Dialogic learning in collaborative investigation

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This article claims that the quality of classroom communication influences the quality of learning. A dialogue can be seen as a conversation with certain qualities: it is a process of inquiry, includes risk-taking, and maintains equality. These qualities can be observed as dialogic acts. From observing teacher-student and student-student relationships in processes of collaboration we have identified different dialogic acts: getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging and evaluating. These acts we include in the Inquiry Co-operation Model (IC-Model). A teaching-learning process rich in dialogic acts in different clusters and combinations provides learning with dialogic qualities. Such learning can emerge in an investigative learning environment. Thus, in this article we develop our understanding of dialogic learning by specifying elements of the IC-Model using an example from the mathematics classroom that takes place in a landscape of investigation.

The traditional mathematics classroom is often organised in a very routinised way: The teacher presents the subject matter from the textbook and introduces an algorithm prior to the students' solving exercises. Then, the students work at the exercises individually, in pairs or groups, as the teacher adopts the role of a consultant assisting the students' work and controlling their results according to the answer book. In this educational framework there is one and only one right answer to a mathematical question. Central elements are exercises, problem solving and correction of mistakes.

This educational frame seems to produce a well-known pattern of classroom communication. The teacher asks a question, the student an-

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swers, and the teacher evaluates the answer. As the teacher knows the answer to his or her question beforehand, the student has to guess what the teacher thinks.[1] Empirical studies of traditional classroom communication have shown severe consequences for student activity of this communication pattern. The students' responses are minimal: They answer with a question, reject their own answer, make arbitrary guesses, ask for another explanation, echo another student's answer, keep silent, occupy themselves with other things etc.[2] It appears that the students give instrumental answers and take minimal responsibility for the process. On the other hand such communication patterns may support a learning of mathematics that aims at learning what is right and wrong in mathematical terms. It is also helpful in order to control what is already known, and as it is well known by teacher and students it can make them feel safe and comfortable in the classroom. In this way it seems that the quality of communication can influence the quality of learning.

One of the most severe consequences of the traditional classroom is the authority that unfolds through the text-book, the answer book and the teacher-student communication, which limit the students' possibilities to take responsibility, to be active and to take ownership of the learning process. We want to challenge the organisation of the traditional mathematics classroom by introducing landscapes of investigation as a frame that allows for inquiry co-operation and for new ways of communicating in the classroom.[3] This brings us to the notion of dialogue.

Dialogue and learning

In everyday speech dialogue can refer to many different things. One can talk about the dialogue between East and West. And it is possible to say that the dialogue in the final act of a play was tedious or lengthy. We want, however, to provide the notion of dialogue with a more specific meaning. We characterise dialogue as a form of communication requiring specific conditions: it is associated with a process of inquiry, includes risk-taking, and maintains equality. This notion of dialogue establishes the initial step of our presentation in *Dialogue and Learning in Mathematics Education* (Alrø & Skovsmose, 2002). In this book we develop theoretical notions and perspectives, which make it possible to connect 'dialogue' and 'learning'. In the present article we intend to recapitulate a few issues in this development. Our concept of dialogue draws on inspiration from different authors, although we do not subscribe to any specific theoretical framework as our foundation.[4]

We understand dialogue as part of an *inquiry process*, its aim being to obtain new insights. During this process, those involved act towards

each other and the subject matter with curiosity, wonder and reflective pondering. Dialogue in this sense is different from instruction, order, and persuasion. Dialogue implies a willingness to question one's understandings and pre-understandings and to examine what is new and different but also what is considered knowledge already acquired. Entering into a dialogue means taking ownership of the process of investigation.

In this concept, a dialogue has no pre-defined direction. The result can not be predicted. It takes place in the space between what is already known and what one might come to know. This means not always knowing where the paths will lead and what might be encountered on the journey. The travel is risky, and so is a dialogue. In everyday language, risk is associated with positive and negative connotations like 'taking a chance'. You take a chance when gambling, and you might win. You take a chance when crossing the street, and you may be run over by a car. A dialogue *includes risk-taking* in terms of unpredictability. When entering a dialogue you may touch issues that are delicate or unforeseen; there is a risk of losing control or steering into a dead end. But at the same time, it is possible to address one's tacit knowledge or to come to see things in new and different ways. It is possible to learn!

A dialogue *maintains equality* including a respect for diversity. This does not mean that a dialogue presupposes similarity or symmetry. We are speaking of interpersonal equality and human respect. In a dialogue there should be no use of power or force, no persuasion of the other, and no winning. The purpose of a dialogue should not be defined or decided by an authority. To be productive, a dialogue develops as a dynamic process between equal communicating partners. This counts for a dialogue in general, but also when dialogue is taking place in an educational setting. Even when the teacher is a more knowing or competent party to the dialogue, classroom conversations can be dialogic. The roles can be different, and so can the competencies. Thus, we understand a dialogue in terms of inquiry, risk-taking and equality.

In what follows we want to show what such dialogic qualities could mean for activities taking place in a mathematics classroom. We refer to an example where the students are invited into a landscape of investigation. There are no prescribed exercises to be solved. The teacher sets the scene by introducing a subject and some potential vantage points from where to start. The students can choose their own paths into the landscape of investigation.

The Batman Project – a landscape of investigation

The Batman Project takes place in a 10th form with students of about 16 years of age.[5] The time-schedule has been changed for the week in question, and the students have mathematics each day between 10 and 12. The teacher has introduced a project that is based on the idea of the class becoming a division of a well-known American company that produces all kinds of sports equipment, called "Run for Your Life". Each day the Danish division receives orders and requests, and soon the classroom has been transformed into a production hall, where the raw material (paper, pencil, glue, pictures etc.) and noise mix into the scenes of a factory with intense activity. For instance, footballs and handballs are going to be produced. They consist of pentagons and hexagons. But how are such balls constructed? A football is bigger than a handball, but does it have more pieces or are the pieces just bigger? What could be the difference between the size of the hexagons of handballs for men, for women and for juniors? The circumference of a handball for women should be 56 cm. How should they produce a ball with exactly this circumference?

One day the Danish division of "Run for Your Life" receives a request from a company "Batman & Co.", interested in table tennis bats. The company wishes to place a large order, but the Danish division does not have such bats on stock. They have to be imported. This could most easily be done from Sweden that has a long tradition with respect to table tennis and a great deal of experience in producing bats with international quality. The Swedish company is ready to sell the bats at 70 Swedish Kronor each, and this appears reasonable as "Batman & Co." has put an upper limit of 89 Danish Kroner per bat.

However, more issues have to be considered. Insurance is estimated at 1,5%. The exchange rate between Danish Kroner and Swedish Kronor is 82,14 (although according to some other information the exchange rate is 81,29). Duty is 8%. The profit for the Danish division is expected to be 25%. Finally, the VAT of 25% should not be forgotten. So, maybe the business is not that straightforward.

Mary and Adam place themselves in front of a computer and open the Excel spreadsheet. They concentrate on sorting out the business problem, and soon their world appears to get smaller and smaller. The sound from the production hall diminishes. In front of them the screen shows the empty spreadsheet, although, set up properly, this could bring clarification to the situation.

It appears reasonable to start out from the 70 SKr, and develop the spreadsheet on the basis of this figure. Insurance and freight of 1,5% have to be added. After some collaborative examination Mary and Adam get to the formula B4 = B3*1,015. Then a transformation from Swedish Kronor

to Danish Kroner has to be made. Mary is surprised that the exchange rate can be that low. Many more things have to be considered, but step-by-step they reach the formulations shown in Figure 1.

	A	B	C
1	COULD BE USED	(formula)	(value)
2			
3	Price in foreign currency	70	70
4	Insurance and freight	=B3*1,015	71,05
5	Rate of exchange	81,29	81,29
6	Total in Dkr	=B4*B5/100	57,756545
7	Duty 8%	=B6*1,08	62,377069
8	Profit in %	25	25
9	Total	=B7+B7*B8/100	77,97133575
10	VAT 25%	=B9*0,25	19,49283394
11	Total price	=B9+B10	97,46416969

Figure 1. *Mary and Adam's 'useful' model.*

These calculations make it obvious that business with "Batman & Co." is not going to be possible. The company was ready to pay 89 DKr per bat, but the calculations show that bats can only be offered for 97.50 DKr each. Or what exactly? Should the asking price be 97.50 DKr or 97.4641... DKr as we have a large order? How many decimals are in fact relevant to consider when we have to deal with big business?

These considerations about numbers of decimals do not occupy Mary and Adam, but the teacher has them in mind when he passes by Mary and Adam, as they are about to finish their spreadsheet. They have done concentrated work on the problem, and they are ready to do some experimentation. Something has to be done about the difference between the offer limit set by "Batman & Co." of 89 DKr and the calculated price of 97.50 DKr, determined by the price of the Swedish supplier. Mary and Adam feel put out by the teacher's question about the number of decimals, and this part of the conversation is concluded by the teacher emphasising that he himself does not know how many decimals should be considered in such a business deal.

Experimentation with the spreadsheet could take place. Perhaps it is possible to make a reduction in the expected profit? Or is it possible to

negotiate prices with the Swedish supplier? Perhaps the exchange rate could change – but how much would it need to change to obtain a significant impact on the result? It appears unrealistic to wait for any significant change, as both Denmark and Sweden are members of the European Union. Perhaps it is better to look for an offer from a different country, Brazil for instance. In that case, it may even make sense to hope for a further reduction of the Brazilian currency. Perhaps it might also be easier to make business with companies from Brazil. But, then, the costs of freight would certainly increase. There are many possibilities to be considered.

The teacher, however, introduces a different possibility. Somehow the spreadsheet could be turned 'upside down'. This would mean starting with the selling price of the bats, namely the 89 DKr, and with this figure as a basis calculating what could be offered to the Swedish supplier. Furthermore, it would be easy to adjust the spreadsheet to the different situations, depending whether the supplier is found in Sweden, Norway or Brazil. It takes some time for Mary and Adam to grasp the idea, but after some consideration and clarification they are ready to face the challenge. They start out constructing the inverse spreadsheet.

It is not easy to make a start. Common sense does not seem to operate in a straightforward way. If the spreadsheet should be organised upside down, the first step should be to remove the VAT since the last step in the original spreadsheet was to add the VAT. Logically, then, beginning by subtracting the VAT, how might that be shown on the spreadsheet? Something like:

$$B2 = B1 - 0{,}25?$$

But this formulation is not promising. Although the VAT has to be subtracted, might the formula expressing this nevertheless contain something with multiplication? Or could it be something with division? Mary suggests that it might be possible to divide by 1,25, but this suggestion is not examined further.

With great difficulty, Mary and Adam manage to organise an expression, containing the following elements of calculation:

$$B2 = B1 - 0{,}25\,B1$$

$$B3 = B2 - 0{,}25\,B2$$

$$B4 = B3 - 0{,}08\,B3$$

Everything, however, has been written together in the same cell of the spreadsheet. The teacher comes across and notices that Mary and Adam are likely to have problems with the set of formulae they are construct-

ing. Naturally, the teacher could decide to let Mary and Adam finish their setting up of the spreadsheet, so that they could do some experimentation with the numbers. Then they would most likely realise that they would have to reconsider their set up. This could become a relevant experience for making the reconstruction. However, other issues are on the agenda. That same day, the class is scheduled to go on an excursion, and the bus will be leaving at 12.00. It makes sense for the teacher to try to put Mary and Adam on the 'right track' quickly.

The teacher decides to interrupt, but Mary and Adam are not particularly interested in being interrupted again. Things are already difficult enough. Better prevent the teacher from stirring up further problems. When the teacher sits down next to them, Mary half turns her back to him. The teacher wants them to reconsider the point that even though the VAT is added as 25%, the percentage that has to be subtracted in order to remove the VAT is not 25%, but 20%. The teacher makes his point by an example. If something costs 40 DKr, and a VAT of 25% is added, then the resulting price would be 50 DKr. However, if we have to remove the VAT of 10 DKr from this 50 DKr, then we have to remove 20%. After some explanations Mary and Adam get the point. They also get the idea of the calculation:

$$0{,}25/(1 + 0{,}25) = 25/125 = 20\%$$

as well as

$$0{,}08/(1 + 0{,}08) = 8/1{,}08 = 0{,}074$$

In fact they might be ready to grasp the general idea that the 'inverse percentage' to x can be calculated as

$$x/(1+x)$$

Now a workable construction of the inverse spreadsheet can begin. Mary has turned back, and shoulder-to-shoulder Mary and Adam make their way through the new sequences of calculations. The teacher disappears into the production hall, and Mary and Adam share their concentration. They arrive at the spreadsheet as shown in Figure 2.

The game is over. Everybody is happy. Mary and Adam clap their hands as if just having made a decisive smash in a volleyball match. On top of the spreadsheet they have written 'pure genius'. The teacher is smiling, and he confirms that they have done an excellent job. The bus is waiting – and, with luck, one day somebody will make the offer of 63,92 DKr to the Swedish supplier.

The student inquiry process has found its route through the landscape of investigation that was introduced by the teacher. The students accepted

	A	B	C
1	PURE GENIUS	(formula)	(value)
2			
3	Maximum price	89	
4	VAT	=B3*0,2	17,8
5	Total	=B3–B4	71,2
6	Profit	=B5*0,2	14,24
7	Total	=B5–B6	56,96
8	Duty	=B7*8/100	4,219259
9	Total	=B7–B8	52,74074
10	Rate of exchange	81,29	81,29
11	Foreign currency	=B9/B10*100	64,87974
12	Insurance and freight	=B11*1,5/101,5	0,958814
13	Maximum price in foreign currency	=B11–B12	63,92093

Figure 2. *Mary and Adam's 'pure genius' model.*

the invitation and took ownership of their learning process. Most of the time they engaged in mutual inquiry that brought them forward. But once in a while they got stuck and found themselves lost in the wilderness. Luckily, the teacher showed up and challenged them in order to find some other paths to follow. Thus, activities of inquiry, of taking risks, and of maintaining equality can be supported by an educational frame as is a landscape of investigation. There is a lot of action going on, and it can be observed as action in the classroom communication.

The inquiry co-operation model

The inquiry process of the students can be seen as learning by doing and talking. They co-operate by means of action and reflection, and as they participate in collaborative work they have to verbalise what they do and think. They have explicitly to co-reflect.

According to speech act theory, to speak means to act. Using language means ways of acting in different contexts [6]. This brings a new dimension to the understanding of meaning that becomes associated with 'use' (and not first of all with 'reference'). It opens a new avenue in the philosophy of language, and it prepares for discourse theory in its many formulations. To us, speech act theory provides a connection between communication and learning, as both can be seen as action [7]. Furthermore, dialogue can

be seen as a particular form of communication, as communication with certain qualities; and such qualities can provide learning with particular qualities. In order to express such ideas in a more explicit form, we introduce the notion of dialogic act.

We think of dialogic acts as being a particular form of speech act. These are speech acts with the qualities of making an inquiry, running a risk and maintaining equality. Thus, persuading, ordering, instructing, correcting, controlling etc. are all speech acts, and certainly acts that are most often used in educational settings, but they are not dialogic. All speech acts can bring about learning. Dialogic acts provide learning with dialogic qualities, and in this case we will talk about dialogic learning.

A dialogue is built up by dialogic acts. However, a dialogue is not simply a 'stream of dialogic acts', but it can be established, developed and maintained by different acts, which, brought together, constitute the dialogue. Naturally, a dialogue is more than the sum of the particular dialogic acts. Perhaps, such acts can better be considered as analytic units by means of which we can observe elements of a dialogue.

From earlier studies where we observed teacher-student communication in an educational setting of inquiry co-operation, we have been able to identify the following series of dialogic acts: getting in contact, locating, identifying, advocating, thinking aloud, reformulating, chal-

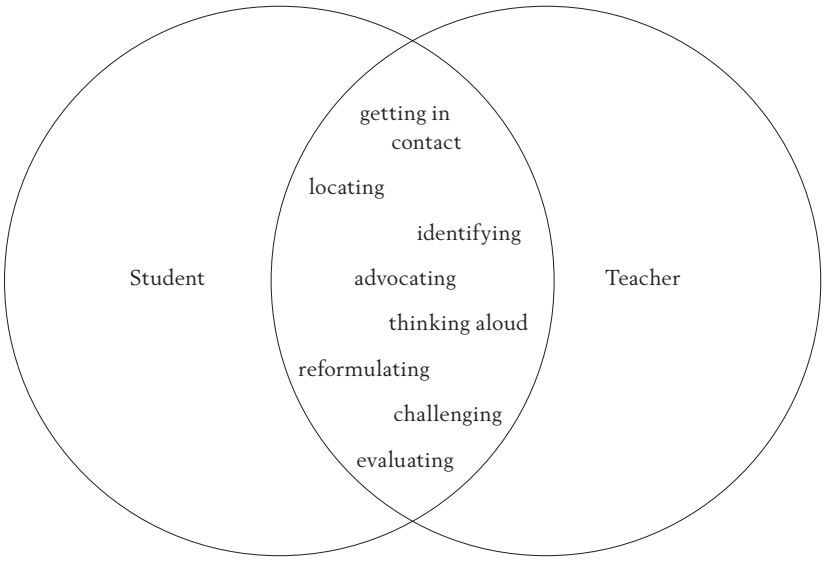

Figure 3. *The inquiry co-operation model.*

lenging and evaluating. These we have put together into what we have called an Inquiry-Cooperation Model (IC-Model) (see Figure 3). When present the components of the IC-Model indicate that a dialogue is taking place.[8]

The inquiry co-operation between Mary and Adam in the Batman Project was rich in dialogic acts, both when they were on their own and with the teacher. We will refer to this example in order to develop the IC-Model and the concept of dialogic learning. In *Dialogue and Learning in Mathematics Education* we have carefully analysed the conversation showing, from the transcripts, how different forms of dialogue emerged. In what follows we do not recapitulate this analysis, but we illustrate the use of what we consider to be examples of dialogic acts and dialogic learning.

Getting in contact

The following sequence took place after the teacher had suggested that Mary and Adam make an inverse spreadsheet. Although this suggestion was initially experienced as an interruption, including an interruption of contact, it soon turned into a challenge that they accepted and contact was re-established:

Adam: Then we'll just make a new one.
Teacher: Do you want to do that?
Mary: Should we try it ... shouldn't we then ... couldn't we just ... can't you go in and change this one ...?
Adam: Then we can just go in and change the rate and ...
Mary: ... in a way so we'll go up and say, that one er ... that one should equal ... something there?
Teacher: Give it a try.
Mary: Yes, should we give it a try?

Getting in contact means tuning in to each other. It means to be present and to be aware of what is taking place in the conversation and of each other's contributions. Getting in contact also means to establish a positive relation between the partners of the dialogue that makes them ready for co-operation. During most of the process, Mary and Adam keep in contact, and they are able to maintain that contact during the whole sequence. The contact can be observed in their inquiring questions, tag-questions and mutual confirmation – not to forget their sense of humour and laughing together.

Sometimes they lose contact, as when one does not really grasp the point of a certain proposal, or when they get problems with respect to

the technicalities of the computer and they start to quarrel. When the teacher interferes with remarks about the number of decimals, Mary experiences this as an interruption, and the loss of contact is marked by her turning her back on the teacher. He interrupted before he had a feeling about what was occupying the students. This is an example of failing to get in contact. However, the teacher managed to re-establish the contact, and, as evidence, the first step may be his remark that not even he himself knew how a company would deal with a greater number of decimals when making budgets.

Then he presented the idea of turning the spreadsheet upside down. This suggestion was met with doubt and questioning, but it was not experienced as an irrelevant interruption as were the comments about the number of decimals. Mary and Adam grasped the challenge, and they were ready to give it a try. The contact between the teacher and the students was re-established. And, furthermore, Mary and Adam were ready to take over the challenge that was incorporated into their process of inquiry. They again became the owners of the process. Getting in contact can mean to establish a contract of co-operation and about ownership. This was what was taking place in the conversation, when the teacher said: "Do you want to do that?" and Mary took over with her remark: "Yes, should we give it a try?"

Getting in contact is not only a process that initiates an inquiry process, it is ongoing. We could talk about getting in contact, maintaining, and re-establishing contact as all being dialogic acts. They may represent the more emotional aspects of an inquiry process. Getting in contact marks the interest in establishing an inquiry process as a shared process. It signals an intention of co-operation, and expresses the collective nature of the process.

Locating

Mary: No, then you write 70 [the original price in Swedish Kronor], and then it's plus the rate, isn't it?
Adam: What if there's another rate and not 70 Kroner?
Mary: Then you write that box there plus the rate, I suppose?

To locate means to find out something new or something of which one was unaware. Teacher and students can locate existing or new perspectives by formulating inquiring questions, i.e. questions that express surprise or wondering. Inquiring questions do not have any simple or direct answer. To locate means, through co-operation, to express and make visible some

perspectives, which might not have surfaced in the conversation. To locate means to explore and try out possibilities. It means to zoom-in on a topic instead of rejecting it. It could, for instance, mean further to explore an algorithm or a suggested way of tackling a problem, though the proposal might appear inadequate, useless, or even simply 'wrong'.

Hypothetical questions and what-if questions can articulate this readiness to make new discoveries. By hypothetical questions the teacher can invite the students into a landscape of investigation. We have observed many examples of the teacher facilitating students' discoveries through hypothetical questions, which turn into students' wondering, exploring, widening and testing questions. Such a mutual questioning may locate new perspectives and illuminate already formulated ones. And when students themselves start formulating what-if questions, it could indicate that they themselves take over the ownership of the process of investigation. Locating is closely connected with ownership.

Mary and Adam made a big effort at locating each other's perspectives. They questioned each other a lot; this brought about preliminary explanations, the testing and confirmation of conjectures. Often they distrusted their own proposals, investigated them anyway and came to an agreement about acceptance or refusal. An example can be found in their comments about how to operate with the transformation of the 70 SKr into Danish Kroner in the spreadsheet (Figure 1). Could it be that the exchange rate should be added? This was a possibility. But 'what if' it was not 70 SKr but a different exchange rate? A change of the numbers was not going to make a difference when operating in the spreadsheet, but Adam's what-if formulation had an important function. Mary suggested that the numbers had to be written just the same, but she added: "I suppose" indicating that there might be something more to be clarified. The process was not closed, more ideas had to be located.

Hypothetical questions can, however, have quite different functions than those only of stimulating an exploration. They can be ironic or controlling. They can set up distance or simply be irrelevant. The function of hypothetical questioning (being a speech act) depends on the context in which it is used. In order to work as dialogic acts hypothetical questions must express wondering and openness.

The openness in an inquiry process also includes the risk that discoveries might be forgotten or ignored, maybe because one comes to concentrate on particular ideas. This happens, for instance, when students refuse suggestions without argumentation, before they are properly located. When group work takes place in the classroom, the teacher is only present for a limited space of time in each group, and this means that he or she cannot grasp every interesting proposal made by the students,

simply because it has not been heard. Relevant perspectives thus run the risk of being ignored because they are not located.

Identifying

Adam	What should it say? It should say ... yes but the VAT is 25.
Mary	Yes, but that's actually the total figure, right? It has to be minus 25. 89 minus 0.25 times [ic][9]
Teacher	How much is the VAT of a total amount?
Adam	A quarter.
Mary	25%.
Adam	A quarter.

By locating perspectives it becomes possible to identify subject matter and make it accessible to the participants in the investigation. Sometimes the participants can point out and maybe reformulate certain proposals and in this way they can start to crystallise a particular mathematical idea.

What-if questions can be followed by why-questions. While a what-if question in many cases can be related to a process of locating, a why-question can be related to an identification of mathematical ideas. (Naturally, why-questions need not literally include the word 'why'.) In the project, the teacher supported Mary and Adam in identifying the principle for constructing an inverse spreadsheet by questioning the percentage of the VAT. If adding the VAT meant adding 25%, what could it mean to subtract the VAT? What questions are important in order to help to crystallise this mathematical idea? This teacher started the process using the question: "How much is the VAT of the total amount?" Both Mary and Adam came up with answers: " ... 25%" and "A quarter". In the following conversation the teacher pushed Adam and Mary to identify the relationship between subtracting 20% from the total, and adding the VAT of 25% to the cost.

As with what-if questions, it is important that a why-question is accompanied by openness. If not, the question can easily be interpreted as an exercise in control, which places the students' ownership of the process in jeopardy. Why-questions can also frustrate the students' curiosity. Naturally, not only mathematical ideas but also priorities and overall perspectives can be identified.

Advocating

The following remark made by Mary occurred when Mary and Adam started the process of organising the inverse spreadsheet. The VAT had

to be removed, and they struggled with trying to find out what the inverse operation to 'adding the VAT' could be. It seemed straightforward that it should mean something with subtraction. However, Mary made the following proposal:

Mary: Yes, otherwise we had to say divided by 1.25, right?

A little later Mary returned to the idea, which was, however, immediately annulled by Adam:

Mary: Then say ... divide
Adam: No, but take a look ...

Learning requires advancing from what one already knows and is able to do. Learning together means to establish an inter-subjective relationship with regard to what is already known. This includes an awareness of already existing perspectives on the issue. Therefore, it is important that it is possible to present views and suggestions for investigations. This can be done in the form of ideas just popping up, but suggestions can also be introduced with some sort of justification. This is the point of advocating. It is possible to advocate a suggestion through argumentation. However, advocating does not mean trying to convince the other about one's own opinion or suggestions. Advocating means examining one's own perspectives, but it can also mean arguing in favour of other's ideas and proposals. The point of advocating is to try out the extent to which a suggestion for investigation can find support in 'good reasons'. Advocating can be considered as an experimental or hypothetical argumentation.

Mary suggested how to remove the VAT. Perhaps, she thought, the operation could mean to 'divide by 1.25'. This seemed a healthy suggestion, but Adam was not ready to consider it. The first time Mary made this proposal, he did not even seem to be listening to what she was saying. The second time she made the proposal, his answer was simply: "No, but take a look ..."

Mary did not advocate her proposal. She only indicated that there might be good reasons for considering the approach by making the suggestion a second time. It would have been possible for her to elaborate through advocacy and Adam could have facilitated this, had he been open to new proposals at that time. He could have invited further justification with questions such as: "Why do you think so? Should we give it a try?" And together they could have sustained a supported advocacy in trying to find reasons which would make sense of 'dividing by 1.25'. Mary could have insisted on further advocacy. As we see dialogue as an inquiry process, it is important that available perspectives are brought into the proc-

ess. Advocacy is the fuel of the investigating process. Had Mary insisted on advocating, the whole process could have taken a new turn.

That Adam did not listen to Mary could be seen as an interruption of the dialogue. Equality was broken, as Adam took upon himself the right to judge what was irrelevant. However, this would be to over-simplify. Adam could be operating with other ideas that needed his full attention. To do so could also be seen as a preparation to provide an input to the process. Thus, dialogue should not be seen as a continuously flowing conversation. Dialogue, as an inquiry process, also contains discontinuities. This is an example of the risk in every dialogue. By being open, there is a risk that some interesting ideas and proposals may get lost.

Thinking aloud

When setting up the original spreadsheet (Figure 1), Mary and Adam worked their way as far as adding profit and VAT. With this step, the spreadsheet would be completed:

Adam: Then you write there, what ... how much the profit is in per cent, right?
Mary: And then you write 25%?
Adam: Yes exactly, then you write [5 sec.] in all ... and there you write equals ...
Mary: ... that one, right?
Adam: ... that one:
Mary: Plus profit, right?

Thinking aloud means expressing the thoughts, ideas and emotions that are included in the inquiry process. Thinking aloud is a way of making public what can only be experienced from within. In this way more elements turn into resources for investigation. To formulate impulsive ideas makes a perspective visible and accessible for a collective challenge or amplification. Sometimes hypothetical questions appear almost as thinking aloud, and they can function as an invitation to further investigation.

Thinking aloud can appear as coded messages, as in the quoted sequence. Here Mary and Adam have to find their way in the setting up of the spreadsheet, and the thinking aloud is short and coded, but nevertheless essential for ensuring the collective nature of the inquiry process and for keeping the process on the right track.

Thinking aloud can take different forms. Naturally, it can operate in a verbalised form, but if we just listen to what Mary and Adam say, we only become aware of some elements of their 'thinking aloud'. Thinking aloud, in particular in mathematics, can also be expressed in sketches,

diagrams, outlines of formulae, or as experiments shown on the screen. Such activities can be accompanied by remarks like: "here" and "just look", so the thinking aloud is not adequately reported by the tape-recorder or by the transcripts. 'Thinking aloud' could mean 'thinking in public', and the 'public making' of thoughts can be facilitated by many different means. This is what was taking place between Mary and Adam, and is an important dialogic act.

Reformulating

Adam: Well, we can, but then we need to enter the formula or something, right?
Mary: We need to enter the formula and change it?

To reformulate means to repeat what has been said, maybe with a slightly different tone of voice or adding something to it. To reformulate can be the same as paraphrasing, which can help to concentrate the attention on some key-formulations. Paraphrasing can also confirm that one has heard what has been said, and this confirmation can also include an invitation to expand on the idea. In this way, the participants in the dialogue substantiate a mutual understanding. However, paraphrasing, depending on the tone of voice, can indicate a difference in perspective. In this sense a reformulation could add a question mark to what has been paraphrased. To reformulate is an important element in 'active listening', where the participants follow each other closely in putting together a shared understanding.[10]

When Adam and Mary were first facing the empty spreadsheet, they had to do something. They could 'enter the formula or something'. Yes, this could be done: "enter the formula and change it". Mary reformulated Adam's suggestion and added something to it. They were together in the process. Both formulation and reformulation includes uncertainty and questioning. Their uncertainty was shared. Sharing uncertainty is as much an important potential for learning as sharing conclusions, discoveries and convictions in an inquiry process.

Reformulating can be initiated with a check-question in order to find out if one has understood the other properly. A phenomenon closely connected to reformulating is 'completing each other's formulations'. This can indicate that one has grasped the outlook of the other, and that one is able to think and reason within this point of view. Completing each other's formulation becomes a signature of sharing thoughts. But this dialogic element can easily turn into something different: a reformulation can be accompanied by scepticism in the facial expression, or a refor-

mulation can be selective of what has been said in order to fit your own understanding and preferences.[11]

There is an important emotional aspect to reformulating because of the signal that it gives that the listener has heard and found relevant what has been said. Reformulating can include both acceptance of a proposed perspective and a way of maintaining collective responsibility. Thus, 'getting in contact' is a dialogic act, which can be followed by other dialogic acts that can ensure 'staying in contact'. Mary and Adam stay in contact during most of the process, and their reformulating helps them to do so.

Challenging

The challenge defining the main part of the Batman Project was set by the teacher, when he suggested that it could be possible to turn the spreadsheet upside down.

Teacher: Could you er ... could you make the spreadsheet in a way so you typed in the Danish Kroner from the start ... this is how much you want to pay ... I mean, kind of turning it upside down?
Adam: You mean ... that we want to pay this much?

Challenging means questioning already established knowledge. It means to question assumptions that for the time being are taken for granted. An advocated assumption or conjecture can be challenged by what-if questions. The teacher did not use a what-if wording when he first presented the idea that the spreadsheet could be turned upside down. His challenge was powerful, mathematically speaking, but formulated in a cautious way: "Could you er ... could you make the spreadsheet in a way so ..."

A too direct challenge can bring a process of investigation directly to a stop. The challenge can also turn into a confrontation. A challenge can be so dramatic that no alternatives appear possible. An important element of challenging as part of a dialogue is to remember that it is part of a continuing process. A challenge is a way of making clear that *other* possibilities are waiting to be located.

A challenge can only be successful when grasped. In the example we noticed that the students were not immediately willing to grasp the teacher's challenge. But we also noticed what great things happened when Mary and Adam realised that 'turning it upside down' could be an exciting step to take. They took ownership of the process and the challenge became a turning point of their investigations. Other challenges probably were ignored, which had been the case when Mary suggested division in order to solve the problem of subtracting the VAT in the inverse spreadsheet.

Evaluating

Mary: Well, should we give this up?
Adam: Yes, no, we'll save it, won't we?
Mary: Yes, it's actually very interesting, we have been quite clever, don't you think?

Evaluation takes place in many ways. Mistakes can be corrected or critiqued, there can be negative or positive feedback, good advice, unconditional support, confirmation or credit can be offered. In the example we saw how the teacher praised the students' work, but we also experienced the students taking ownership of the evaluation: "We have been quite clever, don't you think?" Mary said to Adam after having declared their inverse spreadsheet to be 'pure genius'. They shouted and celebrated and claimed that this day they had really learned something.

Dialogic learning

Learning cannot be observed directly, but so can communication that takes place in educational settings. Thus, we cannot know what Mary and Adam did actually learn. We can only hear Mary claim that she had really learned something, and we can analyse the interaction of teacher and students during the process in order to get a glimpse into the students' learning process.

We talk about dialogic learning when a teaching-learning process includes a rich variety of dialogic acts. The dialogic acts of the IC-Model can occur in different clusters and combinations when teacher and students engage in inquiry co-operation. When they occur they seem to have great influence on the possibilities of teacher and students producing new insights together.[12] Thus, the IC-Model is an indicator of dialogic learning. The IC-Model, however, does not prescribe how to support dialogic learning, it indicates how and when processes of learning are based on inquiry, risk, and equality.

Can the IC-Model be developed further? Using the Batman Project as an example has permitted us to elaborate on the elements of the original IC-Model. As we have shown in the previous section, getting in contact can be elaborated using notions of tuning in, taking care, being present, supporting, confirming, using humour, posing inquiring questions and tag-questions. *Locating* implies the possibility of asking inquiring, wondering, widening, testing questions and check-questions, as well as exploring and trying out strategies or ideas, including raising hypothetical ('what if') questions. *Identifying* requires posing 'why-questions', explaining and crystallising mathematical ideas. *Advocating* has to do with examining

proposals, ideas and subjects by suspending fixed ideas and perspectives through collective reflection. *Thinking aloud* can be expressed in terms of hypothetical questions, verbalising and making public. *Reformulating* can be understood as repeating, paraphrasing, completing each other's utterances and staying in contact. C*hallenging* expects hypothetical questions to be posed, alternatives to be considered and turning points in an investigation to be introduced. *Evaluating* is found in the expression of critique, constructive feedback, confirmation, praise and unconditional support. The IC-Model is dynamic. We could easily imagine many more dialogic acts that could be identified. We could also develop the content of different dialogic acts much further. The point however is that we find that the notion of dialogic acts can give specific meaning to communication in an inquiry co-operation.

How fragile is inquiry co-operation? We have observed elements of the IC-Model in the teacher-student as well as in the student-student relationship, when the parties engaged in an inquiry process. The elements do not occur in a specific order, but they are represented in different clusters and combinations. Further, it is important to realise that the IC-elements often occur in very short sequences. They are seldom present in a whole conversation. Dialogic learning can easily be interrupted by disagreements, fixed perspectives, strategies of persuasion, lack of challenges, quizzing, questioning and other things that are quite common in everyday conversation, not least in the classroom. Such communication was also present in the Batman project, especially when the students got stuck in their work. Dialogic learning is a fragile process.

How can dialogic processes be supported? The conditions in a traditional classroom do not support dialogic learning. This does not mean that no learning takes place in that classroom, but the qualities of learning are different. Dialogic learning is likely to occur in landscapes of investigation that invite inquiry, allow for freedom to define and to choose different roads to exciting experience, but also combine with the risk of getting lost. We see the use of landscapes of investigation as a way of framing such learning processes.

Why try to support dialogic learning? This question brings us to basic perspectives on education and learning. Learning, not least learning of mathematics can be organised on the assumption that mathematics constitutes a form of knowledge that is valuable in itself. In *Dialogue and Learning in Mathematics Education* we talk, however, about a 'challenge of critique'. This emerges from the observation that mathematics in all its many forms cannot be seen as an 'ultimate good' but as a form of knowledge, which can operate in many different contexts, and which is in need of reflection and critique. D'Ambrosio (1994) has emphasised that the

'wonders' as well as the 'horrors' of science and technology have to do with advances in mathematics. This brings about a need for a mathematics education, which does not only try to bring about competence in operating with mathematical notions, but also competence in reflecting on what can be done by means of mathematics. Critical reflections on mathematics become an important element in the development of a 'critical citizenship', and dialogic learning, with its inquiring and reflecting qualities, is important to support such critical learning of mathematics. Thus, we see dialogic learning, in particular in mathematics, as necessary to the development of a critical position. We must emphasise that there might be many reasons for not developing dialogic learning in a context of schooling. The teacher has a responsibility with regard to the students. There could be situations where a quick decision has to be taken for instance when students tease each other. There could be situations that call for instruction and control. There could be many situations where the teacher has to break away from dialogue. Imagine a conversation that has turned into a dialogue between the teacher and certain students about a specific issue, which, however, might have only engaged that particular group. As the teacher has to take into account the interest of all students, there could be limits on how long it is possible to pursue a particular dialogue. Thus, we do not consider an invitation into a landscape of investigation and dialogic learning as being the answer to all kinds of educational challenges. But we find that this learning has certain qualities, of which we need to be aware, in order to challenge routines and facilitate collaborative investigation.

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Notes

1 See Young (1992) and Sinclair and Coulthard (1975).

2 See Lemke (1990) and Alrø (1995).

3 The notion of 'landscape of investigation' is discussed in detail in Alrø and Skovsmose (2002). See also Skovsmose (2002).

4 This concept of dialogue is developed from e.g. Bohm (1996), Cissna og Andersson (1994), Freire (1972), Isaacs (1999), Kristiansen og Bloch-Poulsen (2000), Lindfors (1999), Rogers (1994) and Wells (1999).

5 This project was presented in Alrø and Skovsmose (2002) and in Alrø, Skovsmose and Skånstrøm (2000, 2003). The 10th form is the last year in the Danish Folkeskole. After that the students can choose to continue in different directions within vocational schools. They can choose to enter the Gymnasium (the three year study which prepares for the university). Some of their fellow students have already left the Folkeskole from the 9th year to take the most direct route into the Gymnasium. The 10th form can be useful for students in helping them to make up their minds what to do next in life.

6 Austin (1962) and Searle (1969).

7 For more details about 'learning as action', see Alrø and Skovsmose (2002).

8 See Alrø and Skovsmose (1996). The order of the dialogic acts is as we originally represented them in the IC-Model. Naturally this is not the order in which we suppose the acts in a dialogue are observed, and this is certainly not the order in which we find the dialogic acts in the inquiry process of the Batman project. Therefore, in the following enumeration and illustrations of the dialogic acts, we do not follow the time order of the project.

9 [ic] means incomprehensible from the tape.

10 Rogers and Farson (1969) have introduced the term active listening. The important thing is that listening is done with a sincere interest in the perspective of the other and not as a manipulative technique.

11 Kristiansen and Bloch-Poulsen (2000) and Kristiansen and Alrø (2002) explain the latter with the notion of self-referentiality, which is a basic condition of human communication. Self-referentiality means that we tend to 'translate' new and unfamiliar things about others to our own well-known patterns of understanding and ways of acting. Dialogue on the other hand is about reflecting self-referentiality and trying to move a focus from oneself to the other and to what happens between the parties.

12 For examples, see Alrø and Skovsmose (2002).

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Sammendrag

Artiklens omdrejningspunkt er, at kvaliteten af den kommunikation, som foregår i klasseværelset, har indflydelse på kvaliteten af den læring, som kan finde sted. En dialog kan forstås som en samtale med særlige kvaliteter: En dialog er undersøgende, risikofyldt og ligeværdig. Disse kvaliteter viser sig i form af dialogiske handlinger. Gennem observationer af lærer-elev og elev-elev samtaler i matematikundervisningen har vi identificeret en række dialogiske handlinger: komme i kontakt, opdage, identificere, advokere, tænke højt, reformulere, udfordre, evaluere. Disse handlinger har vi samlet i IC-Modellen (Inquiry Co-operation Model). En undervisnings- og læreproces, der er præget af dialogiske handlinger i forskellige mønstre og kombinationer, har dialogiske kvaliteter. En sådan læring kan opstå i et undersøgende læringsmiljø. Artiklen uddyber og specificerer IC-Modellen med reference til et eksempel fra matematikundervisningen, der udspiller sig i et undersøgelseslandskab.