# Recent developments in Finnish mathematics education

Pekka Kupari

The article addresses some development trends that have taken place in mathematics education of the Finnish comprehensive school over the past 15 years. A particular reason for such analysis arises from the international large-scale assessments on science and mathematics conducted at the turn of the millennium, which showed that Finnish student performance in mathematics is of a high international standard. Good achievement is attributable to a whole network of interrelated factors. The comprehensive school system seems to be successful in providing the majority of its students with a solid foundation for further schooling and for transition to working life. Long-term curricular measures and extensive development schemes have served as important framing factors. Also the high standard of teacher preparation and mathematics teachers' professional skills is a factor that probably contributes to student performance. At the level of students, self-confidence in learning mathematics seems to be closely related to the level of performance. As regards remedies to shortcomings and problems, recognition of national strengths creates a positive atmosphere for improvement.

In recent years, the Finnish school system and its outcomes of high quality and equal distribution have received wide international attention. The international large-scale assessments conducted at the turn of the millennium – TIMSS 1999 and PISA 2000 – revealed that the results of mathematics education in our country are quite good in international comparison. This article discusses some development trends detected in Finnish mathematics education during the past fifteen years. The focus will be mainly on the primary and lower secondary levels, i.e. year grades 1 through 9 in our comprehensive school system. The approach of the

**Pekka Kupari** University of Jyväskylä article is mainly descriptive and the two development areas described below are examples of the educational activities pursued in our country. They also offer some reasons underlying the Finnish success. To give some background, I will first take a brief look at Finnish youngsters' mathematics performance in the TIMSS and PISA assessments.

#### Finnish students' mathematics achievement

International comparisons of educational achievement have become influential in debates over school reforms in many countries all over the world. Such studies have their benefits and also deficiencies. Comparative studies provide policy-makers and educational practitioners with information about the quality of their education system in relation to relevant reference groups of similar nations. Researchers use such findings to develop explanations for why certain countries outperform other countries and why inequality in outcomes appears more serious in some societies than in some others. Although comparative studies have attracted positive attention, they have also drawn critics. How useful are these international comparisons? How authoritative are their results? What is the methodological quality of the most recent international surveys? These important issues have been discussed thoroughly in several articles published during the last few years (e.g. Porter & Gamoran, 2002; Clarke, 2003).

In comparison among the 38 countries participating in TIMSS 1999 (Third International Mathematics and Science Study Repeat) the seventh-grade students of the Finnish comprehensive school did fairly well in mathematics (Mullis, Martin, Gonzales et al., 2000; Kupari, Reinikainen, Nevanpää & Törnroos, 2001).

In Finland also the variance between students was among the smallest across all the participating countries. In terms of the standard deviation of the test scores, the differences between Finnish students were the second smallest. There were relatively few top performers in mathematics, but on the other hand, the group of very low achieving students was small, as well.

The Finnish students' performance profile illustrating their relative achievement for different content areas was reasonably consistent with the average profile of the 14 participating OECD countries. The content areas yielding the best results in Finland included numbers and mathematical operations as well as statistics and probability. In these areas our students performed above the OECD average and close to the top internationally. In the domain of measurement, the performance of the Finnish students were about average among the OECD countries. As a whole, these results show that our seventh-grade students' basic knowledge and skills (e.g. number sense, basic operations, basic geometry, interpretation of charts and tables) were of a good standard. In addition, they did fairly well in tasks dealing with probability (e.g. flipping the coin), for instance, despite the fact that in Finland such matters lie beyond the scope of the curriculum for seventh-grade students.

In the PISA 2000 assessment of mathematical literacy, Finland ranked clearly among the best quarter in the group of 27 OECD countries (OECD, 2001; Välijärvi & Linnakylä, 2002; Välijärvi, Linnakylä, Kupari et al., 2002). Finland displayed the fourth highest mean performance in mathematical literacy. Of the top countries, only Japan statistically significantly outperformed Finland.

Finland's performance in mathematical literacy also showed low variation across students, even to the degree that in Finland the standard deviation for student scores in mathematical literacy was the smallest among the OECD countries. The proportion of weak performers in mathematical literacy was considerably lower in Finland than across OECD countries on average.

From the viewpoint of content areas, performance in Finland was evenly distributed. In both of the content areas assessed (change and relationships; space and shape), the percentages of correct answers in almost all items were higher in Finland than the average across OECD countries. Finnish students did especially well in statistics and, more specifically, in interpreting graphs and diagrams. They also performed above the OECD average in calculating the areas and perimeters of figures. In algebraic contents, by contrast, the Finnish students were at their weakest.

In both of the above-mentioned international assessments the equality aspect of Finnish student achievement was clearly evident. The variance in student performance in Finland was among the smallest across the participating countries. Internationally, Finland had a very small proportion of low achieving students. On the other hand, our share of the very top performers could have been larger. Anyhow, gender differences in mathematics were negligible in Finland. Also regional differences within the country as well as those between urban and rural areas were small. Furthermore, differences between schools accounted for as little as approximately nine per cent of the overall variance in students' mathematics performance. Such degree of between-school variance is internationally very small indeed, and it has remained about the same since the mid-1990s. All these results suggest that high average performance can be achieved by providing all students with similar learning opportunities for mathematics rather than through explicit differentiation at an early age between types of programmes.

TIMSS 1999 together with PISA 2000 also revealed shortcomings and problems in our students' mathematical competence. The domains displaying, in relative terms, the lowest performance were geometry and algebra. For example, students had great difficulties with tasks that required recognition of relevant characteristics, relationships, or patterns, or where a generalisation was requested. These were situations where conceptual knowledge and its connection to procedural knowledge are highlighted (e.g. Hiebert, 1986). For instance, a basic equation in school algebra (12x - 10 = 6x + 32) proved difficult for the Finnish students to solve in TIMSS 1999; only a quarter of the seventh-grade students got it right. As the average percentage of correct answers to this item across 14 OECD countries was 20 percentage points higher than in Finland, there is really a need to examine reasons for this difference.

#### Description of development areas in mathematics education

The results from the above-mentioned assessments give clear evidence of the fact that the mathematical competence of Finnish comprehensive school students is of a fairly good international standard. At the same time this success has produced a somewhat puzzling experience to all those responsible for and making decisions about mathematics education in Finland. Traditionally, we have been used to thinking that the models for educational reforms have to be borrowed from abroad. The sudden change in our role from a country following the examples of others to one serving as a model for others has prompted us to think seriously about the special characteristics and strengths of our education and school system.

Hence, there is a need to review, on the one hand, the development of Finnish mathematics education during the past 15 years and, on the other hand, the strengths of our comprehensive school system in general. In the following we will discuss a couple of development areas in mathematics education that may contribute to the relative success of Finnish students.

## Developments in mathematics curriculum since 1990

In 1987–89 an important committee work was accomplished in Finland on mathematics and science education (the so-called Leikola committee). The final report (Komiteamietintö, 1989) introduced goals for basic education in science and mathematics, highlighted major problem areas, and suggested remedial actions to tackle shortcomings and inadequacies. As for the developmental needs of comprehensive school mathematics education the report mentioned, among other things, 1) diversification of teaching methods, 2) shifting the emphasis from rehearsing routine skills onto development of thinking, 3) connecting instruction to practice and applying what is learnt, and 4) introduction of calculators and computers as instructional aids as early as possible.

In consequence of the committee work the National Board of Education launched in 1990 a development project on mathematics education and curricula, and issued in the beginning of 1994 a new framework curriculum for the comprehensive school (National Board of Education, 1994). This framework curriculum started a new kind of education and curricular culture in Finland. There was a clear shift from a centralised curriculum system to a decentralised system, where instead of uniform national curricula the National Board of Education now issues curricular guidelines, while the Ministry of Education determines the allocation of lesson hours across school subjects, and schools then accordingly make up curricula of their own. Another important change was that learning materials no longer needed the National Board's approval. So, schools were given more freedom and responsibility for their own curricular development, which commenced in 1994–95.

Despite rather strong aspirations for reform, the mathematics section of the framework curriculum for the comprehensive school 1994 included but little changes as compared to the previous guidelines from 1985. The objectives for mathematics education thus continued the line adopted in the early 1980s, which after a "back-to-basics" period emphasised problem solving and application of mathematical knowledge.

During the comprehensive school all students are given the opportunity to get such basic mathematical knowledge and skills which create a foundation for further studies and prepare them to get along in everyday life and in working life. The aim of the comprehensive school mathematics is most of all to develop student's ability to classify, organize and model situations that come up in the surrounding world, with terms he/she has learned.

(National Board of Education, 1994, p. 81)

The main difference from the guidelines of 1985 was that now the objectives and contents of mathematics education were presented in a concise and generic form by school level (about 2 pages in total), whereas previously they had been described in great detail and by grade level. As a result from the school-based curricular process, variation in mathematics curricula has increased in Finland. Since the mid-1990s, there have been differences in schools' mathematics education in terms of the order of presentation (course structure) and elective courses, for instance. Variation has increased in learning materials, as well. Some results of the TIMSS 1999 and PISA 2000 studies can be better understood within the curricular context described above. The relatively weaker performance for algebra and geometry in TIMSS 1999 receive new interpretations when considering how well the items match with the Finnish mathematics curriculum and whether or not the item contents had already been dealt with at the 7<sup>th</sup> grade. Based on expert estimates. alone, roughly a half of the geometry items (21 items in total) were judged to lie outside the scope of curriculum for this age group. More specifically, such contents typically related to congruence and similarity as well as to symmetry and geometric transformations. When the relationships between students' learning opportunities and test performance were analysed in the light of teacher questionnaires, it was found out that the contents of many geometric and algebraic items had been taught very unevenly across Finland. For example, as regards solving a linear equation, 43% of the teachers reported that this had not yet been taught. It seems that in Finland many geometric and algebraic contents are addressed at a later stage than in many other European countries, for example.

The current content area of comprehensive school geometry has long been criticised for its inadequate organisation. For example Silfverberg (1999) regards that the instruction of geometry should not consist of teaching merely computational geometry. He sees that in order to promote students' conceptual knowledge of geometry, changes are needed both in learning materials and in teaching methods. When preparing curricular guidelines, we should also carefully consider what purposes the instruction of geometry serves at the different stages of basic education.

On the other hand, one obvious explanation for the Finnish success in PISA 2000 can be traced back to the goal setting of our mathematics curriculum. As application and problem solving have been strong principles in Finnish mathematics education since the early 1980s (Kupari, 1994), and because the PISA programme puts emphasis on young people's capability to utilise their mathematical skills and knowledge in situations that are as authentic and close to daily-life needs as possible (OECD, 2000), most PISA items can be considered well suited to Finnish students. It is therefore fair to say that curricular objectives have yielded good results.

At the beginning of 2004 the National Board of Education has confirmed a new framework curriculum for both comprehensive education and upper secondary education. The reform was prepared in working groups looking into subject and student evaluation. These working groups consisted of representatives of various expert organisations. A co-operation network consisting of education providers and schools with their principals and teachers participated in curriculum planning. The framework curriculum for grades 1–9 is more detailed than before. It defines the aims and key contents of different subjects and thematic entities, and provides guidelines for student assessment. The objective is uniform basic education, i.e. a continuum through grades 1–9. The new curriculum must be introduced in comprehensive schools by August 1, 2006.

### The LUMA programme

The first Government led by Prime Minister Lipponen set an official objective to raise the Finnish mathematical and scientific competence to an internationally competitive level (Opetusministeriö, 1996). On this basis, the National Board of Education launched a development project in 1995 on mathematics and science instruction, which essentially sought to build an operational network of pilot schools and municipalities for this purpose. In 1996 the Ministry of Education extended the project to cover various parties outside school, as well, and published a wide-ranging development programme, LUMA, for the years 1996–2002, to promote mathematical and scientific competence (LUMA is an acronym for the Finnish 'luonnontieteet ja matematiikka', i.e. science and mathematics.).

The LUMA programme was thus a six-year-long project involving various parties. At the outset there were six, and after an interim evaluation in 1998 seven, main objectives defined for the programme, both quantitative and qualitative ones. To attain these goals, ten projects with several subprojects were commissioned (Opetusministeriö, 2002). In view of mathematics education, especially three of the LUMA objectives deserve a closer look here (Opetusministeriö, 1999, pp. 36-37):

- Pupils and students will have varied knowledge and skills in mathematics and natural sciences, mastering in particular the use of main concepts and the application of knowledge as well as experimental and observation skills. In international comparisons (e.g. PISA and TIMSS 1999), Finland will be among the top quarter of the OECD countries.
- Over 17000 candidates will take advanced mathematics in the matriculation examination each year.
- The number of mathematics and science teachers will meet the needs of educational institutions and other educational activities. Each year, at least 140 subject teacher graduates will have majored in mathematics.

The core operation environment of LUMA consisted of a development and information network involving 78 municipalities and 10 training schools. Governmental contribution focused chiefly on increasing the number of university places for mathematics, science, and technology; on teacher's in-service education as well as on purchasing equipment for the pilot secondary schools. Municipal investments in the action varied considerably. Teachers were the pivotal resource in the programme and they did most of the development work in their own time beside their ordinary job.

Much was achieved in the programme and several useful development activities for mathematics education and teacher preparation were successfully promoted as described below (Opetusministeriö, 2002; Aroluoma, 2001).

Students' enhanced opportunities to learn mathematics can best be seen in schools that have adopted flexible teaching arrangements. In mathematics education such arrangements refer to simultaneous teaching groups with varying compositions. Here students can choose among the parallel groups the one that best corresponds to their current needs and aptitude in terms of syllabus and pace of progress. Such arrangements have been welcomed by parents, students, and teachers alike. Of course, there are practical and organisational problems related to flexible teaching groups, as well, and some schools fear, moreover, the label of streaming. Students' learning opportunities have also been improved by introducing an additional lesson hour for mathematics at the upper grades of the comprehensive school.

Integration of mathematics to science and other application areas has been a prominent feature in the LUMA programme. Teachers' co-operation across different subjects has clearly increased and motivation has improved both among teachers and among students. In addition, integration of mathematics instruction between different school levels has been enhanced.

Teachers' in-service training has all the time been an integral part of the LUMA scheme. Such training has been provided on a regular basis and with quite good resources. Teachers have been offered short-term courses as well as larger study modules leading to university degrees. This has yielded plenty of study credits and teachers' feedback on the education received has been positive.

Already previously, before the extensive in-service efforts in the LUMA project, a very high ratio of the mathematics teachers working in the schools has been professionally qualified. For example, in the TIMSS 1999 study 91 per cent of the mathematics teachers were qualified. The good standard of professional qualifications is further illustrated by the fact that more than three-quarters of the teachers had a very high confidence in their preparation to teach mathematics (Mullis et al., 2000).

Finnish mathematics teachers also have various opportunities to influence the contents and structure of instruction. Firstly, they can contribute to the mathematics curriculum, as the schools and municipalities are responsible for curricular planning and development on the basis of the framework curricula issued by the National Board of Education. Moreover, teachers have a great deal of influence on many essential elements of mathematics instruction. The results of the TIMSS 1999 school questionnaire indicated that when compare to the international average teachers in Finland are vested with clearly greater responsibility for choosing the textbooks used, what homework and student grading policies they adopt, and also for determining and organising course contents (IEA, 2001). Finnish teacher training has been described more closely, for example, in a publication titled "The Finnish success in PISA – and some reasons behind it" (Välijärvi et al., 2002).

Development activities on teacher preparation are very important also in longer terms. As regards classroom teacher preparation, in selecting students for these programmes the aim is to put weight on student grades in mathematics and science achieved in the upper secondary school and in the matriculation examination; to increase the share of mathematics in general studies; to develop study modules for a minor subject degree in science and mathematics, and to increase opportunities for degreeoriented studies. As regards the preparation of specialist subject teachers, learning of the subjects to be taught will be developed so that it will also support teacher's professional growth. Kindergarten teacher programmes, in turn, will include methods and contents to develop mathematical thinking, and for those seeking teacher's qualification with a professional or vocational background there will be individualised Master's programmes. (Opetusministeriö, 1999).

To what extent have the above-described goals of LUMA been achieved? According to the final report by the Ministry of Education, some progress was made towards all these goals and in some respects even to a high degree (Opetusministeriö, 2002):

- In PISA 2000 the 15-year-old students of the Finnish comprehensive school reached the goal set in the LUMA programme (the fourth place among 27 countries, which means a position within the best quarter of the OECD countries). In TIMMS 1999 our seventh-grade students did not quite reach the best OECD quarter, but were still clearly above the international average.
- In 2001 there were 13864 upper secondary school graduates with advanced mathematics courses in Finland. However, because not all of them took advanced mathematics in the matriculation examina-

tion, the objective of 17 000 examinees was still quite far, although even this objective is considered too low in view of the university entrances. Further efforts are therefore needed both to increase the number of students choosing advanced courses and to support beginning university students who may come in with less extensive background knowledge and struggle with their initial studies.

The number of subject teacher graduates majoring in mathematics has clearly increased over the past ten years. Their number came close to the annual objective of 140, as the figures for years 1999 and 2000 were 131 and 127, respectively. In 2001 the number of such graduates plummeted (to as low as 70), however, without no obvious reason attributable to the annual entrance statistics, for example. Still, providing that the overall graduation rate of recent years is sustained and the graduates also choose teacher's career, we will be able to fill with qualified teachers most of the gap left by the retirement of the large age cohorts of subject teachers, which is taking place within the next ten years. Anyway, it would be good to increase the numbers of entrants and graduates for subject teacher preparation.

The LUMA programme was also subjected to an international evaluation. The evaluation team was assigned to review the implementation of the programme from an international perspective and to recommend measures to improve mathematical and scientific competence in the future.

The evaluation team considered the LUMA programme generally successful (Allen, Black & Wallin, 2002). They found that LUMA had essentially influenced many teachers' work. It had helped teachers launch new projects, make use of new ideas in their teaching and start co-operation among and across different teachers, school subjects, schools, universities, polytechnics, and municipalities. According to the evaluation team, many teachers appreciated that the persons in charge of the programme had frequently visited LUMA schools and events. An essential part of the whole project was the LUMA network and its coordinators and the new ideas disseminated over this network. In addition, individual teachers and municipalities played an important role. Teachers' in-service training was also an important channel for disseminating developmental ideas.

Speaking of weaknesses, the evaluators pointed out that the programme had included many teachers for whom it made no difference. In many cases teachers' heavy workload and lack of time were the biggest obstacles for taking interest in LUMA. In the evaluation team's opinion, it is by no means certain how successfully the ideas of LUMA will disseminate in the future, unless specific actions are taken to this effect. Another problem pointed out by the international evaluators was the weak research basis of LUMA. In addition, the team saw some weaknesses in the matriculation examination in its current form, which hinder the realisation of LUMA objectives and the evaluation of their outcomes.

The international evaluation team considered that sustaining the development networks and dissemination of good practices is pivotal also after the programme itself has ended (Allen et al., 2002). Also evaluation and assessment systems should be revised to clearly support the curricular objectives. Especially the matriculation examination should be developed so that it would decrease universities' need for separate entrance tests. The consistency of student assessment by teachers in different schools should be checked at the end of the comprehensive school as well as in the upper secondary schools. The evaluators also suggested that reasons for the deterioration observed in new university students' mathematical skills be investigated and a strategy be developed to solve this problem. In addition, the universities providing subject teacher preparation in mathematics and science should review their programmes for better co-operation between their subject-specific and teacher training departments and for ensuring balanced education.

#### Future prospects

Mathematics education in the Finnish comprehensive school is making good progress. Internationally competitive performance in mathematics is, of course, attributable to a whole network of interrelated factors. The Finnish school system seems to be successful in providing the majority of its students with a solid foundation for further schooling and for transition to working life. The long-term curricular measures and instructional development schemes, like LUMA, have been essential framing factors. Even though it was not possible to establish numerically the effects of the LUMA programme on mathematics performance, it is likely that the programme has created new educational opportunities and aroused new enthusiasm. Also students' self-confidence in learning mathematics as well as their home background seem to be important factors related to their performance (Kupari & Törnroos, 2003). Furthermore, the high standard of mathematics teachers' education and professional skills as well as regionally adequate distribution of qualified teachers are factors that are likely to contribute to student performance. All in all, it is important to note that most of these factors can be influenced by our national policy measures.

Teachers can influence students' self-concepts as learners of mathematics and also their attitudes toward the subject by providing everyone with experiences of success as much as possible, hence reinforcing the notion that the students really are able to learn mathematics. When students see learning sensible and meaningful to themselves, they also go into it and will get better results. At the same time this sets new challenges to teacher preparation. Indeed, teacher-training programmes should include adequate opportunities to deal with the significance of self-concept and learner attitudes. Likewise, they should consider how (prospective) teachers' own views and beliefs shape their teaching.

Evaluations on the current state of Finnish mathematics education give a good starting point for its pedagogical development. The high overall standard of our comprehensive school is an asset that enables us to take care of the low achievers and at the same time to motivate the top performers to use their learning potential to the full. Such positive thinking based on our own strengths offers a fruitful basis on which to build new mathematics education aiming at even better results.

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# Yhteenveto

Artikkelissa tarkastellaan eräitä kehityspiirteitä, joita suomalaisen peruskoulun matematiikan opetuksessa on tapahtunut viimeisen 15 vuoden aikana. Erityisen syyn tällaiseen analyysiin ovat antaneet vuosituhannen taitteessa toteutetut matematiikan ja luonnontieteiden kansainväliset arviointitutkimukset, joiden perusteella suomalaisoppilaiden matematiikan suoritukset ovat hyvää kansainvälistä tasoa. Suoritusten taustalla on tietenkin lukuisten, toisiinsa nivoutuneiden tekijöiden selitvsverkosto. Peruskoulujärjestelmämme näyttää onnistuvan tarjoamaan vankan ja tasa-arvoisen perustan nuorten jatko-opinnoille ja työelämään siirtymiselle. Pitkäjänteiset opetussuunnitelmalliset ratkaisut ja mittavat opetuksen kehittämishankkeet ovat olleet tärkeitä opetuksen kehystekijöitä. Lisäksi matematiikan opettajien korkeatasoinen koulutus ja ammattitaito ovat ilmeisesti myös hyvien suoritusten taustatekijöitä. Oppilastasolla itseluottamus matematiikan oppimiseen näyttää olevan vahvimmin suorituksiin vhtevdessä oleva tekijä. Puutteiden ja ongelmien korjaamiselle omia kansallisia vahvuuksia korostava kehittämisilmapiiri antaa myönteisen lähtökohdan.