A comparative study on students' beliefs concerning their autonomy in doing mathematics

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This paper treats an investigation on beliefs about autonomy held by 260 Finnish and 246 Italian seventh-grade students. The data were collected by means of a questionnaire consisting of 32 closed questions dealing with different aspects of mathematics teaching and learning, and three open questions on students' good and bad experiences in mathematics instruction and their wishes for good instruction. We have confined ourselves to the eight questions of the questionnaire identified as pertaining to the students' beliefs about autonomy in doing mathematics.
The data was collected through the questionnaire allowed to outline the patterns of beliefs in each country and compare patterns with each other. We have identified a core of common patterns in the two countries which are mainly related to the issue of classroom interaction and to the students' need of their teacher's help. The biggest differences between the two countries were found in the items concerning the use of trial-and-error strategies and the possibility that the students solve mathematical problems on their own.

Introduction

In this paper we present a comparative study of students' mathematical beliefs in Finland and Italy. The focus is on students' beliefs about autonomy in doing mathematics. The students in question are about 13-year-olds, which means they are at the compulsory level in the schools of both countries.

Many comparative studies of students' performances in mathematics have been published. Of course these studies are of paramount relevance to know the level and the kind of the mathematical instruction in the countries considered in those studies, nevertheless they give only partial information. Schoenfeld (1983) has pointed out that the boundary between the cognitive domain and the domain of affective (beliefs) is very fuzzy, because:

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> *"the tangible cognitive actions produced by our experimental subjects are often the result of consciously or unconsciously held beliefs about (a) the task at hand, (b) the social environment within which the task takes place, and (c) the individual problem solver's perception of self and his or her relation to the task and the environment." (Ibid. 330)*

As a consequence, to understand and evaluate mathematics instruction in a country we need to investigate both cognitive and affective sides. It seems to us that comparative studies of performances give information on the official side of the school instruction (programs), while beliefs shed light rather on the "everyday" side (classroom practices, teacher behavior, etc.).

The present paper is an example of a comparative study of beliefs held by two cohorts in different countries. Its importance rests both in the insights given on students' beliefs and in the possible model it offers for such studies.

Theoretical background

On mathematical beliefs

The importance of beliefs is earning more and more recognition in mathematics education, as already shown in the wide survey (Törner & Pehkonen, 1996; Pehkonen & Törner, 1999). There is no uniquely accepted, exact definition of what the term 'belief' means. This term is taken from natural language and thus carries a burden of personal meanings that the user wishes to give to it, as it is discussed in Furinghetti & Pehkonen (1999). Different authors have given different characterizations of beliefs and the relation of these entities with other elements such as knowledge and conception (see Abelson, 1979; McLeod, 1992; Nespor, 1987; Pajares, 1992, Thompson, 1992). Schoenfeld (1992, 358) describes beliefs as "an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior". This characterization has a very operative character; since it relates beliefs to behavior, moreover it refers not only to cognitive components, but also to affective components. Our understanding of what a belief is may be further characterized by the following specification of its function in a system, as found in Pehkonen & Törner (1996): (a) beliefs form a background regulating system of our perceptions, thinking, and actions, and therefore, (b) beliefs act as indicators for teaching and learning. Moreover, (c) beliefs can be seen as an inertia force, which may work against change, and as a consequence (d) beliefs have a forecasting character.

Beliefs have their principal origin in social interaction. An individual's mathematical beliefs originate from personal experiences outside of school and in school. The last ones, in turn, consist of perceptions that originate from mathematics teachers, other teachers, schoolmates, learning materials, achievements in mathematics, etc. Especially, mathematics teachers have a big influence, among other things, through their curricular decisions. The prevailing image of mathematics in society influences students' beliefs through their parents, relatives, friends, different kinds of media, job opportunities, etc.

Because of beliefs' interactive character with society, studies on beliefs may concern not only single individuals, but also the whole context in which they operate. This makes it particularly interesting to include beliefs in the comparative studies - as the one presented in this paper - in different countries.

Usually individuals are not conscious of all their beliefs. Accordingly one can distinguish between conscious and unconscious beliefs. In investigating beliefs, one can reach usually only some of the beliefs an individual is willing or is able to share with the researcher. The non-shared beliefs are not easy to grasp, since their nature is hidden (for conscious or unconscious reasons). It is even more difficult to grasp beliefs when an individual has conflicts between his beliefs on a topic and beliefs on this topic that he ascribes to the environment (the interviewer, society, colleagues, friends, etc.). In this case the individual tends to adapt his beliefs to the expectation of the environment. The researcher has to devise specific methods for investigating and analysing hidden or adapted beliefs. The difficulties in this kind of research are efficaciously outlined by the metaphor Berger (1998) uses to describe belief research: the reconstruction of a dinosaur from fossils. In paleaontology [in belief research], researchers find some fossils [observations on an individual's statements and behavior]. From these they try to reconstruct the skeleton [an individual's shared beliefs] and to sketch a model of a dinosaur [an individual's shared and hidden beliefs]. This metaphor illustrates that there are different degrees of "arbitrary deciding" and "inference" in reconstructing an individual's beliefs from his statements and behavior. We will see that this interpretative difficulty is even more present in studies as the one presented here, where the inferences have to be made from mere statistical figures provided by a large-scale questionnaire.

On students' autonomy

The notion of autonomy is largely studied in psychological literature by authors such as E.A. Skinner, R.J. Vallerand, T.G. Plummer. For the purposes of mathematics education we would prefer to consider a characterization of this notion which is strictly linked to teaching/learning processes specific to mathematics such as problem solving, producing conjectures, exploring, etc. According to our understanding students' *autonomy* can be observed usually within learning environments. In the first place it means freedom from their teacher's guidance. Students are left room to make their own decisions e.g. on the rate and the direction of their learning. This has i.e. following consequences: Students

- are let to work independently,
- do not consider the pre-assigned rules as the only sources to perform mathematical tasks,
- are confident in what one is doing for reasons which do not come from an external authority, but rather from an internal rationale,
- are aware of, and draw on, one's own intellectual capabilities to take decisions and judgments based on rational arguments which allow justifying and defending them,
- are free to communicate in classroom,
- play an active role in the classroom community.

We note that with some adjustments our characterization of students' autonomy applies also to the autonomy of mathematics teachers.

Authors in mathematics education stress some aspects of autonomy. As for the construction of knowledge, Confrey (1990, 111) claims that: "To a constructivist, knowledge without belief is contradictory. Thus, I wish to assert, that *personal autonomy is the backbone of the process of construction.*" Consistent with this position, when discussing possible implications of constructivism for mathematics instruction, Confrey lists first "promotion of autonomy and commitment in the students" (ibid, 115) among the elements characterizing her model for teacher professional development.

Other authors stress the positive role of the inquiry approach in promoting autonomy. As discussed by Borasi and Siegel (1994), in the mathematics inquiry approach knowledge is seen as a construction of thinking by means of a process of inquiry motivated by ambiguities, anomalies and contradictions, and conducted within a research group. Understanding comes to be seen as a generating process of the construction of meaning, which requires either social interaction, or personal construction within the motivating situation. Contrast intellectually autonomous in mathematics students (those who are aware of, and draw on, their own intellectual capabilities when making mathematical decisions and judgments) with those who are intellectually heteronymous (those who rely on the pronouncements of an authority to know how to act appropriately). In relation with the practices of the classroom community Yackel and Cobb (1996, 473) say:

> *The link between the growth of intellectual autonomy and the development of an* inquiry mathematics tradition *[italic is ours] becomes apparent when we note that, in such a classroom, the teacher guides the development of a community of validators and thus encourage the* devolution of responsibility. *[Italic is ours]."*

The devolution of responsibility is linked to the notion of teacher's authority. Cooney (1993, 44) distinguishes between the teacher's "authority as a responsibility for classroom management and authority as the determiner of truth". The devolution of responsibility concerns the latter type of authority and has to happen through a negotiation between the teacher and students to establish the rules for the classroom life.

In the Standards it is acknowledged that when students communicate their ideas "they learn to clarify, refine, and consolidate their thinking" (NCTM 1989, 6). Thus - students' autonomy does not mean their isolation from the classroom and from their teacher. On the contrary, we maintain that the more the student is able to interact with the context, especially the teacher and his peers, the more he is autonomous.

From all the issues discussed we see that to study autonomy is important not only *per se*, but also because it gives evidence of a net of strictly related issues that concern mathematics instruction.

On comparative studies

Carrying out comparative studies means not only to find out differences, but also to check whether there are some similar elements, i.e. elements in which the students in both countries have a similar orientation.

In addition to their relative value, comparisons also have an absolute value. Through a comparative study we may be able to see our own system from outside, and this can help us to better determine its weaknesses and strengths. Upon examining the results of a comparative study we might notice that in another country there are students' beliefs which research has proved to have positive or negative consequences in school and then begin to think how to develop or eliminate similar beliefs in our own country. There are also some benefits to be derived from asking whether research findings in mathematics education can be transferred from one country to another. Is it possible, for example, to apply results obtained in the United States directly to Europe? That is something we have believed in, and as a consequence, we have often used results in our own country without questioning them. We know that this has not always been successful (cf. Pehkonen & Lepmann 1993). One cause of failures may have been that we have neglected to ascertain the context in which they could have been successfully applied. And, as stated above, in our beliefs there are important components of the context.

International comparison of students' beliefs

During the last decade, many articles on students' beliefs have been published – especially in the United States. However, the question of the international comparison of students' mathematical beliefs still seems to be an almost unexplored field. Due to the role played by individuals in interacting with society, studies on beliefs may concern not only single individuals, but also the whole context in which they operate. This makes it particularly interesting to include beliefs in the comparative studies - as the one presented in this paper - in different countries.

The main question here is whether there are essential differences in beliefs concerning mathematics teaching in different countries. As mathematics can be understood as a universal discipline, the question arises whether students' beliefs concerning mathematics, and mathematics teaching and learning are also universal, or whether they are, perhaps, context-bound.

However, before the findings presented in some earlier papers (Pehkonen 1995; Pehkonen & Safuanov 1996), almost no research into variations between students' beliefs on an international scale seems to have been done. The Second International Mathematics Study (Kifer & Robitaille 1989) is alone in analysing students' responses to some questions on the affective domain in a background questionnaire. Their study indicated that there are large differences between countries in indicators of mathematical beliefs and attitudes.

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[1] For the history of the questionnaire see Pehkonen (1992).

Methodology

Questionnaire

Since we wanted to consider a large sample, information was gathered with a student questionnaire. The questionnaire[1] was developed for another research project by Bernd Zimmermann (University of Jena, Germany). Its purpose was to clarify students' beliefs concerning mathematics and the teaching and learning of mathematics. The questionnaire consisted of 32 structured items about mathematics teaching, that is, statements regarding which the students were asked to rate their views on a 5-step scale (from 1 = fully agree to 5 = fully disagree). The leading ingress for all items was "Good mathematics teaching includes ...". In addition, students were asked to describe their 1) good experiences, 2) bad experiences, and 3) wishes for good mathematics teaching. The whole questionnaire is discussed e.g. in Furinghetti (1994) and Pehkonen (1992), and it can be found e.g. in the appendix of Pehkonen (1992) as well as Pehkonen & Safuanov (1996).

An English language version of the questionnaire was translated into Finnish and Italian by the authors of the present paper and distributed in the respective countries. A part the obvious differences in geographical conditions, we tried to have two samples quite comparable, by choosing a city and a small town in the area close to these cities in each country. The Finnish sample comprised 15 seventh-grade (13 year-old students) classes from Helsinki and Järvenpää (a small town about 40 km north of Helsinki), altogether 260 students. The Italian sample comprised 10 seventh-grade (13 year-old students) classes from 5 schools, altogether 246 students. About half of the Italian sample was from Genoa, and the other half from two small towns (Recco and Sori) about 20 km east of Genoa. In both cases, the teachers gathered the information by having students fill in the whole questionnaire (32 items) during their mathematics lessons. Each student has filled in the questionnaire independently. The allowed time was about 10-20 minutes within a mathematics lesson.

It should be noted that our sample did not intend to be a random sample, and therefore, the results cannot be generalized to the whole population (here, to all students of the same age group in our countries). Using the language of Cohen & Manion (1994), one may say that our data were gathered on the basis of "a non-probabilistic convenience sampling". For the sake of brevity from now on we shall indicate the population of the Finnish sample as "the Finns" and that of the second as "the Italians" without intending with that to generalize our remarks to all the country.

Selection of the structured items referring to autonomy

To have chosen a questionnaire that was not created by us and which was not aimed at our specific object of study (autonomy) can, at least in principle, be a disadvantage. Certainly it obliged us to a preliminary work of selecting the items suitable to our specific aim. On the other hand we would like to point out advantages of our choice. In other studies we have observed that to present students with items too much focused on the topic of research makes too much explicit the intentions of the researchers and the subject may give answers aimed at satisfying the expectations of the researchers. Moreover we consider that an external author of the questionnaire is bringing an additional perspective to the study. As told before the whole questionnaire of 32 items was given to the students, but for the purposes of the present study we restrict our analysis to the items which we thought would refer to students' autonomy. Initially the following ten items were selected:

GOOD MATHEMATICS TEACHING INCLUDES:

(4) the idea that the students can sometimes make guesses and use trial and error

(13) the idea that students can put forward their own questions and problems for the class to consider

(15) the idea that the teacher helps as soon as possible when there are difficulties

(16) the idea that everything will always be reasoned exactly

(20) the idea that only the mathematically talented students can solve most of the problems

(25) the idea that games can be used to help students learn mathematics

(26) the idea that when solving problems, the teacher explains every stage exactly

(27) the idea that students are led to solve problems on their own without help from the teacher

(31) the idea that also sometimes students are working in small groups

(32) the idea that the teacher always tells the students exactly what they ought to do.

In an earlier paper, Furinghetti (1994) used the same questionnaire to investigate secondary teachers' beliefs concerning good mathematics teaching. She classified the items into five categories. One of them was named "space given to autonomy, personal initiative and personal construction of knowledge" and consisted of the items 4, 13, 15, 16, 26, 27, 31 and 32. This category is quite close to our present selection of the items. Furinghetti put the two additional items included here (20 and 25) into the category of "ideas [held by teachers] on students' learning".

After this first selection, we asked 22 active belief researchers to indicate how proper our selection actually was. We sent the items by e-mail to the researchers in question and asked them to give their opinion on the propriety of our selection by ticking their preference in the following sentences: "The item refers to students' autonomy: 1 = refers clearly, 2 = might refer, 3 = does not refer."

Within three days, we received answers from eleven researchers. In Table 1 we give the frequencies (max. 11) for each item.

	4	13	15	16	20	25	26	27	31	32	?
1 = refers clearly	4	10	5	2	2	1	2	9	5	5	45
2= might refer	7	1	2	8	1	9	6	2	6	-	42
3 = does not refer	-	-	4	1	8	1	3	-	-	6	23

Table 1. The frequencies of researchers' answers

Based on Table 1 some observations were made. The responders' behavior to select the intermediate choice was linked to the character of a questionnaire in general, and especially to a questionnaire on beliefs. Any sentence expressed in colloquial language bears the burden of ambiguity. This is even more so when we are investigating beliefs, since, as pointed out before, we have to grasp also implicit (non-shared) beliefs which are unconscious, or conscious but hidden by the individual. This means that it is not effective to ask directly; instead we are compelled to use "side roads".

Based on the data in Table 1, we saw that item 20 clearly had to be put aside. In the case of the item 32 we saw that the opinions of the researchers were equally divided into two extreme groups, and therefore, we left it away also.

Data analysis

In dealing with the structured items that we have related to students' autonomy, we have used percentage tables and the comparison of distributions (Mann-Whitney U). The Mann-Whitney U test is a standard part of the StatView statistical program package, which is a non-parametric equivalent to the standard t-test. A full account of the statistical treatment that we have adopted is in e.g. Siegel & Castellan (1988). At the moment of the statistical analysis the original response scale (1-2-3-4-5) revealed itself too detailed for the purposes of our study and we reduced it by combining the two response values at the extreme ends of the scale, which yields a three-step scale of *agree* (1 or 2), *neutral* (3), and *disagree* (4 or 5). The responses to open questions were categorized through key topic developed in students' expressions. In these categories we have considered those which are relating to issues linked to autonomy. A detailed description of this method can be found in Pehkonen (1992).

We have used the data to compare the patterns in both countries considering (a) *indecisiveness* (i.e. balance in the three steps of the scale), (b) *degree of agreement or disagreement* (i.e. which of the two extreme steps is prevailing), (c) *number of neutral answers* (i.e. if the step 'neutral' is prevailing). Additionally, we have confronted the consistencies and inconsistencies in the data. Furthermore, we have used statistics to check the statistical significance of the differences between the distributions. In order to facilitate inferences of students' beliefs, we have also combined the quantitative data of the structured items with the qualitative data of the open questions. Also we have used our experience in studying mathematical instruction in our countries to give some further comments and conjectures on the students' reactions.

Findings from the structured items

The findings from the structured items are presented in Table 2. It contains the percentages of agreement – neutral – disagreement, in this order. According to our aim (comparing beliefs in the two countries) we indicated explicitly the error percentage p (i.e. the probability for the error in accepting the difference tested as a true difference) and the level of statistical difference through the star symbols (***/**/*/-). These are usually used to signify the level of significance. In the table there is a horizontal reading (international, i.e. a comparison between the countries) and a vertical reading (national, i.e. an analysis within each country).

Item No.	Statement	Finland	Italy		
	GOOD MATHEMATICS TEACHING INCLUDES	a–n–d %	a–n–d %	P %	sign. level
4	the idea that the student can sometimes make guesses and use trial and error	79–17–5	32–35–32	0.01	***
16	the idea that everything will always be reasoned exactly	53–23–28	77–16–8	0.01	***
27	the idea that students are led to solve problems on their own without help from the teacher	73–20–8	37–27–36	0.01	***
13	the idea that students can put forward their own questions and problems for the class to consider	76–14–10	85–11–4	0.8	**
26	the idea that when solving problems, the teacher explains every stage exactly	72–16–13	63–19–18	3	*
31	the idea that also sometimes students are working in small groups	85–10–5	81–11–8	15	-
25	the idea that games can be used to help students learn mathematics	66–24–10	63–22–15	36	-
15	the idea that the teacher helps as soon as possible when there are difficulties	76–9–16	73–15–11	81	-

- $p > 5\%$, * $p < 5\%$; ** $p < 1\%$; *** $p < 0.1\%$ (by Mann-Whitney U)

*Table 2. Percentages of agreement / disagreement (*a *= agree,* n *= neutral,* d *= disagree,* p *= error percentage)*

Item 4 mentions explicitly one issue that is linked to our discussion on students' autonomy. "Guesses, trial and error" are the key elements in the work in classroom encompassing activities such as conjecturing, exploring, validating, refuting, as it is outlined by Lampert (1990). Finnish students are more in favour of this style. Among Italians there is a balance of the three positions. The findings from item 16 can help to make hypotheses on the reasons of the results of item 4. The key word in item 16 is "exactly". Of course, exactness may stem from the nature of a given task and possibilities of dealing with it. But we take into account that this word may refer to a view of doing mathematics (and of mathematics itself) depending from an authority (the teacher, the book) which contrasts with the "zig-zag between conjectures and arguments for their validity" mentioned by Lampert (1990). Thus the Italian percentages of item 16 suggest that the indecisiveness in item 4 originates from this view. We can guess that the word "exactly" appearing in item 16 is evocative of the myth on the nature of mathematics as a rigid, precise discipline.

We note that the word 'exactly' appears also in item 26: but here the idea that "teacher explains every stage exactly" may evoke factors belonging to the domain of socio-mathematical norms (what students are expecting from the teacher). When we mention socio-mathematical norms we are not necessarily referring to explicitly stipulated norms, but to norms that are implicitly stated by the school tradition. The influence of socio-mathematical norms on students' beliefs may be inferred from the following observation. In item 13, while the position of Finnish students is not sensibly changed in respect to item 4, the Italians responses have different patterns. At a first glance we can conclude that the Italian students seem to have perceived that problem solving and problem posing are distinct activities, as discussed by Silver & Cai (1996). But the different patterns of the Italians in items 4 and 13 have also a pragmatic explanation in that, while posing problems is considered an assessment-free activity; students are used to be assessed when they solve problems. Thus they think it might be dangerous to try ways of solving which teachers might not accept. As far as Italy is concerned this way of thinking emerged also from other investigation carried out by one of the authors (Demattè & Furinghetti, 1999).

We stress that one cause of students' different reactions might be the different teaching style in Finland and Italy. In Italy, mathematics teaching at this age level is based on a teacher-centred style, whereas in Finland most teachers are trying to implement a pupil-centred model of teaching. The higher percentage of neutral answers from the Italian students can be interpreted as a kind of caution on the part of the students in showing their opinions about a situation in which they have, perhaps, not had enough personal experience.

Item 27 is clearly referring to independence from the teacher (see: "without help of the teacher"). Again the Italian percentages are balanced between the three possibilities, in contrast with the clear orientation of the Finnish ones in favour of independence from the teacher.

This first group of the items which we have examined stresses that there are differences in the orientation of the two populations examined: we can summarize them saying that Finnish socio-mathematical norms encompass an orientation towards work autonomous from the teacher, but also expectation that teachers makes clear his strategies in solving problems.

The remaining three items are mainly referring to the interaction among students and between students and teacher. We do not pretend that our population have a clear idea of what "collaborative small group work" can be (see Edwards, & Jones, 1999), even more we think that the positive reaction of both populations to item 31 is mainly based on affective factors. Nevertheless we feel that it is possible to conclude that a wish of "working together" and of working on activities, such as games, that imply a social atmosphere, can be taken for grant. We feel that there is not conflict between this conclusion and the fact that item 25 (about games) has the highest level of indecisiveness. This indecisiveness can be attributed to the fact that students are not clear about the relationship between games and learning mathematics. This lack of clearness could have been stressed by the fact that the students do not have much experience of learning games, and therefore, they were unable to take a position.

The interaction between the student and the teacher is treated in item 15. At a first glance the results of item 15 and item 27 that we discussed before would seem to reveal some inconsistency. Especially in the case of the Finnish students who showed to have the goal of solving problems by their own (item 27), while item 15 reveals them to rely on their teachers' help. Also the Italians are very decided in relying on teachers' help, while they gave very balanced opinion in answering to item 27. Actually this inconsistency may be only apparent. Both items refer to teacher intervention, but the first is related to the devolution of responsibility by the teacher, the second refers to the role of the teacher as a reference point in the classroom interaction.

We shall come back to the discussion of our inferences after the analysis of the findings from open questions which allow integrating the statistical data with the students' expressions.

Findings from the open questions

As reported above, at the end of the questionnaire there were the following open questions (for answering, each of them had three empty rows):

2. What kind of experiences have you had until today (from the elementary level up to now) about mathematics teaching? Explain.

good: ______________________________

bad: ______________________________

3. How would you like mathematics to be taught? ______________

Usually each response contained more than a simple answer such as "I had no good [bad] experiences", but students provided expression describing situations and feelings. In each expression there was reference to different aspects of their school experiences. In Finland, 260 students involved in the research gave altogether 827 expressions in answering to the three open questions, and in Italy the corresponding figures were 246 students and 1029 answers. This means that the average number of expressions per student is 3.2 in Finland and 4.2 in Italy.

Table 3 shows the distribution of expressions contained in the responses between the categories of good experiences, bad experiences, and wishes in Finland and Italy.

	good experiences	*bad experiences*	*wishes*
Finland	35	31	34
Italy	40	21	40

Table 3. The percentage distribution of students' answers to open questions

Answers relevant to students' autonomy

In Table 4 we report the distribution of the answers to the open questions relevant to students' autonomy. The table is the result of a categorization of the students' expressions designed by Pehkonen (1992) to overcome the difficulties of analysing and quantifying answers to open questions. This categorization was made *a posteriori*, after having analysed the students' protocols. The responses were first grouped into six main classes: (1) teacher/teaching, (2) mathematical topics, (3) learning control, (4) student, (5) interaction and working forms, (6) resources. The class (5) appeared to be the one containing the aspects concerning autonomy we were looking for. At the interior of this class it was possible to classify the responses in four sub-categories named "group work", "working together", "independent working", and "discussions". As the reader can see the students referred spontaneously, among others, to issues we considered in the discussion of the structured items. Taking into account the responses in the four sub-categories referring to, we obtained Table 4.

	good experiences	*bad experiences*	*wishes*	Σ
Finland	*2*	*3*	*30*	*35*
Italy	*8*	*1*	*17*	*26*

Table 4. The distribution of the answers relevant to students' autonomy

Most of the wishes in both countries concerned group work. Only in Italy the wish for discussion was expressed in relation to group work, e.g. "*students have to be divided into groups where they can discuss their ideas and better understand the topic*". Only in Finland did the students express the wish for differentiation in teaching, e.g. "*those students learning quicker should be given more demanding tasks*". One explanation for the amount of wishes for students' autonomy in Finland might be due to the Finnish school reality where in many classes students feel that they do not have an opportunity to learn because their peers disrupt the classes.

This understanding is supported by these two wishes: "*if some students are quick, the teacher does not let them work ahead, since the others calculate more slowly*", and "*[the slow students] disturb those who like to work*".

Discussion on similarities and discrepancies

In the questionnaire, we asked the students to express their views on mathematics teaching and learning. This process implied for them a reflection on aspects of classroom life which, we hoped, revealed their mathematical beliefs. This can be considered as a starting level for an activity based on metacognition. We are aware that such activities are unusual for students, since they are not accustomed to answer such questionnaires. The students were, perhaps for the first time in their life asked, to analyse their way of thinking.

Students were very willing to fill in the questionnaire, as can be seen, for example, in the richness of the comments they gave to the open questions, as well as in the variation of the answers to the structured items. Our comparative analysis of the findings has shown that there is an inner consistency in the students' answers.

Although we cannot generalized the results to the whole population in question, since we have used "a non-probabilistic convience sample" (cf. Cohen & Manion 1994), we would like to sketch the patterns emerging from the data: I.e. the students' answers to the structured and open questions, in order to have an overall idea on what is considered good mathematics teaching from the point of view of students' autonomy in the two countries. We have identified a core of common patterns in the two countries which is comprised of the three items (15, 25, 31). They are strongly related to the issue of classroom interaction. In particular, two of them (25, 31) can be referred to the need to have some form of communication among peers in classroom. This can happen through the construction of knowledge via working in group or learning games. The answers to the open questions by the Italians students are very explicit in this concern: learning games are seen as an occasion to work together. This association can be exemplified with an Italian student's wish: "*I would like mathematics to be taught through games and by working in groups in such a way that it is less boring.*"

The data about item 15 said that in both countries pupils believe that good teaching involves that the teacher has to help students in performing mathematical tasks. This is also confirmed by the percentages for item 26. The kind of help expected from the teacher, e.g. mere prompts to apply in a given situation or a conversation/ discussion which may lead students to find the way on their own, emerge from other data. For example, concerning beliefs on the teacher's role, the difference in students' answers in item 27 suggests that Italian students definitely show a greater dependence upon the teacher than the Finns. The words of an Italian student express the terms of this dependency and how it is expressed in discussions with their peers: "*The teacher has to be patient, to help students in understanding topics and the students have to commit themselves, together with the teacher, to develop experiments and exercises*". This belief can be interpreted as having a significant affective component.

The biggest differences between the countries were found in the items concerning the use of trial-and-error strategies and the possibility that the students solve mathematical problems on their own (items 4 and 27).

The difficulties of inferring on beliefs, that in previous sections we have outlined through the metaphor of the dinosaur, prevent us to take our inferences as a complete and absolute description of students' beliefs. Nevertheless we find that our data allow us to outline some character of beliefs emerging in the two countries. Combining discrepant and stable patterns from structured and open items we can sketch a virtual picture of the Finnish and Italian classroom where the ideal teaching for 13 years old students of the two countries is carried out.

The Finnish picture shows a classroom in which students are engaged in solving problems in a quite autonomous way. In the Italian classroom the teacher is seen as the holder of the truth. If we adopt the metaphor of Henry Pollak as reported in Lampert (1990, 41-42), the Finns are more oriented "to move around in mathematical territory in a flexible manner". Italians are more oriented to a mathematical activity which is "like walking on a path that is carefully laid out through the woods [and not like coming] up against any cliffs or thickets". We have seen that an inference we can make from the data (item 16) is that Italian students held the belief that mathematics is a pure, rigid discipline. It is likely that this belief be unconscious, anyway it affects the students' behavior in classroom. This fact is an example of what Furinghetti (1994) terms ghost in classroom, i.e. unconscious beliefs in action.

Conclusions

It seems to us that the research results presented here may be considered as one model of the use of comparative studies to explore and understand aspects of mathematics education.

One significant aspect is mainly concerning beliefs, which are difficult to study, especially those which are hidden. As shown in the analysis of the results, our study offers an additional method (comparative studies) which may be of help in grasping such beliefs. The possibility of comparing data is a kind of magnifying lens to look at the data collected (the fossils in the metaphor of the dinosaur) to reconstruct the students' beliefs (the dinosaur of the metaphor).

There are also practical aspects. We have personally experimented with an interesting implication of our study in a training course for in-service teachers, the intent of which was to make teachers reflect on their style of teaching. The presentation of the results of the questionnaire proved to be a good means for starting the discussion (Furinghetti, 1994). One fact turned out to be very important, namely that some results were absolutely unforeseen by the teachers attending the course. Thus the questionnaire offered the occasion to rethink one's personal views on teaching and to reshape some beliefs.

Another practical aspect of our study concerns the national system of a country. Beliefs have a powerful impact on our thinking and action, and they work for the rationality of our decisions. Thus to know students' beliefs is vitally important to those who are involved in the management of national educational systems, such as curriculum developers, teacher educators, administrators, etc. Important influences on mathematical beliefs may be seen a priori as nationally defined by the issues characterizing the instructional system, i.e. university education, curricular arrangements, country-specific educational forms and structures, teacher education policies and recruitment. An international comparison offers an opportunity to clarify the influence of these national factors on students' beliefs. This clarification will provide hints for effective implementation of curricular innovations and the limits for transferring results obtained from one country to another.

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