Mathematical belief research in Finland

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In this paper we give a brief overview of the present state of belief research done in Finland. The Finnish research on mathematical beliefs has recently flourished because of three different sources of funding provided by the Academy of Finland: 1) a national graduate school for mathematics, physics, and chemistry teachers, 2) a series of international MAVI workshops, and 3) research projects on affective issues in mathematics. The Finnish research has contributed especially to understanding the development of beliefs during teacher education and to the development of theoretical foundation of belief research as well as some new methodological solutions.

In the industrialised countries, everybody seems to know what is mathematics. But when the question is put forward, one gets different answers depending on the respondent in question. School children understand mathematics differently from their mathematics teachers, and teachers of other subjects will explain it again differently. Still another description is received e.g. from a "man-on-street". And mathematics professors have their own view of mathematics.

This big variety of answers to the question "What is mathematics?" hints that there is not only one understanding of mathematics, but several different views of mathematics. And not in the sense that there is only one right view of mathematics and the others were *wrong*. Philosophers of mathematics (e.g. Ernest, 1991; Hersh, 1997) have introduced several *right* views of mathematics that are also accepted among mathematicians. This state of art with the constructivist view of learning has led researchers of mathematics education to investigate teachers' and pupils'

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views of mathematics and their implication for mathematics teaching and learning.

What are beliefs?

The affective domain was for a long time a neglected area in research of mathematics education. About three decades ago an individual's attitude toward mathematics was brought up as one of the central research topics in mathematics education; the well-known Fennema-Sherman attitude scale (Fennema & Sherman, 1976) represents this phase. One may state that McLeod and Adams initiated a new phase with their famous book (McLeod & Adams, 1989), and in the recent research the constructs of the affective domain have been further elaborated. The focus of research has changed from broadly defined attitudes to more specific sub-concepts: emotions, narrowly defined attitudes, values, and most commonly beliefs. Within belief research in mathematics education, further subcategories have been distinguished; for example, 1) beliefs about mathematics education (mathematics as subject, mathematical learning and problem solving, mathematics teaching in general), 2) beliefs about self (self-efficacy, control, task-value, goal-orientation), and 3) beliefs about the social context (social and socio-mathematical norms in the class) (Op't Eynde, De Corte & Verschaffel, 2002). Another direction of sharpening has been mathematics itself. Today we are no longer considering attitudes or beliefs toward mathematics as an entity, but researchers distinguish e.g. attitudes or beliefs on geometry or on problem solving.

Within research in school, the prevalent understanding of learning has emphasised cognitive academic achievements. Affective by-results that are in connection with an individual's meta-cognitions, however, determine, to a large extent, how good a learner of mathematics one has become. Especially during the last decade, researchers around the world have paid more and more attention to mathematics learning as a process. Such a view highlights the importance of meta-cognition and affect, especially in the form of pupils' and teachers' beliefs. Beliefs seem to be situated in the "twilight zone" between the cognitive and affective domain, and thus, have a component in both domains.

Individuals continuously receive perceptions from the world around them. According to their experiences and perceptions, people draw conclusions about different phenomena and their nature. The individual's personal knowledge that includes one's beliefs is a compound of these conclusions. Since beliefs seem to form and change in social environment, people compare these beliefs with their new experiences and with the beliefs of other individuals, and thus their beliefs are under continuous

evaluation and change. When one adopts a new belief, this will automatically form a part of the larger structure of their personal knowledge, of their belief system, since beliefs never appear fully independently. Thus, the individual's belief system is a compound of one's conscious or non-conscious beliefs, hypotheses or expectations and their combinations.

Although beliefs are popular as a topic of study, the theoretical concept of "belief" has not yet been dealt with thoroughly. The main difficulty has been the inability to distinguish beliefs from knowledge, and the question does not still seem to be clarified (e.g. Abelson, 1979; Thompson 1992). These problems are discussed in a recent paper of Pehkonen & Pietilä (2003). Another difficult issue has been the distinction between beliefs and other affective variables such as emotions, attitudes, and values. These affective variables have often been located on a continuum, stretching from intense and fluid emotions to stable and cool beliefs (McLeod 1992). Such a view, however, has been also criticised, for example, for exclusion of mild or long-lasting emotional states and emotions that arise regularly (Evans, Hannula, Philippou & Zan, 2003).

As an implication of this fuzziness in the definition, one might mean different matters with beliefs, depending on the discipline and the researchers who deal with them. For example, beliefs are considered equal to concepts, meanings, propositions, rules, preferences or mental images (Thompson, 1992). At other occasions, beliefs are seen in much broader sense as "mental constructs that represent the codification of people's experiences and understandings" and that shape their perception and cognition in any set of circumstances (Schoenfeld 1998, p. 19). In social psychology the impressions of and reactions to other people are typically divided into beliefs, expectations and attitudes. For them, beliefs are statements thought to be true, whether they are true or not. Expectations are explicit or implicit predictions about people's future behaviour, and attitudes are emotional reactions to them. (Brophy & Evertson 1981) These questions of fuzziness in defining beliefs are dealt with more broadly e.g. in the published paper of Furinghetti & Pehkonen (2002).

On one hand, beliefs can be thought to form one part of an individual's meta-cognition (e.g. Schoenfeld 1987). On the other hand, some people try to define beliefs through attitudes (e.g. Törner & Grigutsch 1994). In the different definitions, a truth-value is usually attributed to a belief (i.e. beliefs are kind of knowledge), and they are often seen to include an emotional component. Sometimes the definitions say also something about the stability and intensity of beliefs and about the nature of their origin or warranty. Here an individual's beliefs are understood in a rather wide sense as his/her subjective, experience-based, often implicit knowledge and emotions on some matter or state of art. Such a characterisation is

very near the one given e.g. in the published paper of Lester, Garofalo and Kroll (1989). In the literature, the term conception is often used parallel to beliefs. In the case of conceptions, we understand that the cognitive component of beliefs is stressed, whereas in basic (primitive) beliefs the affective component is emphasised.

The spectrum of an individual's beliefs is very wide, and they are usually grouped into clusters of beliefs. The belief clusters influence each other. Some beliefs depend on other ones, for the individual more important beliefs. Here Green (1971) uses the term "the quasi-logical structure of beliefs" which means that the individual himself/herself defines the ordering rules. Thus, beliefs form belief systems that have a quasi-logical structure, and that might be in connection with other belief systems or might not. Therefore, the term *belief system* is used as a metaphor to represent how the individual's beliefs are structured. The affective dimension of beliefs influences the role and meaning of each belief in the individual's belief system.

In the literature, one can find several overviews of belief research (e.g. Schoenfeld, 1992; Thompson, 1992; Pehkonen, 1994; Op't Eynde et al., 2002). The continuing interest on beliefs shows that there are still many unanswered questions. Recently the first book on mathematical belief research (Leder, Pehkonen & Törner, 2002) was published, in order to give an overview on different research perspectives and on research done.

Belief research in Finland

In Finland there are some research domains in mathematics education where several studies are concentrated giving support to each other. Such a school of research has been formed during the 1990's around research on mathematics-related beliefs. The systematic research on mathematical beliefs was started in the middle of the 1980's in the United States (cf. Thompson, 1992), since their meaning was pointed out by several researchers for teaching and learning (e.g. Schoenfeld, 1983; Silver, 1985). About a decade later research on mathematics-related beliefs and conceptions was started in Finland (cf. Pehkonen, 1991).

Internationally, the observations made in the implementation of mathematical problem solving in schools seemed to be a starting point for belief research in mathematics education. Teachers and students showed unexpected reluctance to use problem solving in ways suggested by researchers and teacher educators. This was the case also in research of Pehkonen (cf. Pehkonen & Zimmermann, 1990, in Finnish), and therefore, the point of emphasis in his research was moved from problem solving to the struc-

ture and meaning of teachers' and students' beliefs (e.g. Pehkonen, 1991, 1993, 1994, 1995).

MAVI Activities

Belief research in Finland was strengthened, when the Finnish-German co-operational research enterprise MAVI (= Mathematical Views) was launched in the middle of the 1990's. MAVI activities were financed by the Academy of Finland and DAAD (= Deutsche Akademischer Austausdienst) for three years 1996–98 (cf. Pehkonen & Törner, 1998). The main activities were joint workshops in both countries, where especially young researchers were supported. From these workshops seven reports were published in the publication series of University of Duisburg and University of Helsinki. Within the Finnish MAVI participants, there were mainly doctoral students but also some experienced researchers, such as Sinikka Lindgren (Lindgren, 1995) from the University of Tampere.

Gradually MAVI workshops became so popular that some foreign researchers regularly took part in them. And at the end of the 1990's, the MAVI workshops grew into a joint European enterprise, which can be seen e.g. in the latest published MAVI–11 report (cf. Di Martino, 2002). The MAVI meetings offer a safe environment for doctoral students to present their ideas, to get feedback, and to learn to write research reports in concise form in English.

Another consequence of the Finnish-German joint MAVI activities was an international specialist conference in Oberwolfach in November 1999 that was also initiated and organized by the same persons (cf. Pehkonen & Törner, 1999). In the Oberwolfach Institute, a group of about twenty selected international specialists in belief research discussed, over several days, their understanding of beliefs and related topics. This meeting resulted in the recently published book on beliefs (cf. Leder et al., 2002).

Dissertations on Mathematical Beliefs

The Academy of Finland initiated funding for a national graduate school for mathematics, physics, and chemistry teachers in 1995 for six years. Several graduate students chose mathematical beliefs as the topic for their doctoral thesis. Many of these dissertations were elaborated and/or finished, at least partly, within the MAVI environment: Kupari (1999), Huhtala (2000), Malmivuori (2001), Pietilä (2002), Soro (2002), and Hannula (2004). Hannula and Malmivuori have written their theses in English, while the others have used Finnish. Additionally there are some

further dissertations that are nearly finished (e.g. Attorps, 2004). But there are also dissertations on mathematical beliefs in Finland, which have been accomplished outside of MAVI: Kaasila (2000) and Perkkilä (2002), both in Finnish.

When we have a closer look at the topics of the dissertations, teachers' beliefs seem to be the most popular domain: Pekka Kupari (1999), in Finnish, charted teachers' beliefs with a survey trying to sketch a picture on their view of mathematics. Raimo Kaasila (2000) and Anu Pietilä (2002), both in Finnish, used qualitative methods to follow the development of students' beliefs in elementary teacher education (cf. also Kaasila, 2002 and Pietilä, 2002). Päivi Perkkilä (2002), in Finnish, combined quantitative and qualitative methods to sketch firstly a general outlook of in-service teachers' views on mathematics in elementary school, and then deepened the approach through interviewing a group of teachers (cf. also Perkkilä, 2003). Riitta Soro (2002), in Finnish, developed a new instrument to measure quantitatively mathematics teachers' gender-related beliefs, and administered a survey for lower secondary school mathematics teachers on their gender-sensitiveness (cf. also Soro, 2002).

Sinikka Huhtala (2000), in Finnish, used a grounded theory approach to explore what mathematics means to those practical nurse students who have difficulties to meet the formal requirements of their schooling in drug calculations (cf. also Huhtala, 2002). Marja-Liisa Malmivuori (2001) accomplished an extensive exploration of the literature, and made a theoretical synthesis of the dynamics of affect, cognition, and social environment in self-regulation of learning thus providing a larger context of theory to embed belief research into. Markku Hannula (2004) developed certain aspects of a theory on the birth and change of pupils' mathematical attitudes and beliefs (cf. also Hannula, 1998, 2002a, 2002b).

Research projects on beliefs

Starting parallel with the graduate school and MAVI activities, there have been several research projects on mathematical beliefs in Finland financially sponsored by the Academy of Finland, and applied and organised by Erkki Pehkonen. A brief description of each of them delivers an overview of research done (in the parenthesis are the years of financing given).

Development on pupils' mathematical beliefs (1996–98): The participants of the project were pupils in the lower secondary school (about 13–15 year-olds). The project had a double aim. Firstly, there was a large-scale survey of pupils' mathematical skills and potential as well as a belief questionnaire that was administered twice. Secondly, there was an idea to follow the development of pupils' mathematical beliefs during

the lower secondary classes, and this was implemented as an ethnographic study of the development of pupils in one class (Hannula, Malmivuori & Pehkonen, 1996).

Professors' beliefs (1996): In this project, the Finnish mathematics professors' conceptions of mathematics and its teaching were analysed. The seven mathematics professors in Finnish universities who are responsible for teacher education at departments of mathematics were interviewed individually. One interesting partial result was that they all expressed (spontaneously) a similar conception that our school mathematics emphasises computational skills too much, and not enough mathematical structure, e.g. proofs (Pehkonen, 1999).

Teacher knowledge on open problems (1999): The project focused on mapping lower secondary mathematics teachers' pedagogical content knowledge, in the case of open problems. A randomly selected representative group of mathematics teachers in lower secondary schools in Finland answered a survey. The study was also expanded into a case study of teachers' and pupils' beliefs on onpen problems in one school (Pehkonen & Vaulamo, 1999).

Understanding and self-confidence (2001–03): The aim of this project was to explore the connections between pupils' developing understanding of mathematics and their level of self-confidence in grades 5–8 of the comprehensive school. The study included a quantitative survey for randomly selected Finnish mathematics classes (grade 5 or 7). A small group of classes were selected to a longitudinal part of the study, and from each of them some pupils participate also a qualitative study. In reports, there have been analysed, for example, pupils' conceptions of fractions (Hannula, 2003b), pupils' understanding of infinity (Hannula, Maijala, Pehkonen & Soro, 2002), and gender differences in pupils' self-confidence in mathematics (Nurmi, Hannula, Maijala, & Pehkonen, 2003). Results of the longitudinal part will be presented at the next PME meeting (cf. Hannula, Maijala & Pehkonen, 2004).

Elementary teachers' mathematics (2003–06): The longitudinal study, started only recently, has its aim at revealing the development of elementary teacher students' mathematical view during their studies in three different Finnish teacher education departments (Helsinki, Rovaniemi, and Turku). A quantitative survey on students' mathematical understanding and their beliefs was administered at the beginning of students' studies (autumn 2003), and the measures will be repeated each semester. Twenty students are selected for a more focused qualitative part of the follow-up study. Preliminary results will be presented at the next PME meeting (cf. Kaasila, Laine, Hannula & Pehkonen, 2004;

Laine, Huhtala, Kaasila, Hannula & Pehkonen, 2004; Pehkonen, Hannula, Kaasila & Laine, 2004).

Discussion

Belief research in Finland cannot be described as a coherent enterprise. There is diversity in theoretical and methodological choices made. Yet, we can find some general trends.

Studies on theoretical foundations

There has been a significant effort to develop the theoretical foundation of belief research. One approach has been to clarify the characterizations given to beliefs (Furinghetti & Pehkonen, 2002; Pehkonen & Pietilä, 2003). The result of these efforts show that the concept of belief is so complicated and fuzzy that even international specialists in belief research cannot find consensus on the characterization (cf. Furinghetti & Pehkonen, 2002). Therefore, Pietilä (2002) suggests the use of the concept "view of mathematics" that does not try to distinguish between the problematic concepts of belief, conception and knowledge.

Some researchers were interested in the broader theoretical context of affect-cognition interaction and the dynamics of self-regulation and the development of beliefs, as Huhtala (2000), Malmivuori (2001), Hannula (1998, 2002b, 2004), and to some extent also Pietilä (2002). These works highlight the importance of intention and reflection as well as the dilemma between conscious and unconscious processes. For example, both Malmivuori and Hannula emphasise the role of beliefs both in automatic and reflective self-regulation.

Methodological questions

Characteristic for many of the research projects on mathematical beliefs in Finland is their longitudinal nature, and the combination of qualitative and quantitative methods. The Finnish research on mathematical beliefs opens up also some new methodological approaches.

Malmivuori and Pehkonen (1996) as well as Soro (2002) developed and tested new questionnaires to measure beliefs. For example, Soro (2000, 2002) introduced an innovative scale to measure "mathematics as a male domain". Instead of agreeing or disagreeing with a given statement the respondent had to choose if a statement about a student would apply usually to a boy or to a girl, more often to one than the other, or as often to both genders. Similar ideas on a special structure of a gender

scale can be read also e.g. in the papers of Australian researchers (e.g. Forgasz & Leder, 2000).

Among the different qualitative methods, there were approaches that have not been used much in mathematics education. Most notably diverging from the usual have probably been the repertory grid technique (Hoskonen, 1999), the narrative analysis (Kaasila, 2000, 2002), and use of fiction-writing techniques (Hannula, 2003a).

Hoskonen (1999) used Kelly's theory on personal development (Kelly, 1955) – and the research method he developed, the so-called repertory grid technique – to reveal her pupils' beliefs. Kelly developed the repertory grid technique for assessing an individual's personal constructs. When during an interview, a person is asked to categorize elements, he is thought to organize the patterns in his built-in filter. The categories obtained are constructs. The constructs are bipolar in the sense that every construct has its opposite, e.g. honest vs. dishonest. Persons make sense out of the world by simultaneously noting similarities and differences between the elements. With Kelly's repertory grid technique, Hoskonen (1999) was able to construct a ninth-grader's view of what it means to be good in mathematics.

Narrative analysis is well known from behavioural sciences (e.g. Riessman, 1993). Kaasila (2000) uses narrative analysis for his research data. His main point is to interpret students' interviews from a narrative view point, i.e. what kind of story can be seen in the interpretation of their memory pictures. In teacher pre-service, the narrative method can be used, as follows: Students are offered opportunities to tell about their school-time memories, and they may share their experiences with other students. For example, if a student remembers from his/her mathematical past mainly failures, and if he/she sees in his/her mathematical future only dreads, he unconsciously interprets his/her mathematical life from the viewpoint of a tragic story. When the student reflects incidents in his/her own mathematical life, and sees that its interpretation can be changed, this may free him/her to look after new viewpoints into his/her mathematical past and future. At the same time the student's view on himself/herself as learner and teacher of mathematics may be improved.

Hannula (2003a) took another view of narratives as a method of reporting qualitative research. Research on affect often tries to convey something of very personal experiences. How can the mathematically able understand the anxiety and helplessness often experienced by students when they face mathematics? He suggested that fiction-writing techniques could help both the writer and the reader of a scientific report

to understand and empathically relate with the personal experiences that are being studied.

Also the Dionné triangle method used and discussed in Pehkonen and Törner (2004) is worth mentioning. In order to single out teachers' view of mathematics, Dionné (1984) asked them to distribute a total of 30 points among three categories representing the three main perspectives of mathematics (traditional, formalist, constructivist). Pehkonen and Törner (2004) enlarged the method by asking teachers to mark beside the numerical presentation additionally their view into an equilateral triangle. Each corner of the triangle represents one perspective, and thus the numerical presentation can be understood as barycentric coordinates of the point in the triangle. These both methods of data gathering (Dionné numbers, and Dionné triangle) were compared.

Implications for school teaching

Most belief researchers have had a possibility to influence teachers' views as teacher educators of pre-service and in-service courses. In the first place, Finnish belief research has increased general awareness of the meaning of beliefs for mathematics teaching and learning. At least some teachers are trying to use also new approaches, e.g. different kinds of open-ended problems, in order to enlarge their pupils' view of mathematics. However, the practice in schools seems to be very resistant toward change (e.g. Perkkilä, 2003).

The Finnish success in international comparison studies, as TIMSS and PISA, has caused large interest in Finnish school system and teaching methods. It is very complicated to find an answer to all the enquiries concerning Finnish success. In the publication (Välijärvi, Linnakylä, Kupari, Reinikainen & Arffman, 2002), the PISA researchers do suggest that Finnish good results are due to several factors that are additionally interrelated. Such factors have to do, at least, with comprehensive pedagogy, students' own interests and leisure activities, the structure of the education system, teacher education, school practices, and, finally, Finnish culture.

The Finnish State invests money into research in all fields through the Academy of Finland, and the investments are comparative with those of other highly developed industrialised countries. The high standard of Finnish belief research which is surely above the international average, can be seen in the number of research projects financed by the Academy of Finland, since there the competition is hard, and the acceptance rate of the applications is generally very low, about 15 %.

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Sammanfattning

Artikeln ger en kort översikt över tillståndet hos forskningen om matematiska föreställningar i Finland. Den finska forskningen om matematiska föreställningar har blomstrat de senaste åren på grund av tre olika resurser från Finlands Akademi: 1) En nationell forskarskola i matematik, fysik och kemi för lärare, 2) en serie av internationella MAVI-verkstäder och 3) forskningsprojekt om affektiva aspekter på matematik. Den finska forskningen har i synnerhet befrämjat förståelsen av föreställningarnas utveckling under lärarutbildning och utvecklingen av den teoretiska grunden för föreställnigsforskning, samt erbjudit några nya metoder.