Methodological Considerations on

Investigating Teachers' Beliefs

of Mathematics and Its Teaching

Erkki Pehkonen & Günter Törner

Our primary concerns are the methodological considerations on investigating teachers' beliefs of mathematics teaching using a quantitative as well as qualitative approach. However, a discussion of this type cannot be completely detached from the textual determination that beliefs result from. Hereby, Dionne's and Ernest's characterizations of beliefs on mathematics served as a theoretical background; the dominant perspectives on mathematics can be described as toolbox aspect, system aspect and process aspect. Originally, our test subjects numbered a total of 13 experienced German mathematics teachers. However, we will limit the scope of the discussion to research on six representative persons. We used three data-gathering methods: questionnaires, videotaped interviews and graphical as well as numerical self-estimations, respectively. In our investigation a comparison of the self-estimations stands in the foreground. Since this information is mainly overlapping, partly redundant and likely contradictory, we have to question the data in order to describe the beliefs of the teachers. The research was conducted during the spring and summer of 1994.

1 Theoretical framework

It is imperative that any research conducted on teaching and learning within a framework of constructivism (e.g. Davis & al., 1990) should take into account teachers' and pupils' beliefs on mathematics and its teaching if we are to completely understand their behavior (Noddings, 1990). Even in the early 1980s there was evidence that these beliefs are part of different philosophies of teaching mathematics. It was Lerman (1983) who made it obvious that different philosophies also lead to different teaching practices (e.g. Ernest, 1989; Furinghetti, 1996; Lerman, 1983; Schoenfeld 1992).

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1.1 Definition of beliefs and the impact of beliefs

The purpose of this subsection is to give an overview of the terminology used in this paper. For a survey of research on teachers' beliefs, the reader is referred to the research syntheses of Thompson (1992) and Pehkonen (1994). However, there are hardly any articles focusing on mathematical beliefs of German mathematics teachers.

The concept of *mathematical beliefs* has many definitions in the literature. We adopt mainly Schoenfeld's (1998) definition that defines beliefs as mental constructs that represent the codification of people's experiences and understandings. We point out that affective loadings are also included (McLeod 1992). In general, indisputable ground for the beliefs may not necessarily be found in objective considerations. One feature of beliefs is that they can be held with varying degrees of conviction (Thompson, 1992). Beliefs cover personal convictions mixed with facts and external knowledge, so the subjective certitude of the beliefs ranges from truth-like facts to vague assumptions. However, the individual is not in all cases aware of the truth-degree of the beliefs.

The process of how and for what reasons a belief is adopted and defined by the individual him/herself is not well understood. The adoption of a belief may be based on some generally known facts as well as beliefs, and on logical conclusions drawn from them. But in each case, the individual makes his or her own choice of the facts and beliefs to be used as reasons and his and her own evaluation of the acceptability of the belief in question. This question has been partly discussed in Schoenfeld's book (1998). So, often there seems to be no objective distinction between facts and beliefs for the individual.

The individual compares his/her beliefs with new experiences and beliefs of other individuals, and thus beliefs are subjected to a continuous evaluation and to change. When an individual adopts a new belief, it will automatically form a part of the larger structure of his/her personal knowledge and of the belief system, since beliefs never appear fully independently (Green, 1971).

Green pointed it out in 1971 that beliefs always come in sets or groups. Thus we assume that an individual's beliefs form a structure. We will call this construct a belief system or more generally his/her *views of mathematics.* This wide spectrum of beliefs related to mathematics contains four main components that are also relevant for mathematics teaching: (1) beliefs about mathematics, (2) beliefs about oneself as a user of mathematics, (3) beliefs about teaching mathematics, and (4) beliefs about learning mathematics. These main groups of beliefs, in turn, can be split into smaller units. It is evident that these "dimensions of beliefs" are interrelated. For more details on these ideas see Pehkonen (1995).

Törner & Grigutsch (1994) and Grigutsch, Raatz & Törner (1998) attempted to investigate structures of belief systems using factorial analysis. They used the term "mathematical world view" which originally can be found in Schoenfeld's discussions (1985).

1.2 Beliefs on Mathematics and its Teaching

There are numerous papers focusing on the aspect of the nature of mathematics in terms of a belief system. For his research, Dionne (1984) used the following three perspectives of mathematics:

A Mathematics is seen as a set of skills (*traditional* perspective): Doing mathematics is doing calculations, using rules, procedures and formulas.

B Mathematics is seen as logic and rigor (*formalist* perspective): Doing mathematics is writing rigorous proofs, using a precise and rigorous language and using unifying concepts.

C Mathematics is seen as a constructive process (*constructivist* perspective): Doing mathematics is developing thought processes, building rules and formulas from experiences of reality and finding relations between different notions.

It seems obvious that these perspectives reflect guiding aspects of mathematics. However, different persons might evaluate these components differently and so Dionne let his test subjects assign weights to these "dimensions". Ernest (1991) describes three similar views on mathematics: instrumentalist, Platonist and problem solving. These categories correspond more or less to the three perspectives outlined by Dionne (1984) that are mentioned above. Of course, there might be further relevant perspectives characterizing mathematics (see the book of quotations by Schmalz (1993)), e.g. mathematics as an art. However, these three seem to be the leading ones in school mathematics.

Furthermore, the same three-component model was used ten years later by Törner & Grigutsch (1994) using factorial analysis in a quantitative research project, calling (A) the *toolbox aspect*, (B) the *system aspect* and (C) the *process aspect*. Recent research by these authors shows that there exists at least a fourth basic component in an individual's view of mathematics, namely the role of application within mathematics (Grigutsch, Raatz & Törner, 1998).

2 The design of the research

2.1 The general setting

The literature contains abundant contributions concerning teachers' beliefs (see Thompson, 1992), but only marginal attention is dedicated to the corresponding methodological questions concerning the research into these beliefs.

Doubtlessly dependent on the favored methodological paradigm of the time, there are quantitative as well as qualitative investigations on teachers' beliefs; the research in each case depends heavily on the available resources.

With respect to quantitative investigations, finding large, statistically homogeneous, sufficient target populations, in which quantitative tools can be introduced, is known to be rather tedious (e.g. Grigutsch, Raatz & Törner, 1998 with N = 300). Administrative restrictions exercised by schools make generating such a population difficult. As a rule, permission has to be granted by the local school authorities to enable such a survey to be conducted. One must rely primarily on private contacts with individual teachers, which could possibly produce an inconsistent representation. On the other hand, in order to keep the questionnaire as brief as possible, the scope of the observed parameters has to be limited, in particular concerning the subjectivity of the researchers.

Qualitative methods employing natural interviewing techniques (e.g. Lincoln & Guba, 1985) are, as a rule, extremely time consuming. This leads to an actual limitation of either the number of test persons or the time allotted for each test person. Therefore, research papers often restrict their discussion to a very small number of test subjects, say three or four (see Schoenfeld, 1998).

Since the setting for the results to be presented here was viewed as a phenomenological, cross-sectional inquiry and, therefore, had to consider teachers' biographies from the four different German school forms, we decided to mix quantitative as well as qualitative tools. This approach can be called methodological triangulation (see Cohen & Manion, 1994, p. 233). Finally, the time given for the entire project was set at two months. In this case, the inquiries had to be limited in order to provide time for further, suitable questions after a temporary evaluation was concluded.

In the videotaped interviews, the teachers were given as much freedom to report on their everyday classroom situations as they wanted. The questionnaire was sent to the teachers beforehand. After one or two weeks we interviewed the teachers and collected the filled-in questionnaire (see Section 2.3.2). Furthermore, we gave the teachers an opportunity to discuss the relevant items and their answers with the authors. Again, this discussion was videotaped. Finally, asking the test subjects to categorize their views on mathematics using the wording of Dionne–and therefore we call the polarization "Dionnian" (see Section 1.2)–led to a very restricted, low dimensional characterization of the test subjects. Since the test subjects mailed us their answers, we had no chance of re-discussing the results.

We viewed these various restrictions, which at a first glance appeared to be disadvantages, as a challenge in which our primary research could contain a methodological concentration and which, therefore, could be used to our advantage. This could be accomplished, namely, by placing the compatibility and consistency of the teachers' statements of the time at our disposal.

Therefore, in this paper we introduce a method of self-estimation, which entails data collection aimed at investigating teachers' views of mathematics. Next, we present some condensing methods in order to evaluate numerical data using Hamming distances of closeness (see Table 3.3), which are well known in coding theory. We constantly compare the results with the verbal statements coming from the interviews and questionnaires. To our knowledge, these methods (graphical self-estimation, Hamming distance) have not been reported previously in mathematics education literature.

2.2 The research questions

With respect to the considerations above, our main research question therefore reads:

1 How well does information from different methodological sources and using different methodological tools to investigate teachers' beliefs of mathematics fit together? In particular, to what extent do the data attained by the Dionne method offer relevant information concerning self-estimation?

Through this question (1), the next problem is obvious:

2 Which method is best suited to which aspect?

Since our theoretical framework on views of mathematics relied heavily on the Dionne-Ernest-categorization (Section 1.2), with question (2) we should be able to investigate the validity of the information that is obtained with numerical self-estimation (the Dionnian method), as compared to the questionnaire and interview method. Hereby, in particular, the validity of the graphical self-estimation employed by us is to be tested and analyzed.

2.3 Data gathering

As to research methodology, we wanted to emphasize triangulation in data gathering, i.e. the use of many simultaneous data-gathering methods (e.g. Denzin, 1970; Cohen & Manion, 1994). The reason for this decision was the fact that the research methods seem to act as a filter through which a researcher experiences his/her surrounding selectively. The simultaneous use of several data-gathering methods adds to the researcher's possibilities to grasp complex reality. In this study we used the information obtained through interviews, questionnaires and self-estimations.

2.3.1 Interviews

The teachers were asked to report on their careers in detail, stressing particular core ideas and tendencies. We generated four main questions as follows:

– Describe your "history" as a mathematics teacher.

– How did you teach in the very beginning?

– How do you teach today?

– Can you name some factors which might have had an influence on changing your teaching methods?

The teachers interviewed were provided with some additional questions, according to the situation.

The set of interviews extended over two months (May, June, 1994), taking place in the teachers' homes or schools. The interviews were videotaped, ranging from 40 to 60 minutes in length. Both authors were present at each interview in order to obtain two different viewpoints of the situation.

2.3.2 Questionnaire

The questionnaire contained thirteen statements about teaching principles in school mathematics (see Appendix 1). In order to activate the teachers, who were unknown to us, we sent the questionnaire beforehand to introduce the teachers to be interviewed to the theme. The thirteen items of the questionnaire represent aspects that had emerged as the result of a factor analysis in another study on teachers' conceptions, and were based on previous questionnaire results (Pehkonen & Lepmann, 1995). The teachers were asked to react to these thirteen statements on a five-step scale: 1 = fully agree, 2 = agree, 3 = don't know, 4 = disagree, 5 = fully disagree. The process of answering and discussing the items of the questionnaire was videotaped and served as a data source.

Each interview, including the comments while filling out the questionnaire, was discussed and evaluated thoroughly the same day. In addition, since all sessions were videotaped, the authors were subsequently able to review the tapes.

2.3.3 The teachers' profiles

Based on the information from the interview and the questionnaire, we constructed a profile of each teacher that included the following components:

- Time and place of the interview,
- Position within respective school,
- Teaching experience,
- View of mathematics (today),
- Own view of his/her personal change,
- Change factors mentioned,
- Comments regarding the questionnaire.

We gave the teachers an opportunity to express their opinions about their profiles in order to validate our interpretations. Thus, in July 1994 each teacher received his/her profile by mail for reviewing. They had two weeks to respond, if they thought we had reached incorrect conclusions.

The teachers were satisfied with our interpretations of their mathematical conceptions. Only one teacher of the thirteen wished to make one small addition. He requested the addition of one detail with regard to his views of mathematics concerning change.

2.3.4 Self-estimation

As in Dionne's paper (1984), we asked the teachers to distribute a total of 30 points among three categories representing their perspectives of mathematics. These points, the so-called Dionne-parameters, were written in tabular form.

Since there was some confusion in our earlier attempts with the Dionne numbers, with respect to the view of mathematics under consideration, we explicitly allowed the teacher to distinguish between his *real* and *ideal* view of mathematics. The Dionne-parameters compel the interviewee to give an unambiguous statement without room for variation. Through the distinction between the real and ideal view, however, the teacher is given free space to answer more freely. This, in turn, enables us to pay some specific attention to the effects of transition in the self-estimations, with respect to their real and ideal view of mathematics respectively. By means of this twofold data acquisition, we have also attained information on the objectives propagated by the teachers. More recent research approaches (Schoenfeld, 1998) focus in particular on the interplay of knowledge, beliefs and goals.

The self-estimation request forms (see Appendix 2) were mailed to the teachers in August 1994. The teachers were asked to fill them out and return them within one month. A 100 % response rate was achieved.

2.4 Test subjects

With the aid of the local school authorities in Northrhine-Westfalia in the area of Düsseldorf and Duisburg, we sought possible test persons. Our aim was to interview approximately ten middle-school teachers in the spring of 1994. The teachers were expected to have had at least 10 years teaching experience. Furthermore, they were expected to be innovative in their teaching, at least according to the school administrators who provided us with their addresses. Originally, we wanted to include in this research secondary teachers from each of the school forms in Germany (*Gymnasium*, *Realschule*, *Hauptschule*, *Gesamtschule*; see Appendix 3). Because of varying teaching practices, different curricula etc., the teachers in the various schools might think differently about teaching mathematics.

Our test subjects were 13 experienced German teachers of mathematics, five of whom were from the *Gymnasium*, two from the *Realschule*, one from a *Hauptschule*, and five from the *Gesamtschule*. Three teachers were female; all the others were male. It was not easy to find teachers from the *Realschule* and the *Hauptschule* who were willing to be interviewed. This reflects the fact that the *Hauptschule* must unwillingly play the role of a "left-overs'" school, as, in particular, a high percentage of children from groups with social deficiencies tend to attend the *Hauptschule*.

Here we will restrict ourselves to the results of only six teachers. We find the restriction justifiable for the following reasons:

- Firstly, it is necessary to condense enormous amount of available material.

- Secondly, our main focus is on the phenomena surrounding methodological investigations and not on the diversity of different teachers' views on mathematics and its teaching.

- Thirdly, these six teachers cover the main effects observed in the test group. The six teachers were chosen to represent all the test persons (13 teachers in all), reflecting different formal aspects (see below).

In order to maintain confidentiality, we will refrain from mentioning the test subjects' gender and will give each teacher a male pseudonym: *Dylan*, *Harry*, *Henry*, *Joseph*, *Ken*, and *Larry*.

- Formal Teacher Certification: Teachers *Dylan*, *Henry*, *Ken* and *Larry* graduated from university with the same degree, completed an upper secondary level degree (see Appendix 3) and are qualified to teach mathematics, particularly from grades five to thirteen. Teachers *Joseph* and *Harry* obtained the lower secondary level degree, enabling them to teach at the lower secondary level (grades five to ten).

- Teacher Profiles: On the basis of the interviews there are two primarily mathematically centered teachers (*Ken*, *Larry*), two rather innovative teachers (*Harry*, *Dylan*) and two predominantly rigid-thinking teachers (*Joseph*, *Henry*).

- School Forms: The teachers were from different school forms: Gymnasium (*Ken*, *Larry*), Gesamtschule (*Dylan*, *Henry*), Realschule (*Joseph*) and Hauptschule (*Harry*).

Other facets of our research, e.g. change factors for professional development, are reported elsewhere (Pehkonen & Törner, 1999). Within each of the two (qualification) groups, the teachers should be comparable with respect to their mathematical competence.

2.5 Methodological considerations regarding the data analysis

Using a questionnaire methodology, researchers usually remain on the surface level of beliefs. Only conscious beliefs may be extracted. Furthermore, the test persons may choose only those conceptions which they think are appropriate to respond to the statements and which they are willing to reveal. In addition, it cannot be ignored that the belief system of the researcher greatly limits the results of his inquiry. As one of the authors is primarily a didactician and the other a mathematician, we believe to have found a fruitful balance in our research.

Although the above also applies to other techniques, with interviews an attempt may be made to go deeper, as well as to find out about the subconscious beliefs which lie behind the explicated conceptions. Since the structured interview often remains on the same level as a good questionnaire, the interviews here were conducted according to the methods of a theme interview that allows the test subject much freedom. We had some main questions, which we showed the teachers a few days beforehand, and which formed the core of the discussion. During the interview we asked additional questions if we felt that we had not yet extracted ”all the answers” to our main question. The narrative mode of interviews encouraged the teachers to reflect on their past experiences and on the feelings associated with them.

Whereas the interviews are of primary importance in our complete research (Pehkonen & Törner, 1999), in particular to change factors, the twofold self-estimations provide us with information balancing different categories of mathematics and mathematics teaching. It is the interaction of the two frameworks, namely the real and ideal teaching, which allows some insight, in particular since we use two different representations of the transition process, the numerical and the figurative.

The nature of the data gathering leads to inductive (not deductive) data analysis, since it is in this way more likely to identify new phenomena from the data. The inductive data analysis differs, however, from the conventional semantic analysis.

Obviously, our three-fold investigation on related topics led to overlapping information. The question arose as to how to handle redundant, possibly contradictory information. First of all, we tried to fit this information together as closely as possible, like a puzzle. Secondly, if the statements, however, only correspond to each other conditionally, we prefer that the self-estimation and the questionnaire remain our predominant source in the foreground of this paper, whereas the interviews could serve us by completing and explaining information. Thirdly, we are now in a position to estimate the advantages and disadvantages of independence of the respective methods, due to the abundance and variety of information and the possibility of distinguishing between the test persons. This is especially true with reference to the two statements of the self-estimation, and both the numerical as well as the pictorial estimation.

3 The results

We will present the results of the various surveys in the following section. We will include specific remarks and explanations about the interviewees when necessary. The global assessment of the results obtained by different methods will be discussed in Section 3.4.

3.1 Characterizations through interviews

In addition to the formal characterization of the teachers (see Section 2.4), it seems helpful to us to present an initial short introduction of the six teachers in question by using quotations originating in the interviews. At the same time, these quotations deal with the central statements from the perspective of the interview.

Dylan's view of mathematics and its teaching are revealed in the following quotation[1]: "I do not regard mathematics as dry, I find it fascinating. To me, mathematics is alive and I derive pleasure from it." And furthermore: "It is always of importance to me that the teacher makes the textbook understandable to the students ... That forms a visible line throughout my teaching, up to the graduation examination." Whereas the next quotation shows that for him formalism is not important: "Proof, just for the sake of proof I regard as arrogant abundance. Pillars could support mathematics just as well as solid walls."

Harry's mathematical view is strongly process-oriented: "I disapprove of any product-orientation, I regard the process as being too important" is a statement he mentioned several times. The core idea of his teaching is described by "We have to find a way to meet the demands of the teaching-market". *Harry* and his students have fun and derive pleasure from mathematics, "It is a decisive factor that education should be enjoyed by my students as well as by me."

Henry's view of mathematics is toolbox-centered: "What works well is giving formulas. When you give the formulas which are present in the classwork, you get the results." His statements are reminiscent of a factory worker who manufactures products. Another notable point is his frequent use of the word "thing" when referring to mathematical contents. Furthermore, he, himself, offers another metaphor, namely that of a nursery school teacher who, "... leads small children by the hand through a garden without leaving the path, in order not to confront unexpected things". This teacher makes a tired, uninvolved impression.

Joseph's view on mathematics can only be indirectly considered as belonging to teaching mathematics. He understands his teaching as a continuous use of worksheets, which points to the toolbox aspect. His principle of teaching is reflected through the following quotation: "The

[1] The second author has done the translation from German to English.

teacher-centered lesson has proven to be successful because questions could be answered and problems solved with regard to the whole class. *Joseph* makes a worn-out, resigned impression, and is often preoccupied with his own thoughts.

Ken is always in the process of keeping his mathematical knowledge up-to-date on an elaborate level. His view can be considered as well reflected, detailed and balanced. This interpretation is supported by the following quotation: ”In the beginning, I was strongly structurally-oriented; fractions were dealt with rather formalistically... and Freudenthal was not much of a help either because he, too, is very rule-oriented in this regard”. He continues, ”My openness implies that I try to involve more visual elements than formalism, which compels...” and further ”I came to realize that visuality stays in the memory longer, as well as association...” As a teacher, he is realistic, pragmatic and at the same time lacking illusions: ”I suspect that my teaching was teacher-oriented in the beginning, and I suppose it still is today”.

Larry regards mathematics as ”a colorful structure that allows the formal contents to be dealt with abstractly and systematically”. He is still in contact with his former university and would like to obtain some new impulses from there. His teaching is moderately teacher-oriented, but his starting point, in general, is mathematics-oriented. His knowledge of mathematical topics is convincing. Furthermore, during the interview there was no mention of teaching in groups; he in fact expresses several objections to project work.

Further quotations are integrated in the discussion following in the next subsections. Finally, in Pehkonen & Törner (1999) focusing, however, on the change aspect, one can find additional interview results.

3.2 Questionnaire data

In the questionnaire (Appendix 1), there were thirteen statements representing important teaching principles connected with different views on mathematics. The teachers were asked to grade them on a five-step scale. The teachers' responses are given in Table 3.1.

The teachers discussed these thirteen items as an appendix to the interviews, since we wanted to understand the full meaning of their statements. Item (3) turned out to be an extremely difficult one to respond to. The statement confused the teachers, since creativity and logic were put in opposition to each other. During the interview, nearly everyone wondered how the statement should be understood. And five teachers eventually refused to take a position on this statement. One of them was *Larry*, who in addition hesitated to respond to two further statements. His reaction reflected not his inability to answer the question, but his unwillingness to make short comprehensive or polarizing statements. Therefore, in addition to a blank answer with respect to (3), he also left item (4) and (11) blank. As a result of probable misinterpretations of item (3), we have deleted this item in the further discussion.

Table 3.1 Responses to the questionnaire

	Question number												
Teacher	1	2	3	4	5	6	7	8	9	10	11	12	13
Dylan	2	1	–	1	5	5	2	5	1	1	2	2	1
Harry	1	1	4	2	5	5	2	4	1	5	4	2	1
Henry	2	1	4	2	5	3	2	2	1	4	3	2	1
Joseph	1	1	2	1	3	4	2	3	1	5	2	2	2
Ken	2	1	4	1	5	2	1	4	1	5	4	1	1
Larry	2	1	–	–	5	2	1	4	2	4	–	1	2

Note. 1 = fully agree, 2 = agree, 3 = don't know, 4 = disagree, 5 = fully disagree, – = no answer.

It should be noted that *Henry* and *Joseph* in particular were indecisive. This applies to *Henry* with respect to items (6) (= the role of proofs), and (11) (= the role of calculation) and furthermore to *Joseph* with respect to items (8) (= mathematics for talented) and (5) (= the role of process versus product-orientation). (This pattern is supported also by the conversations within the interviews.)

By grouping the responses "fully agree" and "agree" under the plus sign plus (+), the responses "disagree" and "fully disagree" under the minus sign (–), and leaving out item (3) we obtain Table 3.2 where the neutral response (don't know or no answer) is represented by the sign ±.

We use the Hamming distance d to measure the internal relatedness of the teachers' answers (e.g. Hill, 1986, p. 5). The Hamming distance (over the ternary alphabet +, –, ±) between two vectors is defined as the number of places at which these vectors differ. Here we read the rows in Table 3.2 as 12-dimensional vectors whose entries consist of three elements, namely the symbols +, – and ± denoting "agreement", "disagreement" and "neutrality" respectively. Thus, the Hamming distance between two vectors counts the number of places where the answers differ and therefore represents a parameter for substantial differences in the answers (Table 3.3).

Table 3.2 The tendencies of the responses to the questionnaire

	Question number											
Teacher	1	2	4	5	6	7	8	9	10	11	12	13
Dylan	+	+	+	–	–	+	–	+	+	+	+	+
Harry	+	+	+	–	–	+	–	+	–	–	+	+
Henry	+	+	+	–	±	+	+	+	–	±	+	+
Joseph	+	+	+	±	–	+	±	+	–	+	+	+
Ken	+	+	+	–	+	+	–	+	–	–	+	+
Larry	+	+	±	–	+	+	–	+	–	±	+	+

Note. + = "agreement", – = "disagreement", ± = "neutrality".

Table 3.3 The "Hamming distance" between teachers on the basis of Table 3.2.

	Teacher				
Teacher	Harry	Henry	Joseph	Ken	Larry
Dylan	2	4	3	3	4
Harry		3	3	1	3
Henry			4	3	3
Joseph				3	5
Ken					2

It is natural to focus on those teachers whose answers are close to each other; i.e., who are similar by their profile through the questionnaire. By taking the smallest distances ($d \leq 2$) we derive Figure 3.4. Obviously, this figure evokes the question how to explain the closeness of the teachers' estimations. We offer some speculations that seem to make sense. The closeness of *Ken* and *Larry* ($d = 2$) is not surprising. It can be explained by the fact that both are mathematically highly qualified secondary school teachers at the same type school with balanced views on mathematics.

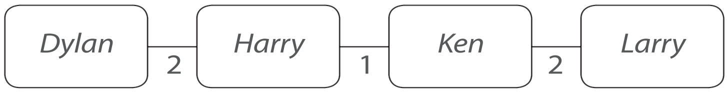


Figure 3.4 The graph of "neighboring teachers", derived from Table 3.3

The closeness of *Harry* and *Ken* is a striking point, since they are quite different teachers with respect to their mathematical qualifications. However, this result may be explained by the fact that both teachers have elaborated views on mathematics teaching. Clues by the questionnaire data cannot be found (see Section 3.3). Finally, the small distance d between *Dylan* and *Harry* reflects the fact that both teachers have in particular an innovative student-centered view on mathematics teaching that became obvious through the interviews.

3.3. Self-estimations

It is indisputable that Dionne's three-pole polarization, corresponding to the Ernestian categorization, must be seen only as a primitive model, in particular as through this approach itself the dimensionality of the perspectives is limited to three. Nevertheless, in spite of its simplicity, it still possesses a high degree of clarification, especially as a first approach to the problem of becoming aware of and identifying different views on mathematics. When collecting both the numerical and the graphic self-evaluation, one has access to different kinds of data.

3.3.1 The numerical self-estimation

In Table 3.5 we give the scores which teachers attribute to three components of the view of mathematics (see Section 1.2), and which was asked for in the letter sent to the teachers during the second round of the study (see Appendix 2).

Each teacher, with the exception of *Ken*, wanted to change, more or less, towards "process"; also, *Harry* and *Larry* wanted to emphasize the process aspect even more. At a first glance, the following is not surprising and became obvious through the table: None of the teachers chose an extreme position, neither in their real view nor in their ideal view. Thus Dionne's polarization is in a particular way balancing. Each teacher regarded the process aspect as the most important factor in his ideal view. *Harry* gave the process aspect the highest loading, which corresponded with his quotations in the interview, also regarding real teaching. It is noteworthy that the estimations of *Ken* and *Larry* were about the same regarding their real teaching, although they had not met each other before.

Table 3.5. Scoring of the self-estimation.

	Real			Ideal		
Teacher	Tool	System	Process	Tool	System	Process
Dylan	**15**	5	10	5	5	**20**
Harry	9	1	**20**	4	1	**25**
Henry	**14**	8	8	6	**12**	**12**
Joseph	**15**	3	12	10	5	**15**
Ken	8	10	**12**	10	8	**12**
Larry	9	9	**12**	6	9	**15**

Note. Tool = mathematics as a toolbox, System = system aspect of mathematics, Process = process aspect of mathematics. The leading positions are marked in bold.

With respect to the ideal view of mathematics by *Dylan* and *Harry*, however, there are small differences concerning the role of systems and structures in mathematics (*Dylan*, System = 5; *Harry*, System = 1). Note that the ”Hamming-distance”, according to Table 3.3, is 1. Minor differences may originate in their different academic careers. However, on the basis of the figures for the real classroom lesson, the assessment of *Dylan* (Process = 10) is considerably more rational than of *Harry* (Process = 20). Perhaps this discrepancy is explained by the fact that *Dylan*, in contrast to *Harry* has completed a higher university degree (see Appendix 3), so *Dylan's* mathematical horizon can be regarded as broader and so his estimation of what is happening in school is more modest.
Since there is no objective scale for the three mentioned aspects, the absolute numbers should not be overestimated. Moreover, it seems natural to us that primarily the weights set by the teachers, not the exact scoring, indicate their understanding of mathematics teaching, which leads to a linear ordering of the components. Thus, we derive Table 3.6.

3.3.2 Triangular approach

In Figure 3.7 we illustrate the marks within the equilateral triangle that were taken from the teachers' original responses (see Section 2.3.4 and Appendix 2).

Table 3.6. The ranking of the components derived from Table 3.5

	View of mathematics	
Teacher	Real	Ideal
Dylan	T > P > S	P > T = S
Harry	P > T > S	P > T > S
Henry	T > P = S	P = S > T
Joseph	T > P > S	P > T > S
Ken	P > S > T	P > T > S
Larry	P > T = S	P > S > T

Note. T = tool, S = system and P = process aspect.

There are three features which come to mind at first glance: (1) the distribution of the respective positioning, (2) the tendencies of change which are represented as vector arrows and (3) the magnitude of assumed change.

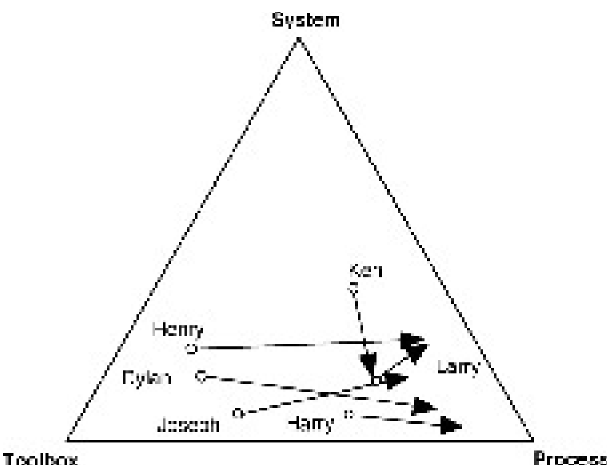


Figure 3.7 The self-estimation data in graphical form as given by the teachers. Arrows (real to ideal) indicating ten- dencies were drawn by the authors.
Note. The idea to show the tendency with arrows is due to Peter Berger (1995).

First, the predominant tendency of change for the teachers makes clear the importance of the process aspect. The two Gymnasium teachers *Ken* and *Larry* show some slight differences, in particular *Larry*. It should be noted that *Larry* does not estimate the necessity of change in his own classes as very high. Perhaps, *Larry* is satisfied with his teaching style.

Furthermore, the interview reveals that he does not believe in the possibility of fundamental change in the present system. Possibly the daily disturbances that take place in his classes lead only to marginal frustration, because for him (as is with *Ken*) mathematics exists outside of the classroom and, as a result, is a pure and philosophical discipline worthy of respect. Equally, he has fitted himself into the system.

Secondly, the diagram points on the whole to three classes in view of the arrow lengths: *Dylan*, *Joseph* and *Henry*, then *Ken* and *Harry*, whereby *Larry* takes the last position. These observations do not contradict the interview data when intervening feelings are included as a measure for change. In view of *Ken* and *Larry* we have already given explanations above.

3.3.3. Comparison of the self-estimations

The question arises as to which data should be more seriously regarded: the graphical or the numerical data, or whether both ”messages” are equivalent. Of course, the representation modes are equivalent in a mathematical sense, each three-element-distribution can be calculated as a vector in the triangle and vice versa.

Next, we believe that the teachers are not trained to transform numerical data into barycentrial coordinates. However, since the teachers have studied mathematics they should be able to handle graphics. Furthermore, we don't believe that the teachers attempted to present direct translations of both data sources.

Our hypothesis is that both representations have their own messages and may cover partly different aspects. For example, *Henry* estimated his mathematics view by Toolbox = 14, System = 8, Process = 8, thus the aspects of System and Process are playing an equivalent but lower estimated role. In the ideal teaching his scores are Toolbox = 6, System = 12, Process = 12. This feature is not reflected in the graphical representation where the System aspect remains unchanged. However, the length of the vector indicates his feeling that his real teaching differs greatly from ideal teaching.

Apparently there are some inconsistencies in *Ken's* numerical and graphical data. His estimations of real and ideal teaching show some interchanging of the roles of Toolbox and System, which should be represented by a reflection of positions within the equilateral triangle. On the other hand, *Ken's* arrow in the graphical mode calls for some change, in particular, towards more Process aspect and less System.

Joseph's data in both surveys also deviate considerably. If we compare the change vectors (from real to ideal) of *Joseph* and *Dylan* in Table 3.5, the vector of the latter is more than twice as long as that of the former. However, the lengths of the arrows in the graphical representation are in the same region of length. Under consideration of the presentations given in the interviews, this graphical information seems to reflect the situations more adequately.

Whereas it is easier to realize the tendencies and the direction of the changes in the graphical mode, the table may show some clues or patterns as to how the changes should take place. Note that all three, *Dylan* (System = 5), *Harry* (System = 1), and *Larry* (System = 9) would not be likely to change the absolute value of the factor System; they only prefer an exchange between the Tool aspect in favor of the Process aspect. It must remain an open question whether or not it is an intentional exchange, or perhaps whether it is just merely a strategy to treat the data that is to follow the Dionnian categorization and to distribute 30 points twice among three entries. These arguments again support our hypothesis that the derived Table 3.6 is no less important than the absolute values.

3.4 Evaluation of the information from the different sources

In the previous section we have already compared the self-evaluations so that a further discussion of these approaches is unnecessary. The question remains what correlation the results of the interviews, questionnaires, and Dionnian self-evaluations have.

3.4.1 Self-estimation versus interview data

Arguments and quotations from the interviews support many entries in the graph as well as in the table. In spite of this, the resulting analysis is not always self-explanatory and only the statements from the interviews can enrich this quantitative information in a qualitative sense. The problems are twofold: first the categories asked by the Dionnian approach heavily depend on the interpretation by the subjects; they have open meanings. Secondly, the figures provided by the test subjects are, trivially to say, highly subjective.

Of course, there is no canonic meaning and implication of what the toolbox aspect means. However, the teachers' description in the interview is fitting to this line: *Dylan* points out the importance of a student's ability to handle school books, whereby *Joseph* and *Henry* believe that routine exercises are the only possible way to motivate the majority of the students within a class. Again, the three persons mentioned are precisely those teachers who are not quite satisfied with their own teaching of mathematics and would like to change their situation, however through quite different ways. *Dylan*, *Joseph* and *Henry* are found, with their implemented lessons, in the toolbox corner, which is again confirmed by some corresponding quotes in the interview (see Section 3.1) and underlined by Figure 3.7 as well as in Table 3.5.

Apparently, with the exception of *Ken*, all teachers placed the system aspect at least second in their real teaching. This can be understood through the interviews: In the past, *Ken* was extremely in favor of the formalism aspect; this conception may not have disappeared completely. It is remarkable that *Larry* has given formalism and toolbox an equal ranking. This fact may probably be explained through their teaching career in a Gymnasium and its mathematics curriculum.

On the other hand, with respect to ideal teaching, the process aspect is ranked first by all of the teachers, and, in the case of *Henry*, on the same level as the system aspect. The interview data certify these observations where *Henry* favors a stronger dominance of formalistic aspects. This teacher seems to be obliged to mathematics, which he has been taught at the university to be of structural importance. In the interview, the authors got the impression that he feels somewhat guilty since his current situation makes it impossible to present this subject in an adequate manner, in his opinion.

The interviews depicted *Ken* and *Larry* as highly qualified in mathematics (see Section 3.1). Also, Figure 3.5 showed the teachers as being close in their estimations of mathematics (see Section 3.2). They are more or less satisfied with the real classroom situation. *Larry* points out that mathematics lessons are generally not very encouraging and motivating because of the subject. And therefore, *Larry* "… is continuously looking for external stimulation". *Larry* hereby underlines his need for external stimulation for variation of his own lessons.

On the basis of the interview (see 3.1), we may classify *Dylan* and *Harry* as the most innovative among these six teachers. However, using the method of self-estimations, it is by no means evident that these persons play such a striking role. Vice versa, it is unthinkable that the information from the self-evaluations is derived only from the interview data!

The ranking Table 3.6 derived from the scoring of self-estimation (see Table 3.5) would also be too rigid to allow detailed conclusions. Through the additional information we obtained through the interviews it becomes clear that a pure ranking would not be especially meaningful. On the basis of our data and using Table 3.6, test persons can be identified who, according to our additional knowledge, show different beliefs. In other words, determining beliefs is of little evaluative value when one asks the interviewees for rankings.

3.4.2 Self-estimation versus questionnaire data

The statements compliant to Dionne's model, as explicit as they may be, cannot be derived using the questionnaire method. In principle it is conceivable that teachers with similar answers on the questionnaire would show similarities in the self-evaluations. The patterns from Tables 3.1–3.4 are, however, not represented in the Dionne data. From that it must be inferred that the evaluation dimension of mathematics and the learning of mathematics must be discussed, as they cannot be represented in the simple Dionne model.

3.4.3 Questionnaire data versus interview data

Considerations hitherto have shown that few patterns clarifying the quality or pertinence of a particular question are generated through the questionnaire. For insights can also be insinuated by the results of the questionnaire itself, for example how differentiated the interviewee's responses to the single questions were, if balanced, rigorous, undecided, etc. However, the explanations for such resulting patterns (cf. Figure 3.4) are rather speculative on the basis of the interview information and the conclusions are in no way beyond dispute. Also their sums are quite unspecific.

3.4.5 Summary

The differentiation in Dionne's paper which in our realized form proved itself favorable, namely the strict differentiation between real and ideal instruction, is not represented in our questionnaire and is not always clearly separate in the interview statements.

4 Conclusion

The original underlying hypothesis of our research question (1) (see Section 2.2), namely that our methodical approach is to be understood as a triangulation, had to be revised in part. The collected data is only partly redundant, although it merges into a complete picture that could not have been drawn in such detail through any of the three approaches alone. In other words, the results of the various methods complement each other in both quantitative and qualitative respects. When one neglects both versions of the self-estimation, the resulting information can be assigned to various aspects of beliefs. By doing so it becomes clear that a view of mathematics and its teaching can barely be described through the Dionnian parameters.

The confrontation of the Dionnian parameter (real instruction versus ideal instruction) does, however, include information that surpasses any idealizing description of mathematics. Here at the latest we find interfaces to the interview data and in part also to the questionnaire data. In view of question (2) (see Section 2.2) we can state that there is no best (indirect) method that can investigate teachers' beliefs.

Firstly, the open interview as well as the open discussion of the items of the questionnaire show once more that closed questionnaires are of limited use. Since we opened our procedure, we had the possibility to correct some misunderstandings; e.g. we had to omit one item whose discussion showed the inadequacy of the questioning.

Secondly, the interviews lead to some interesting central quotations describing main features of the teachers' beliefs. However, it is not easy to condense the verbal profile as well as the responses in the questionnaire and to "coordinatize" the teachers' positions in order to compare them objectively and quantitatively as the Dionnian approach pretends to do.

Thirdly, although the Dionnian method appears to be a very rough quantitative tool ignoring some details and seemingly unaware of other details, this method leads to some figures which contain worthwhile information. Of course one cannot overlook that the Dionne approach (in both versions employed here) projects a highly dimensioned world of attitudes into a 2- or 3-dimensional variety. Such a procedure inevitably leads to serious reductions. The fact that the questionnaire data are not compatible is part of the results of our investigation. Individual conclusions about the persons, even when based on Dionnian parameters, still contain many uncertainties, unless one couples these results with the statements in the interviews.

Nevertheless, we highly recommend using the tabular as well as the graphical mode of investigation in the sense of Dionne. The noticeable inconsistency should not be overrated because both sources of data make allowances for varying emphasis.

Metrical aspects play an especially strengthened role in pictorial illustrations. The examinee can highlight his basic discrepancies, and, finally, a direction of change will become evident in relation to the three components represented by the three corners of the triangle. Whereas the graph informs one about the magnitude of a change, the table may show the pattern of change.

The interview statements in this survey thus fulfill the central function of being explanatory and authoritative. As expressed before, one should clearly distinguish between real and ideal teaching. For it is the pair of the two vectors which tells a story! The additional expenditure for the test subjects is of a marginal magnitude. Finally, we believe to have proven that additional collection of graphical data is hardly more costly and is of important explanatory value. Intervening feelings and effects are revealed, e.g., in the length of the arrows more clearly than in the numerical data.

References

Berger, P. (1995). Teachers' beliefs about computers and computer science. In G. Törner (Ed.), *Current state of research on mathematical beliefs. Proceedings of the MAVI workshop* (pp. 15-22). Duisburg: Schriftenreihe des Fachbereichs Mathematik, Gerhard-Mercator-University Gesamthochschule Duisburg.

Cohen, L. & Manion, L. (1994). *Research methods in education*. London & New York: Routledge.

Davis, R. B., Maher, C. A. & Noddings, N. (Eds.) (1990). *Constructivist views on the teaching and learning of mathematics. JRME Monograph Number 4*. Reston, VA: NCTM.

Denzin, N. K. (1970). *The research act in sociology: A theoretical introduction to sociological method*. London: The Butterworth Group.

Dionne, J. (1984). The perception of mathematics among elementary school teachers. In J. Moser (Ed.), *Proceedings of the 6th annual meeting of the North American chapter of the International group for the psychology of mathematics education (PME)* (pp. 223–228). Madison, WI: University of Wisconsin.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: a model. *Journal of Education for Teaching 15 (1)*, 13–33.

Ernest, P. (1991). *The philosophy of mathematics education*. Hampshire, UK: The Falmer Press.

Furinghetti, F. (1996). A theoretical framework for teachers' conceptions. In E. Pehkonen (Ed.), *Current state of research on mathematical beliefs III. Proceedings of the MAVI-3 workshop* (pp. 19–25). Helsinki: University of Helsinki. Department of Teacher Education.

Green, T. F. (1971). *The activities of teaching.* Tokyo: McGraw-Hill Kogakusha.

Grigutsch, S., Raatz, U. & Törner, G. (1998). Einstellungen gegenüber Mathematik bei Mathematiklehrern. *Journal für Mathematik-Didaktik 19 (1)*, 3–45.

Hill, R. (1986). *A first course in coding theory.* Oxford: Clarendon Press.

Lerman, S. (1983). Problem-solving or knowledge-centered: the influence of philosophy on mathematics teaching. *Int. J. Math. Educ. Sci. Technol. 14 (1)*, 59–66.

Lincoln, Y. S. & Guba, E. G. (1985). *Naturalistic inquiry.* Beverly Hills, CA: SAGE publications.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 575–596). New York: Macmillan.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis et al. (Eds.), *Constructivist views on the teaching and learning of mathematics. JRME Monograph Number 4* (pp. 7–18). Reston: NCTM.

Pehkonen, E. (1994). On teachers' beliefs and changing mathematics teaching. *Journal für Mathematik-Didaktik 15 (3/4)*, 177–209.

Pehkonen, E. (1995). *Pupils' view of mathematics: Initial report for an international comparison project. Research Report 152*. Helsinki: University of Helsinki, Department of Teacher Education.

Pehkonen, E. & Lepmann, L. (1995). *Vergleich der Auffassungen von Lehrern über den Mathematikunterricht in Estland und Finnland. Research Report 139.* Helsinki: University of Helsinki, Department of Teacher Education.

Pehkonen, E. & Törner, G. (1999). Teachers' professional development: what are the key change factors for mathematics teachers? *European Journal of Teacher Education 22 (2/3)*, 261–277.

Robitaille, D. F. (1997). *National contexts for mathematics and science education. An encyclopedia of the education systems participating in TIMSS*. Vancouver: Pacific Educational Press.

Schmalz, R. (1993). *Out of the mouth of mathematicians. A quotation book for philomaths.* Washington, DC: The Mathematical Association.

Schoenfeld, A. H. (1985). *Mathematical problem solving.* Orlando, FL: Academic Press.

Schoenfeld, A. H. (1998). *Toward a theory of teaching-in-context (draft version)*. Retrieved from http://www-gse.berkeley.edu/faculty/aschoenfeld/TeachInContext/tic.html

Schoenfeld, A. H. (1992). Learning to think mathematically: problem solving, metacognition, and sense making in mathematics. In D.A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 334–370). New York: Macmillan.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 127–146). New York: Macmillan.

Törner, G. & Grigutsch, S. (1994). "Mathematische Weltbilder" bei Studienanfängern – eine Erhebung. *Journal für Mathematik-Didaktik 15 (3/4)*, 211–251.

Appendix 1

A Questionnaire for Teachers:
Conceptions of Teaching Mathematics

Through the following questionnaire, we would like to get a profile of your ideas and conceptions concerning teaching mathematics. These are some statements on teaching mathematics. Circle the option which best describes your opinion. The choices are:

1 = fully agree 2 = agree 3 = don't know
4 = disagree 5 = fully disagree

1	1	2	3	4	5	In teaching mathematics, one should use varied exercises and applications above all else.
2	1	2	3	4	5	Mathematics in school necessarily requires a concrete dimension; abstract mathematics alone is not enough.
3	1	2	3	4	5	Logic is promoted in teaching mathematics, whereas creativity and originality are not stressed.
4	1	2	3	4	5	Problem orientation should be the core of teaching mathematics.
5	1	2	3	4	5	In teaching mathematics, finished products take priority, not the process by which they are achieved.
6	1	2	3	4	5	Doing mathematics means: working through the proofs carefully.
7	1	2	3	4	5	Teaching mathematics provides an excellent opportunity to promote the development of the pupils' thinking.
8	1	2	3	4	5	Mathematics teaching is especially meant for talented pupils.
9	1	2	3	4	5	One should always make sure to visualize aspects of teaching mathematics.
10	1	2	3	4	5	Indisputable formality takes priority in mathematics.
11	1	2	3	4	5	Learning calculation techniques is the core of teaching mathematics.
12	1	2	3	4	5	While doing mathematics, understanding the topic is the most important idea.
13	1	2	3	4	5	In teaching mathematics, one should often realize projects without subjects limits.

These aspects of mathematics teaching, referred to in the questionnaire, will be discussed in detail during the interview.

Appendix 2

The letter to the teachers

Starting point: a rough classification of mathematical views consist of the following three perspectives, which are part of every view of mathematics and the teaching of mathematics:

T Mathematics is a large toolbox: Doing mathematics means working with figures, applying rules and procedures and using formulas.

S Mathematics is a formal, rigorous system: Doing mathematics means providing evidence, arguing with clear and concise language and working to reach universal concepts.

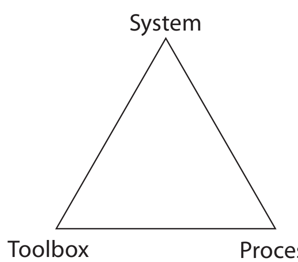
P Mathematics is a constructive process: Doing mathematics means learning to think, deriving formulas, applying reality to Mathematics and working with concrete problems.

Question 1: Distribute a total of 30 points corresponding to your estimation of the factors, T, S, and P in which you value your …

	T	S	P
… real teaching of mathematics			
… ideal teaching of mathematics			

For additional comments please use the reverse side of this page.

Question 2: Acknowledge your position on the three factors mentioned above by marking points within the equilateral triangle below.
x = real teaching of mathematics
o = ideal teaching of mathematics



For additional comments please use the reverse side of this page.
Thank you very much!

Appendix 3

The educational system in Germany

The school system in Germany

In order to provide the reader with the framework of our investigation (see also the survey article on the German education system in Robitaille (1997)), we shall give a short description of the German school system, of which the interviewed teachers are a part.

The basic structure of the public school system in Germany is run by the federal government and determined largely by the local governments of the various States (*Bundesland*). Since this research was conducted in the State of Northrhine-Westfalia, it will suffice to focus on the school system there. There are slight differences between the states, but the main components of the system are basically the same in all of them.

The features can be described as follows: school is compulsory for ten years, and it is necessary to attend school for thirteen years in order to qualify for university study. After four years of elementary school (*Grundschule*), pupils face the following four options: to attend the *Hauptschule* with graduation after the completion of the tenth grade; to opt for the *Realschule* (which originally prepared its pupils mainly for the service industry) and graduate after the tenth grade; to go to the *Gymnasium* (academic high school) and graduate after a total of nine further years to continue through to the matriculation examination; or to spend six, eight or nine years in a *Gesamtschule* (comprehensive high school) with the option of leaving after grade 10, grade 12 or grade 13. It is possible to transfer from the *Hauptschule* or the *Realschule* to the *Gymnasium* or *Gesamtschule* after grade ten, however generally not without problems, since the students have to accommodate to the new curriculum.

In the school year 1993–94, the distribution of pupils in Grade 10 in different school forms in Northrhine-Westfalia, according to the local school administration in Düsseldorf, was as follows, *Gymnasium* 30 %, *Realschule* 25 %, *Hauptschule* 30 %, and *Gesamtschule* 15 %.

German mathematics teachers' qualifications

In the Federal Republic of Germany, there are two main academic levels with regard to teacher qualifications, depending on the school form the teacher is qualified for, namely the upper secondary level or the lower secondary level. Only the first degree is sufficient to qualify him/her to teach mathematics up to grade 13 at a Gymnasium. Already at the beginning of his/her teacher education which takes place in a university, the teacher student has to opt for one of the degrees.

At the beginning, teachers seeking a qualification for the *upper secondary level* have to attend the same lectures as the mathematics students looking for a master's (or diploma) degree. They also have to study a second subject. Only ten percent of the compulsory lessons focus on mathematics education. The degree the students obtain is more or less equivalent to a master's degree in mathematics. On the other hand, prospective teachers wishing to become teachers at the *lower secondary level* are trained through separate courses in which mathematical learning is to a great extent limited to school-relevant subject contents.

Prospective teacher students of both types leave university having passed through a first examination. They then continue their instruction in a two-year prospective teacher in-service training in a seminar under the auspices of the school administration. The period is finished with a second examination. After this final exam they can apply for a teacher position at a school of the type for which they have been qualified.

In Northrhine-Westfalia e.g., however, teachers are trained for a specific school level. Thus, a teacher qualified for the Secondary I level (ages 10–16) can, e.g., teach in all four Secondary I school forms. And a teacher for the Secondary II level (ages 16–19) can teach at a Gymnasium or a Vocational College or a composite school form for both the general and vocational education (*Kollegschulen*).

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Yhteenveto

Meidän lähtökohtamme muodostavat metodologiset tarkastelut opettajien matematiikka-uskomusten tutkimisessa sekä kvantitatiivisin että kvalitatiivisin menetelmin. Tällaista keskustelua ei voida kuitenkaan käydä liittämättä sitä joidenkin uskomusten määrittelyyn. Tässä Dionnén ja Ernestin luokittelut matematiikkauskomuksille muodostivat teoreettisen taustan; matematiikan keskeisimmät näkemykset voidaan kuvailla työkalupakki-ajattelu, systeemiaspekti ja prosessiaspekti. Alkuaan koehenkilöitämme olivat 13 kokenutta saksalaista opettajaa. Mutta tässä rajoitamme keskustelumme vain kuutta edustavaa henkilöä koskeviin tutkimustuloksiin. Käytämme kolmea tiedonkeruumenetelmää: kyselylomake, videonauhoitetut haastattelut ja itsearviot, sekä graafisesti että numeerisesti. Tutkimuksessamme on itsearviointien vertailu etusijalla. Koska saatu informaatio on suurelta osalta päällekkäistä ja osittain ristiriitaista, on meidän kyseenalaistettava saatu tieto, jotta voisimme kuvata opettajien uskomuksia. Tutkimus toteutettiin keväällä ja kesällä 1994.