organisational parts of the universities by a reform in 1977. This way, two different educational traditions were brought together – the teacher training college tradition (experience-based knowledge) and the university tradition (research-based knowledge) – creating tensions still present today: "Different knowledge traditions and ideals on what is most important for the education of teachers are this way often set up against each other, which among other things can be seen in the view of scientific knowledge and tried experience" (SOU1999:63, p. 12; our translation). The question of who is educating future teachers is also related to this tension and the didactic divide between disciplinary and pedagogical knowledge.

During the 1980s, teacher education was discussed, both professionally and politically, by the report from the teacher education committee called LUT74 (SOU1978: 86) and because of claimed low results (see e.g. Lindblad & Stukát, 1981). As a consequence of poor IEA study outcomes for Sweden (Skolöverstyrelsen, 1983), a report presented suggestions for improving mathematics teaching at all levels (Ds U 1986:5). There was a claim that teachers for primary school were not sufficiently specialised in their subjects and a wish that teachers follow their pupils over a longer time period. A reformed teacher education programme started in 1988 (Prop. 1984/85: 122), where teachers for compulsory school were trained for school years 1–7 or 4–9, either in mother tongue and social science or in mathematics and science. For mathematics the pre-requisites were raised compared to previously. The intention was to

give compulsory school its own specialised teachers, with stronger subject matter knowledge than class teachers, but at the same time strengthen the didactical focus of this knowledge by making connections between the different parts of the education (Prop. 1984/85:122, p. 11). We see this as a deliberate attempt to bridge the didactic divide between disciplinary and pedagogical knowledge.

As the reform was implemented, however, the subject matter knowledge taught for student teachers for the lower grades was much in the tradition of the previous teacher training colleges, while student teachers for the upper grades often studied traditional university courses with a focus on substantial knowledge. Strong organisational constraints and the structure of the professional competence of the teacher educators thus interfered with the purpose of the renewal.

The education for teachers for upper secondary school was much the same as before 1988, except that the year of practical pedagogy was split into two parts so that students would not have to wait 3 years to meet their subject as an object of teaching in real school situations. We interpret this as an organisational change to overcome the didactic divide between disciplinary and pedagogical knowledge present in the previous programme. The importance of the ongoing developmental work "aiming at an increased integration between subject theory, methods, and practice" (Prop. 1984/85:122, p. 26; our translation) was also stressed. However, the reformed teacher education still stayed within the tensions between experience-based and research-based knowledge, failing to develop a professional subject matter didactics (Gran, 1995; HSV, 1996; Ds 1996:16). These failures as well as recent changes in society, school, and views of learning led the government to appoint a new teacher education committee (called LUK97), instructed to propose a renewal of teacher education. In the report from the committee we can identify a deliberate attempt to bridge the didactic divide: "The committee suggests that subject matter knowledge and didactics shall not be separated" (SOU 1999:63, p. 14; our translation). The committee questions the traditional division of teacher knowledge into the separate knowledge areas subject matter, pedagogy, methods, subject matter didactics, and practice (ibid., p. 71).

Based on this report, and reactions to it, a new common structure for all teacher education was introduced (Prop. 1999/2000:135), characterised by an integration of three education areas: a *general education domain*, *directions*, and *specialisations*[5]. The "general education domain" contains knowledge for the profession of teaching common to all teachers such as learning, socialisation, and interdisciplinary studies. In a "direction" a subject or subject field the teacher intends to work with is studied. A "spe-

cialisation" aims for deepening of the direction studies or for broadening studies of other topics. Parts of the direction and of the general education domain must be school-based (*Verksamhetsförlagd utbildning*, VFU), consisting of teaching practice and other work in schools, including practice related course tasks. A diploma thesis is compulsory, where covered knowledge is to be related to the profession of teaching. The education, with a general entrance combined with special requirements for different directions, will lead to a teacher's diploma for primary school, middle/lower secondary school, or for upper secondary school, depending on depth and length of studies [6]. A mathematics teacher in Sweden will after the completed programme teach more than only mathematics.

The openness and flexibility in this design allows students to find individual combinations and for teachers at any level to be specialists in, for example, mathematics. A new kind of teacher for primary school may thus emerge, with a broader and deeper knowledge in mathematics and mathematics education. By the idea of a direction, with a regulated integration of disciplinary and pedagogical knowledge through the VFU concept, we can observe an explicit attempt to overcome the didactic divide:

> It is of great importance that the school-based part of teacher education gets a new qualitative content ... and contributes to that students to a higher degree relate subject matter knowledge to learning processes and the selection of content for teaching.
>
> (Prop. 1999/2000:135, p. 11; our translation)

The aim is to create a closer connection between the studies of the different content topics, related didactical issues, and the task of teaching the same topics in school.

An example of a new programme

The implementation of the reformed teacher education programme started in 2001 and is characterised by great local variations, due to the open formats of the design. For our discussion of the focused didactic divide in relation to the new programme, we find it helpful to refer to a practice that has grown out of the reform. When the new programme started at Linköping University, as an example, there was a long tradition to build on with a teacher training institution established in the middle of the 19th century. The aims of the present programme are stated in four main student goals: to develop practical teacher, subject matter, and critical knowledge, and personal maturity. It starts with one year of study within the general education domain, where all beginning student teachers (K–12)

study together. At the end of this year the student chooses a direction to study the whole second year, including ten weeks of VFU related to the subject(s) of the direction. The programme offers, at the department of mathematics, two different directions: *Mathematics for compulsory school*, and *Mathematics for secondary school*, aiming to give the students basic relevant subject matter knowledge for and beyond the intended school level, including mathematics didactics courses in connection to periods of VFU. After finishing the second year, some students continue their studies in mathematics with a deepening specialisation, while others go directly to their second direction or other specialisations. Finally, the diploma thesis may be completed for example on a topic in mathematics education, supervised at the department of mathematics.

There are obviously many ways to organise VFU, and implementations differ widely between directions. For the directions mentioned above there is a shorter period of VFU for mathematics in the autumn and a longer one in the spring semester, connected to theoretical and methodological studies in the didactics of mathematics. To illustrate some of the inherent potentials in this way of organising teacher education, one example of a working format used within the VFU study is given here, titled "How is a mathematical concept handled?". Based on the completed first mathematics course of the direction, in the didactics course some key mathematical concepts for school mathematics were highlighted. During the VFU period, students were asked to observe in mathematics classes how a mathematical concept was handled in educational activities with pupils. In their report, students were asked to write a synthesis on the concept, including a concept map, describe a task for pupils to work on the concept, and report on how the concept was treated in the observed teaching sessions in class. The purpose of this task was to relate reflections on the mathematical content studied to how it is being authentically taught in class. The idea of a direction thus invites a practice that explicitly aims to bridge the didactic divide between disciplinary and pedagogical knowledge.

Critical issues

Some problem areas identified for teacher education concern recruitment, pre-knowledge of students, composition and structure of the education, the quality of teacher educators' competence, and changes in expected competence of teachers (see e.g. SOU 1999:63). In the above discussion we have touched upon three tensions influencing the practice and development of teacher education, i.e. the tensions between tradition and renewal, experience-based and research-based knowledge, and

theoretical and practical components of teacher education, respectively. These tensions relate strongly to the problem areas mentioned but also have links between themselves. We can thus identify the following three critical issues in relation to the development of the structure and content of mathematics teacher education: views on what should constitute teacher knowledge, back-up research, and the creation of communities of learning. The didactic divide defined above is at the core of all these critical domains.

The didactic divide is also strongly related to the links between the university and schools, and the professional development of teachers and teacher educators, an area that is only beginning to be researched (e.g. Zaslavsky & Leikin, 1999; Tzur, 2001). The problems related to the didactic divide thus ask for collaboration across the different levels of organisations involved in teacher education (cf. Krainer, 2003).

As an example, it causes problems for a student teacher if the supervisor during school practice cannot accept the ideas that the student wants to try. Creating communities of learning, based on co-learning partnership (Jaworski, 2002), where mentors[7], student teachers and teacher educators work together within the organisational frame of a direction, may be one way to contribute to the overcoming of the didactic divide between disciplinary and pedagogical knowledge. To develop these teams is, however, a difficult process. The persons are physically apart from each other, have different working conditions and what is valued and rewarded in their work is different (Wheeler, 1986).

Research and mathematics teacher education

In Sweden few research studies have been focused on mathematics teacher education. One report on written tests investigated students' mistakes during problem solving and showed that the requirements for entering teacher education for compulsory school were set too low at the time (Lindblad, 1978). This started a debate that later led to changes making the prerequisites more demanding (Ds U 1986:5). In a similar study conducted about ten years later it is claimed that questions about teacher competence cannot be isolated to details about substantive knowledge (Ljung, 1987), a result in line with international research (e.g., Mewborn, 2000; Kilpatrick et al., 2001).

Later more interpretative case studies appeared. In a study on student teachers' understanding of mathematical modelling, Lingefjärd (2000) found that students, by a transformation of authority, become uncritical of the results they get from the computer and graphing calculator. In a longitudinal study Grevholm has focused on the conceptual development

of student teachers of mathematics (e.g. Grevholm, 1999, 2000; Hansson & Grevholm, 2003), who often have vague and poorly developed mathematics concepts. During the studies they begin to use a professional language but the first years they have difficulties to communicate mathematics verbally. Also Holmquist (2004) analyses concept images of student teachers in a longitudinal study with a focus on geometry. Views on mathematical problem solving in class (Wyndhamn, Riesbeck & Schoultz, 2000) and reasons to study mathematics at school (Bjerneby Häll, 2002) seem to develop from non-reflected and instrumental to more mature and balanced ideas in student teachers as they pass through their education, indicating that the education has an effect. In addition, overviews including research on mathematics teacher education have appeared (Mouwitz, 2001; Grevholm, 2002).

This research has some relevance to the didactic divide discussed here, though not explicitly addressing the issue. One tentative overall conclusion to draw is that the disciplinary knowledge studied during teacher education needs to become an object of reflection by the student teachers to a greater extent, in order not to be isolated, by a didactic divide, from the pedagogical considerations when used in teaching.

It is clear that the government in Sweden wants to stress the responsibility for a research based teacher education and the fact that the work has to be done inside the academic world. In the Government bill for the renewal of teacher education (Prop. 1999/2000: 135), the need for subject didactics research is directly pointed at, thus seeing the didactic divide as critical for the development of teacher education. These research areas have had great difficulties in Sweden to establish themselves compared to the situation in many other countries (Bergsten, 2002; Björkqvist, 2003). There is a vicious circle that might provide one explanation to this scarcity of national research. Since teacher educators in general have not had a research degree, there have been, as a consequence, almost no professors in teacher education to promote research.

An empirical study

In this section we report from an empirical study using data from student teachers enrolled in the new teacher education programme in Sweden. We set out to investigate how the new construct of a direction is comprehended from the students' perspective, with a focus on the didactic divide. We wanted to investigate what teacher competencies mathematics student teachers find important and how they manage to handle them. In particular, we were interested of their views on the relation between disciplinary and pedagogical knowledge in mathematics, as spontane-

ously reported from authentic teaching situations. To accomplish this, we wanted the student teachers, during their direction VFU studies, to reflect on themselves as teachers of mathematics. We found the format of writing an essay appropriate for this purpose.

During the second semester VFU period in mathematics, the student teachers had full teaching responsibility in one mathematics class for at least three weeks, supervised by a mentor, i.e. the ordinary teacher of the class. As a VFU task, students kept a reflecting diary about their teaching and videotaped one of their teaching sessions. Based on a discussion of this videotape with their mentor, and on the diary, reflections from the didactics course and literature, the student wrote, as an examination assignment, an essay called "This is how I am as a mathematics teacher". [8]

We report here on data from all 46 short essays that were submitted for the examination, 12 of which were written by student teachers for the lower and 16 for the upper grades of compulsory school, and 18 for upper secondary school. The essays covered on an average three full type written pages each. The discussion here will focus mainly on those points of views that were common across school levels.

A phenomenographic approach was used to analyse the written data. In this approach, the aim of the research is to describe the variation of qualitatively different ways of experiencing, conceptualising, or understanding phenomena in the world. With the perspective of the experiencing subject in focus, it is therefore, when analysing written data, necessary not to use pre-conceived categories. Out from a general overview of the material, expressions relevant to the focus of the research are identified and compared to explore the space of variation, described in categories of the outcome (Marton, 1993).

After initial readings we identified the following five main categories, which were chosen as a structuring tool for the further analysis of the observed variations: *importance of VFU experiences*, *ways of being*, *ways of working*, *things to improve*, and *what is important for a teacher*. During repeated readings the contributions from each individual paper within these categories were listed along with examples of quotations, sorted by each of the three school levels, thus providing a structural basis for the final analysis.

VFU experiences and ways of being

It is expressed in almost every essay that these weeks of VFU were of very high value for the professional training to become a teacher of mathematics, and were also enjoyable and inspiring. As an argument, some students note that not until now had they understood what was required to be able

to teach mathematics, and that the object of a full teaching responsibility for a three weeks period was the key motivating factor to engage in a process that turned out to be so rewarding. This commitment is visible in all essays, in the engagement, language and scope of the writings. All agree as well on the high value of the outcome of the concluding discussion with the mentor, and with the visiting teacher from the university. In particular at the secondary school level, a great value is also set to the observation and discussion of the video recordings that were made of a teaching session during the VFU weeks. One student wrote that "it makes it easier to see what I need to think of in front of the pupils". By these responses, we find that the student teachers see themselves as active participants in communities of learning.

Most of these student teachers describe themselves as relaxed and secure in the teacher's role, with a general positive attitude. Many students express a concern about the difficult balance between establishing a relaxed friendly relation in parallel to the need of owing respect as a teacher. A good contact with pupils is by some seen as a necessity to feel secure in the teacher's role. To be receptive to pupils' reactions, moods, and ways of thinking, is another part of a teacher's repertoire that most students pay great attention to in the essays, often with an explicit wish to develop it further. However, there is no tendency to hide away one's general personal traits, or as one student teacher for primary school expresses it: "How I am as a mathematics teacher is to me very much the same as how I am as a human being". Others find it difficult to give a description of a personal teacher profile.

Ways of working and things to improve

Aspects of ways of working successfully in class that are stressed by most of theses students regard variation, the use of realistic problems and concrete materials, and the importance of developing a rich mathematical language. Visualization is especially important because many pupils have problems to follow verbal logical reasoning and the special language of mathematics. One student explains how the use of concrete material is helpful not only for the pupil but also for the teacher: "Concrete materials help me to explain, and help the pupils to understand". For training to use language to better understand a problem, one student teacher always let the pupil use his/her own words to describe the particular problem before giving response to his/her question. Some students stress the problems they encountered to make students "talk mathematics" with each other, even when they were put in situations with such a purpose.

One aspect of teaching that the student teachers find important to develop further, is the ability to show a more mature leadership in the classroom, give more precise instructions, avoiding ambiguous language use. Another aspect concerns the difficulty to begin the class in a way that gives a good and active working climate, and to end the class so that the pupils feel they have accomplished something together. Also the way to effectively use the white board is seen as important and difficult, for the secondary level by almost all the student teachers. A last common concern is the problem to make whole class presentations viable for as many pupils as possible. The students seem to be aware of problems with too long presentations, as expressed by the words: "For me 20 minutes was not so much but for them it was an eternity".

What is important for a teacher

There are three issues that stand out from almost all essays about what is seen as the most important aspects for a teacher of mathematics: a good contact with pupils, a good planning for teaching, and self reflection concerning the role as a teacher. On the first of these points, the student teachers for different school levels use different language but refer to the same fundamental basis for successful work as a teacher: a mutually positive relation to the pupils, built on friendship and respect. For the primary school teachers a common phrase used is to "see" the pupil, at every teaching session, so that he/she feels that the teacher cares. Lower secondary teachers stress the dynamic role of the teacher: to build on pupils' own thoughts to create a meaningful learning activity, and to be aware of sudden changes to prevent negative moods to take over, is possible only with a good teacher-pupil contact. The student teachers for the upper secondary level seem to use the phrase "good contact with pupils" more in general terms. There is a deep concern among the student teachers that the pupils feel well in the mathematics class, that they actively take part in what is happening, that the teacher is there by their side: "What I have discovered during my teaching practice is that what makes me feel I had a successful class is if I made the pupils forget about the invisible wall between pupils and teachers".

For the compulsory school level, the importance of a good planning is mostly discussed in terms of making it possible to find ways to catch the attention of and inspire the pupils, in order to make the basic work in the class more active and meaningful. Teachers for upper secondary school mention two different reasons for spending much time on the planning work. One is to be prepared for the variety of questions that may arise because of the more advanced mathematics at this level. Another reason

concerns a wish to feel free in relation to the subject matter to be taught, in order to be able to broaden the perspectives and engage in a dialogue during presentations. Both of these aspects relate to the need of good subject matter knowledge, something that was stressed much more by this category of student teachers. A good knowledge of mathematics is mentioned as an important premise for a profitable VFU study. We here observe links between the student teachers' own disciplinary knowledge in mathematics and their concerns for the pupils.

Finally, the importance of being aware of your own views on the teaching and learning process and of mathematics, and how you act and develop as a teacher, is frequently touched upon in the essays. This particular task of writing the essay is seen as valuable to develop such self awareness, and by some also as the task that opened up the eyes to see that this is important.

Discussion of the empirical results

The validity of the information given in essays of this type may be questioned by their examination function. To what extent did some of the students try to "impress" by showing an interest and commitment greater than was actually the case? Considering the strong uniformity of this commitment across all essays as well as in the anonymous evaluation sheets after the completed course, the risk for such a misinterpretation of this validity is here judged to be small. Further support for this conclusion is supplied by direct personal impression and discussions during one of the author's visits at the schools[9], and the free group discussions among the students that were organised in class after the completed VFU period.

The student teachers writing these essays were all in their second year of the study programme, and engaged in the ongoing first direction study, i.e. their first deeper study of one of their chosen main subjects. For most of them this VFU was also their first experience to have a full responsibility of a class for a longer period, which may be one reason behind the overall positive engagement and comments in the essays. We see this engagement also as a witness of the importance to let the student teachers meet authentic teaching situations early in their studies, in the subject of their interest.

These student teachers were strongly pupil oriented in their comments. However, they put equal attention to the social and learning/understanding aspects of their classroom work, indicating a balanced view between general educational and subject matter concerns in relation to the pupils. For this practice to be rewarding, the preceding theoretical courses and VFU experience from the first year of general education stud-

ies therefore seem crucial, along with a basic study in the taught subject. Even if it was stated by some students that they on some occasions would have needed deeper knowledge of mathematics, the essays bear no witness of a deep divide between disciplinary and pedagogical knowledge. Ways of working which they value, such as variation and a rich mathematical language, and what they find important for a mathematics teacher, such as good planning, confirm this.

The data show that it is a critical factor to feel secure in the teacher's role, which requires a well balanced competence among the different aspects of teacher knowledge. For these students, being novice teachers, along with good teacher-pupil relationships, a thorough planning seems to be a basis for this sense of security, from which they can feel more free, improvise and adapt adequately to unexpected sequels that open up in class. From the essays it can be concluded that, to be able to make this planning, firm subject matter knowledge of mathematics is necessary, including its pedagogical knowledge. In this case, a course in the didactics of mathematics immediately preceded the VFU period, during which the students could concentrate on the teaching, and on pupils' learning, of the mathematical topics studied earlier during disciplinary courses of the direction.

Using a professional language, based on integration of disciplinary and pedagogical knowledge, the teacher can build a friendly, respectful mathematical conversation with the pupils. Being able to listen and understand students' mathematical thinking makes the teacher prepared to respond in a variety of ways, thus supporting the mathematical development of the pupils. Data presented here show that student teachers do understand the importance of such interactions and professional language in the mathematics classroom. We hypothesize that this understanding has been promoted by the attempts to overcome the didactic divide by the close connection of the disciplinary and pedagogical studies made possible within the design of the direction.

Summary and conclusions

Reforms of educational systems at a national level are extremely complex enterprises, with a multitude of aims, causes and effects coming into play. In this paper we have focused on one issue that we argue has been critical to the development of the structure and content of teacher education in Sweden, i.e. the didactic divide between disciplinary and pedagogical knowledge. Mathematics is one of the core subjects where this divide traditionally stands out clearly, not only in Sweden but also as

an internationally observed phenomenon (e.g. Boero et al., 1996; Comiti & Ball, 1996; Ball & Bass, 2000).

By looking at the development of teacher education in Sweden, we have observed that even if the backgrounds to renewal relate strongly to societal change, including changes in the school system and general views on teaching and learning, when it comes to the content and its organisation within teacher programmes, the didactic divide between disciplinary and pedagogical knowledge has been a major focus. Already in the committee work (e.g. SOU1965:29) underlying the regulative framework from 1967 (Prop. 1967:4), a holistic view of teachers' work across school levels and areas was expressed. However, due to separation of programmes and of actors in teacher education, a didactic divide was one of the problems that set up the need for a renewal of teacher education (SOU1978:86). The tensions between experience-based and research-based knowledge remained also after the reform in 1988 (Prop. 1984/85:122), even if strong attempts had been made to integrate the traditional four strands of teacher education (i.e. subject theory, pedagogy, methods, and practice): "A good teacher education must include all these parts and it is necessary to have connections between them during the whole education" (ibid., p. 11; our translation). The failures to establish a professional subject matter didactics was only one of the results of the permanence of the didactic divide. A critical issue pointed at by evaluations of the programme was the weak link to research. It became apparent that a deeper structural change of teacher education, including its formal organisation within university, was needed to resolve some of the tensions mentioned.

Such structural and organisational change was a key feature of the most recent reform in teacher education in Sweden (Prop. 1999/2000: 135), even if we find much the same basic ideas of integration to overcome the didactic divide as the previous reform. Along with an emphasis on common competencies needed for all categories of teachers, expressing a holistic view on teachers' work, the new framework promotes a multilevelled integration of the three education areas and, within these, a progression of VFU studies. It aims to integrate disciplinary and pedagogical knowledge by its conception of a direction, which we see as the fulcrum balancing general teacher competencies, studied in the general education domain, and deepened disciplinary knowledge studied in specialisations. From the student teachers' essays we conclude that it has been possible to develop a commitment to the teaching of mathematics, based on a pupil-centred view on classroom work, within this framework. The essays also show that when the students reflect on themselves as mathematics teachers, they do not separate the disciplinary and pedagogical aspects of their knowledge but relate it to the demands of the pupils and the classroom

situation. In addition, the data indicate that some of the problems students report, for example when making whole class presentations, relate to the level of their disciplinary knowledge of mathematics and, as a consequence, problems to integrate this with pedagogical knowledge.

The idea of integration as a solution to problems of connecting different parts of an education programme as complex as the one in focus here, is tempting to follow but hard to implement successfully, as we have observed in the recent history of teacher education in Sweden. The conception of a didactic divide, and different attempts to resolve the tensions it causes, cuts across this and many other critical issues in teacher education.

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Notes

1 Examples of such studies include the *TIMSS Video study* and *The Learner's Perspective Study* (see the web pages http://nces.ed.gov/pubs99/timssvid/ and http://www.edfac.unimelb.edu.au/DSME/lps/). [available 2004-04-21]

2 A common feature of these two outlines of competencies for teachers of mathematics is that they explicitly parallel educational target competencies described for students of mathematics, promoting a holistic view of the mathematics education enterprise also expressed in the *Professional Standards for Teaching of Mathematics* (NCTM, 1991). See also Krainer (2003).

3 This includes knowledge of how to sequence topics and use materials in teaching.

4 For further details on the school and teacher education systems in Sweden, see Bergsten et al. (2003).

5 Our translations of the new Swedish terminology used in (Prop. 1999/2000:135): 'allmänt utbildningsområde' (general education area), 'inriktning' (direction), and 'specialisering' (specialisation).

6 For primary school level the education is three and a half years, for secondary school four and a half years (with deepening specialisations).

7 A teacher supervising student teachers' school practice is called a 'mentor'.

8 Back at the university, this was also followed up by whole class discussions on themes that were highlighted by observations from some of the video tapes.

9 As a part of the teaching in the course in connection to the VFU period, the author supervised some student teachers' work in class during their teaching practice, including follow-up discussions where also the mentors took part.

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Sammanfattning

Utifrån perspektivet tradition och förnyelse ges en diskussion kring reformeringen av lärarutbildningen i Sverige. Som kritiska faktorer för utbildningen av lärare i matematik ses behovet av forskning och en balans mellan olika innehållsaspekter. Begreppet ”didaktisk klyfta” (”didactic divide”) mellan ämneskunskap och pedagogisk kunskap används som ett analys-verktyg för att beskriva centrala idéer bakom de senaste reformerna. Empiriska data från lärarstuderande i det nya lärarprogrammet belyser hur dess struktur och innehåll bidrar till att överbrygga den didaktiska klyftan.