Some Problems of Socio-Cultural Research in Mathematics Teaching and Learning

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In the spirit of the conference, this article is not to argue for a socio-cultural perspective. I have been working in this area for close on a decade now and although I sometimes engage in discussions about theoretical perspectives (see Lerman, 1996; 2000) the intention of this paper is somewhat different. Instead I will first summarise some of what I consider the key notions of socio-cultural theory for education in general terms. I will then describe some of the aspects of a socio-cultural perspective on mathematics teaching and learning with which currently I struggle in my work.

Starting Points

I start from the point of view that people are products of the multiple cultural and social situations in which we are born, grow up and develop. These include gender, ethnicity, class, sexual identity, religion, local community, etc. Of course all theories of learning consider social factors to perform a crucial role, and it is useful to distinguish between theories which see social factors as *causative* of learning and those which see social factors as *constitutive* of learning (Smith, 1993). For constructivists social factors are the most common and significant interactions that can trigger disequilibrium in an individual's cognitive system. Accommodation or assimilation then lead to cognitive reorganisation, which is how Piaget defined learning. At the heart of that theory is the notion of the individual coming to situations with context-free knowledge, state of mind, and identity, in which events may or may not trigger disequilibrium. I have argued elsewhere (Lerman, 1998) that one cannot engage with issues of culture, power, or context from this perspective. Socio-cultural theories on the other hand take social, cultural and historical factors to be constitutive of learning in that who one becomes in the range of social, cultural situations in which we grow and develop is constituted by those social, cultural situations. A child does not choose to be a gendered, she or he is formed as gendered from the first day of her or his life. Consider, as a second example, the distinct meaning of elements

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of the landscape for Navajos (Pinxten, 1994), in both general terms and mathematical terms. "For the Navajo Indian 'movement' is a constituent of nearly all spatial distinctions, including such 'static' or structural notions (at least in the Western view) as angles or volumes" (Pinxten, 1994, p. 90). Meanings that we have are not chosen by us, nor triggered by interactions; those meanings are acquired by us from culture, as we become persons. Indeed we become human through communication (Vygotsky, 1924/1979) as the world is mediated by others, such as adults, teachers, peers, texts, through tools and signs. Language and meanings precede us and we become enculturated in multiple social settings. We internalise meanings (see below) from the social plane, in Leont'ev's sense that the mental plane is formed in the process. For research, therefore, the unit of analysis must be one that incorporates affect and culture with cognition.

Regarding individuality, I have written elsewhere (Lerman, in press, a):

> *A discursive, cultural psychology locates its interpretation of the individual at the intersection of overlapping language games in which the person has developed and thus is necessarily rooted in the study of cultures and histories. Individuality is the uniqueness of each person's collection of multiple subjectivities, through the many overlapping and separate identities of gender, ethnicity, class, size, age, etc., to say nothing of the 'unknowable' elements of the unconscious.*

The rich sense of *social* that we have inherited, particularly from Vygotsky, provides us with the principles of a mechanism for interpreting how the cultural becomes the individual's. Vygotsky's own theories demand, however, that we locate his work historically and culturally, taking account both of the nature of the time and place in which he lived and worked, his own personal history, and the fact that he died in 1934. What is often called neo-Vygotskian work, that is, developing his ideas in relation to more recent theoretical frameworks and to historical/cultural research, is therefore actually Vygotskian, in this sense. I will elaborate a little on some of these ideas, and other key elements in socio-cultural theory.

Key elements in Socio-cultural Theory

Instruction

Given the fundamental notion that consciousness comes about through communication, and Vygotsky's insistence that instruction begins from the very first day of a child's life, teaching becomes inseparable from learning, and is often written paired as teaching-learning. This is not to raise any of the old debates, either about priority of learning over teaching, or that teaching does not necessarily lead to learning. It is to

challenge child-centred theories, however, which ignore or downplay the inherent imbalance of knowledge/power. Teaching, in this sense, should be taken much more widely than the activities of the teacher in the classroom. She or he may be a more informed peer; a parent who has no explicit intention to teach; or a master creating, together with the apprentice, a zone of proximal development. The teacher may be a text, a production of the culture from which one can learn; or indeed a teacher whose explicit intention is to enable the student to do something, be someone or know something that he or she could not do, could not be or did not know. All human development is led by learning *from others*, from the culture that precedes us. Several issues are raised by the pair teaching-learning, linked to the notion of the zone of proximal development, and mediation, and these are described below.

Mediation

The world and what things mean are mediated for us by others. Meanings signify, therefore, they are not identical with the empirical object, but are known only through language. By analogy with physical tools, cultural tools transform us internally because they form and transform the world and enable us to see and to act differently. Just as one's thinking of acting in the world is transformed by learning about a hammer and its purpose, so too a ruler, the natural numbers, and the notion of drag in dynamic geometries become tools which transform us and how we act in the world mathematically. The teacher, which may be a peer, another adult, or a textbook, are central in providing these tools and their history, and they then mediate the world for the learner. Vygotsky operationalised Marx's dictum of the ascent from the abstract to the concrete through the notion of mediation in the zone of proximal development. Work currently taking place in algebra teaching and learning in early primary years (Falcao, 2000) is an example of this in the context of mathematics.

Internalisation

As mentioned above, the process of internalisation is to be seen as part of the transformative nature of learning. Piaget pointed out that the idea of knowledge of the nature of the empirical world coming about by forcing itself into our minds makes no sense, but nor does the idea of knowledge as innate. Vygotsky's insistence on the social origins of the individual's knowledge, through intersubjectivity rather than interaction (Steffe & Thompson, 2000; Lerman, 2000a), offers a different resolution to the epistemological problem than Piaget. Internalisation is how one becomes a person in each of the social and cultural situations in which

we grow and develop. "The process of internalization is not the transferral of an external to a pre-existing, internal "plane of consciousness"; it is the process in which this plane is formed" (Leont'ev, 1981, p. 57). Steffe and Thompson (2000) have argued that interaction is prior to intersubjectivity, whereas for Vygotsky intersubjectivity is prior (Lerman, 2000a). When seen as equilibration (interaction) or the zone of proximal development (intersubjectivity), it becomes clear that this issue is central to the difference between constructivism and socio-cultural theories.

Learning leads development

Learning therefore leads development. Development is not restricted by maturational processes, such that what one can know is limited by which age-related developmental stage one has reached. Development is a consequence of the range of social and cultural situations that one has experienced and which have constituted the individual. There are stages to a child's life, but these are social stages, of play, of the school child, and of the developing adult. Vygotsky's well-known notion of the zone of proximal development (zpd) provides a mechanism for learning which brings together the power of authority, the goals and needs of the child and what each participant brings to the learning situation (see Meira & Lerman, submitted). Regarding the term 'mechanism', Lave argues that learning may be represented as increasing participation in communities of practice (Lave, 1996). In searching for a theory of learning that would be "a liberating analytical tool" (1996, p. 156) for discussing learning as social practice, she lists the following:

- *Telos*: that is, a direction of movement or change of learning (*not* the same as goal directed activity),
- *Subject-world relation*: a general specification of relations between subjects and the social world (not necessarily to be construed as learners and things to-be-learned),
- *Learning mechanisms*: ways by which learning comes about (p. 156)

Whatever mechanism is used, whether it is used as an explanatory framework or as an ontological statement, it must take account of the differences between workplace apprenticeships and the classroom, as well as being able to account for both. Where Piaget offers equilibration as the mechanism for learning, Vygotsky proposes the zone of proximal development. For Lave learning is transformation through increasing participation in social practices, and a mechanism for learning would

need to take account of the goals of the individual in joining, or being coerced into joining the social practice, and the specificities of the practice in terms of situated meanings and situated ways of being. The mechanism would need to take account of the factors that contribute to the individual trajectory through the practice, including what an individual brings to a practice in terms of their prior network of experiences, and the regulating effects of the practice. Vygotsky was not directly concerned with social practices. At the time of the Russian revolution the singular discourse of dialectical materialism, and the drive for progress from a feudal society to communism did not allow for the availability of other theoretical resources. His early death in 1934, at the age of 38, precluded any engagement with more relativistic social theories.

Vygotsky's work is generally taken to be about the individual learning in a social context, but I have suggested in this section that his theories make it clear that the zpd offers more than that. First, in that consciousness is a product of communication, which always takes place in a historically, culturally and geographically specific location, individuality has to be seen as emerging in social practice(s). Second, I have argued that all learning is from others, and as a consequence meanings signify, they describe the world as it is seen through the eyes of those socio-cultural practices. In his discussion of inner speech Vygotsky makes it clear that it is the process of the development of internal controls, metacognition, that is, the internalisation of the adult. Again, these are mechanisms that are located in social contexts. Finally, the zpd is a product of the learning activity (Davydov, 1988), not a fixed 'field' that the child brings with her or him to a learning situation. The zpd is therefore a product of the previous network of experiences of the individuals, including the teacher, the goals of teacher and learners, and the specificity of the learning itself. Individual trajectories are therefore key elements in the emergence (or not) of zpds (Meira & Lerman, in press).

Multiple socio-cultural situations

Both the notion of power/knowledge and the necessity of authority, and the multiple situations in which we grow up, of gender, class, ethnicity, religion, language, social groupings, place in family, size and so on, mean that we have multiple aspects to our identity - better to speak of identities. Paths through contexts are not determined by those contexts since, through resistance and through the range of identities that make us unique individuals, individual trajectories are described. A particular

social situation may elicit, or call up, an identity or positioning (Evans, in press), such as a student initially silenced by a sarcastic teacher, for example, but one may also shift to a discourse of resistance or another discourse, through chains of signification, which one may have mastered, and in which one may feel powerful rather than powerless.

Some problems

This brief overview of some key elements of socio-cultural theory as it relates to education is avowedly idiosyncratic (see Lerman, 2000a for a fuller version); there remain considerable problems in socio-cultural research with which I work in my own research.

- Sociologists analyse why children fail and how schools reproduce disadvantage (Ball, 1993; Apple, 1998, Bernstein, 1996, etc.). How can we bring the macro-theories of sociology meaningfully into our micro-analysis of the classroom? How do we bring together the influence of social class, for example, and what happens in a particular mathematics classroom? I have described classroom research as a particular focus of a zoom lens (Lerman, 1998), in an attempt to suggest that we need to develop ways of carrying out analyses of classroom interactions that take account of more than the children's utterances in the moment, followed by the researcher's inferences about what the child might know or intend by what s/he says. Draw back in the zoom, and the researcher looks at education in a particular society, at whole schools, or whole classrooms; zoom back in and one focuses on some children, or some interactions. The point is that research must find a way to take account of the other elements which come into focus throughout the zoom, wherever one chooses to stop. The same goes for studies of teachers' utterances.
- Apprenticeship models of learning are much studied and discussed today in our community. It appears, therefore, that postmodernism, Vygotskian theories, including activity theory, and learning as increasing participation in communities of practice are all core socio-cultural theories[1]. Each has its proponents who would claim priority of their theory. Can/should they be integrated theoretically? Of course it may just be that we are into a time of fragmenting theories and loyalties, and diversification of communities and ideas, and this may be a good thing. The idea that we may be able to arrive at 'the

[1] I know that some writers see postmodernism as an extension of individualistic theories. On the contrary, I think that postmodernism denies discourses of any essence of identity which could be called individuality, and looks instead to the discursive practices in which identities are produced.

answer' in the social sciences in general and education in particular could be seen as an attempt to model research in these domains inappropriately on images of the natural sciences (also challenged by social studies of scientific knowledge).

- What can we do about the continuing failure of so many students in mathematics, and the correspondence of that failure to socio-economic class, economic deprivation, some minority ethnic communities, etc.? Can we make our research matter?
- All of the above could be about language learning, or anything else. What's special about mathematics?

In the next four sections I will offer some of the ideas I have been working in, in each of these areas. The conference stimulated much fruitful discussion and I hope readers will react with their own ideas in this journal and in other journals and meetings.

Macro/micro issues

In some recent analyses of classroom activities, both of mathematics and of science, I have tried to incorporate information surrounding particular interactions, regarding the social relationships between the students and the framing of the activities by the teacher (Meira & Lerman, submitted; Lerman, in press) but I have not been systematic in any way that I feel to be satisfactory. Others (dos Santos & Matos, 1998) have been more successful, but I feel the need for frameworks to facilitate research that brings the range of levels of analysis together. Bernstein (e.g. 1996) offers one such possibility (see also Daniels, 1993). He argues that dominant groups in society influence the dominant official educational discourse. It is up to teachers, in the process of recontextualisation into the classroom, to comply or resist that official discourse. One can then analyse the dominant view and individual classrooms, in terms of classification and framing, the supporting psychological paradigms, forms of assessment, curriculum, pacing of activities and so on, to see the effects of the official discourses. His theory also demonstrates how students are positioned differently by discourses, and he puts a particular emphasis on social class. His perspective has proved very useful in our own community in, for instance: studies of the role of context in mathematics questions (Cooper and Dunne, 1999) which indicate that working class children are disadvantaged by everyday contexts; the positioning produced by textbooks written for different abilities rather than simply reacting to

perceived differing needs (Dowling, 1998); models of teacher education that give us another way of looking at the gap between what students produce in teacher education courses and how they teach when they go into schools (Ensor, 1999). Currently we[2] are using Bernstein's framework to examine how to produce readings, from interviews with teachers, which indicate how teachers are positioned in these relationships of resistance and compliance when assessing what students produce as mathematical investigations. We (Lerman & Tsatsaroni, 1998) are also working on a study of the productions of the mathematics education research community using Bernstein's framework. In this way we can use Bernstein's work to examine the relationship between ourselves, in the field of knowledge production, and the official field of discourse, and again what is recontextualised by teachers in the classroom, and by us in teacher education.

What other tools are there for analyses that bring together macro and micro issues in teaching and learning?

Proliferation of theories

I have recently written a paper (Lerman, 2000b) in which I gave an overview of what I called the social turn in mathematics education research. In that paper I tried to bring together some of these bodies of work. In the concluding section I wrote:

> *Perhaps the greatest challenge for research in mathematics education (and education/social sciences in general) from perspectives that can be described as being within the social turn is to develop accounts that bring together agency, individual trajectories (Apple, 1991), and the cultural, historical and social origins of the ways people think, behave, reason and understand the world. Any such analysis must not ignore either: it should not reduce individual functioning to social and cultural determinism nor place the source of meaning making in the individual. In order to develop such accounts researchers can choose to begin from the development of the individual and explain the influences of culture, or from the cultural and explain individuality and agency (Gone, Miller & Rappaport, 1999). I have argued here for the latter. In my review I have used Lave and Wenger's situated theories as a foundation and attempted to open spaces, through critique, for the development of their theories for our needs in mathematics education research. I have argued for consideration of the regulating effects of*

[2] Morgan & Lerman (2000), part of the activities of the project: " Teaching and Learning – Mathematical Thinking" which has been supported by Fundacao Ciencia Tecnologia, Grant no. PRAXIS/P/CED/13015/98, and also involves João-Filipe Matos (Project Director), Susana Carreira, Madalena Santos, Jeff Evans and Anna Tsatsaroni.

discursive practices. I have discussed the multiple practices at play in the mathematics classroom, most of which are not the intention of the teacher. As a result, the notions of mastery and legitimate peripheral participation need careful analysis in order to extend them to the classroom, and I have suggested that narrative methods of research are proving to be most fruitful in research. I have suggested that Vygotsky's notion of the zone of proximal development, when set within a discursive/cultural psychology that was not fully available to him, in terms of intellectual resources, during his lifetime, can perhaps provide the mechanism of learning to study the process of people 'becoming kinds of persons'.

Others may feel that the multiple frameworks that are offered by these different perspectives are fruitful by virtue of their multiplicity and attempts to unify them are misguided.

Producing failure

We have the theoretical resources to describe and explain student failure in mathematics in sociological and socio-cultural terms (Bernstein, Apple, Lave and Wenger etc.). I would like to think that we should be able to frame strategies that can ameliorate failure, but these theories demonstrate how deeply inequality reaches into education. We must look at how education is perceived by young people, their parents, peers etc. We must look at the role of teachers in society, their status and their levels of pay. We must look at how mathematics is perceived by young people, parents, the media, and other elements of society that are significant for them. We must also recognise how education is set up to separate, to make sure there are enough failures to enable a small number to succeed. This manifests itself in the differential levels of resources given to different schools, by government strategies that give power to the neo-liberal groups, governments that are themselves empowered by neo-liberals of course, and by the self-fulfilling expectations of schools in different ethnic and socio-economically advantaged or disadvantaged communities. But we must also look to what we teach and how we teach it. I have to say that I don't think the answer lies in a reform-orientated curriculum. Power and authority do not disappear in an inquiry classroom, or anywhere else for that matter. Power just becomes hidden by not being spoken about. Many studies using Bernstein's framework indicate this, as in his discussion of visible and invisible pedagogy. But neither does the answer lie in more traditional curricula and teaching/ learning styles. Such a simplistic dichotomy is not helpful to education with an equity orientation.

What's special about mathematics?

Finally, I will make a few remarks about mathematics. Most of what I have written applies to education generally. What is so special about mathematics? In once sense, nothing, in that learning theories, inequality, apprenticeship etc. are all about the developing child and apply to all aspects of education. But clearly there are some special features to *mathematics* education, just as there are special features to all school subjects. Some features are rather depressing of course. More people fail at it, are bewildered by it and even hate it, than any other school subject. It is socially acceptable to say that one is non-numerate, whereas it is not acceptable to say that one is illiterate. We have great difficulty justifying mathematics content to students who ask, "What is this for? How will it help me in later life?" except by calling on what someone called 'deferred purpose': "Take my word for it, you'll see why later".

Perhaps Jo Boaler's book (Boaler, 1997) offers some insight here, at least in relation to one aspect of the problem. She demonstrates that it is possible to construct a curriculum which students enjoy, which they see as closely related to their everyday lives, and in which they learn to be as capable of the acquisition of skills as students from textbook based environments. The fact that the problem-based school in her study changed to being textbook based after the study returns me to the political issues I mentioned above.

We certainly need to continue studying the specific nature of mathematical thinking and how to teach and learn it. I am convinced that socio-cultural perspectives offer different and very fruitful directions here. Seeing tools and signs as mediating learning, and seeing learning as the taking over of the socio-cultural, in Wittgenstein's sense,

> *Words are connected with the primitive, the natural, expressions of the sensation and used in their place. A child has hurt himself and cries, and then adults talk to him and teach him exclamations and later sentences. They teach the children new pain behaviour. "So you are saying that the word 'pain' really means crying?" — on the contrary: the verbal expression of pain replaces crying and does not describe it".*
>
> *(Wittgenstein, 1967, p. 89)*

and as described by Vygotsky

> *He [Vygotsky] argued that when the infant cries or reaches for an object, the adult attributes meaning to that behaviour. Though the infant has no communicative intent, these acts nonetheless function to communicate the infant's needs to his caretaker. Here, as in the adult's attempts to interact with the infant, the infant is included in communicative social activity before*

> *he has the capacity to use or respond adequately to communicative devices. Vygotsky argued that this provides the foundation for the transformation of the infant's behaviours into intentional indicative gestures."*
>
> *(Minick, 1987, p. 28)*

are providing researchers, such as those already mentioned in this paper, and others, including Bartolini Bussi, Mariotti, Crawford, De Abreu, Forman, van Oers, with new possibilities for interpreting teaching and learning and therefore enabling the design of teaching and learning. Indeed, in some of their recent papers, the Italian researchers mentioned bringing the two together as tool-and-result, following Newman & Holzman (1993).

The practices of classroom mathematics, the significations of terms, their meaning and sense, indeed becoming used to what constitutes legitimate activity in school mathematics, are realised in that context. A further key feature here is what Vygotsky called, taken from Marx, the ascent from the abstract to the concrete, as mentioned above. Vygotsky drew on this notion in his theory of the acquisition of scientific concepts, and one development of this perspective has been towards the teaching of general principles to students, with particular questions being seen as instances in which the general principles need to be identified and applied (Galperin, 1969; Talyzina, 1981). This runs contrary to the usual tendency to work inductively from a range of everyday examples to general principles. In studies of young children learning algebra, researchers (Lins, 1994; Falcao, 2000) have built on Davydov's work, engaging children in letter as variable from the start of their induction into algebraic thinking/speaking, in contrast to the more familiar algebra as succeeding and generalising arithmetic.

In another study, this time of students in three inner city secondary schools in the UK, across all levels of achievement according to national tests, principles for calculations of rates of processes were taught (Day, forthcoming). Students were shown a generalised model for the conception of the values involved and the relations between them, in the form of a visual structure for combining given data, such as time and rate, and using it to calculate the quantity. The teaching was orientated towards success in using the model for a range of problems of increasing complexity, both set by the teacher and invented by the students. That success was measured by a dynamic assessment procedure based on the amount and type of assistance they required. Results show a level of achievement and change in attitude, across the ability range, which has surprised the classroom teachers and ourselves, and we have certainly found that the results support the argument for a 'theoretical learning approach' (Karpov & Haywood, 1998).

Walkerdine's (1988) approach, of seeing mathematics teaching and learning as regulation in a particular discourse rather than the eliciting of accepted school mathematical concepts from children's prior experiences, fits well, in my view, with the socio-cultural framework I am seeking. She describes a transition from significations in the home environment to those required in the school mathematics classroom, through the shift along chains of signifiers and signified, orchestrated by the teacher. Thus the discursive resources offered by the teacher to mediate children's learning of mathematics and the metaphoric and metonymic connections of those meanings become focuses for research.

I feel sure (fairly sure) that thinking of the construction of mathematical concepts as an unnatural process that has to come from culture, whether Walkerdine's version of the process or another, is going to prove to be productive. I think we have seen very little research and teaching-learning development from this point of view so far. We have, of course[3], a vast range of deep studies in the design of mathematical tasks and the analysis of children's learning of mathematics from the Piagetian inheritance, particularly that of the constructivist researchers of the last fifteen years, even if there is a need to reinterpret the research findings from the perspective of socio-cultural theory.

A case study

Finally, to illustrate some of the points I have made here, I will give a brief example of some analysis of classroom transcripts. There is insufficient space to present enough analysis to cover all the issues in socio-cultural research on which I am currently working. The transcript comes from the data collected by the Classroom Learning Project, under the directorship of David Clarke (Clarke, in press). One camera is focused on the teacher and another on a pair of students chosen by the teacher for the researchers. Immediately after the lessons the students who were the subjects were interviewed, whilst looking at the videos of themselves. The researchers posed the questions. Later, the teacher was also interviewed using video-stimulated recall. The full data set, transcribed, was available to a number of researchers for analysis, under the terms of the research project.

The teacher in this extract set some ratio questions, a 'ratio pep test', to all the students in the class, telling them to cross out the ones which

[3] My thanks to Jere Confrey for pointing out how rarely socio-cultural theorists, and I include myself here, recognise this body of work.

contained algebraic terms. She then called several of the students to the front of the class, the ones who elsewhere she referred to as 'those who like working ahead'. She gave these students some extra instructions on cancelling algebraic terms in fractions and ratios, which later she called an 'algebra trick', so that they could also answer the crossed-out parts of the question. At the end of the transcript her "Bye" (utterance 15) sent them back to their desks to work on all the ratio questions.

1. T: Just working ahead a little bit?... OK. Now, I'm going to think of three numbers, right?, x is going to be 7, y is going to be 9, and uh, m is going to be 3. OK? Now I'm going to multiply x by 5. I would write it as 5x. OK? I'm going to multiply y by 5, how would I write it?
2. Ss: 5y.
3. T: OK. And I'm going to multiply m by 8.
4. Ss: 8m.
5. T: All right. Now, I'm now going to divide x by 5. Now what's going to happen if I do that?
6. Ss: Be the same number.
7. T: Ah. It's going to go back to the same number. All right. I'm now going to take this, um, I multiplied m by 8, I'm now going to divide it by m. What am I going to be left with?
8. S: Eight.
9. T: Um. Is it?
10. S: Yes.
11. T: Right, m is 3, 8 times 3 is 24, divided by 3, brings it back to 8. Do you notice that this one you told me brought it back to 7. Seven times—5 times 7 is 35, divided by 5 is 7. Good. And this one here you told me went to 8. Now can you see a pattern?
12. S: Yup.
13. T: Right, if anything's on the top I'm multiplying, if anything's underneath I'm dividing. So this is actually a multiply by, and this is actually a divide by. Can you see how they cancel each other out?
14. Ss: Yeah.
15. T: So really you say 5 into 5 goes once, and 5 into 5 goes once, so really I've got 1x over 1, which is just x. And this one here is I've got n, which is a number and I'm going to divide it by itself. They cancel out and give me 1, so I've just got 8... Bye.

The teacher spoke quite quickly, the whole teaching episode lasting 1 minute 48 seconds. These students are apparently familiar with episodes of this kind and the conversation is quite abbreviated. We can describe the teacher's discourse as fast pacing, and strongly framed in terms of precise rules and language for the mathematical task. There is no concern with other discourses, such as everyday mathematics. In the earlier section of the lesson, before the ratio pep test, teacher and students

were discussing how to interpret velocity-time graphs. The task was strongly classified and framed, everyday examples such as skiing and drag-racing being harnessed and expressed in classroom mathematical terms.

The resources she offered the students in working on ratios with algebraic terms were: substitution of numbers for letters, as a legitimate way of demonstrating underlying patterns, through her own work (particularly in utterance 11); and cancelling out terms that are common to numerators and denominators (utterances 13 and 15). These procedures and the associated language mediate the task for the students. From this episode we do not know how much of these strategies have been taught before. The teacher does not distinguish between fractions and ratios, which causes some confusion for at least one of the students. The teacher is later confronted by the confusion, and is forced to make a pedagogic choice at one moment between clarifying the distinction or not.

The relationship established is friendly and informal. The students called out are positioned as more able, although the social norms of the classroom, the teacher's 'rules', enable other students to volunteer to join the group if they wish. Thus the boundaries of ability are not overtly strongly marked, although it seems that they are still of considerable significance in the classroom. The use of 'I' is interesting, contrasting with the often used 'we' (Pimm, 1987). It establishes the teacher as single authority and instructor, as expert, but perhaps also offers a view of the potential apprenticeship of the students. The appropriate mathematical activity can be acquired by the students if they follow the teacher's instructions; it is presented as technique that can be acquired. Using 'we' can appear to express membership of an exclusive club.

It should be noted that this description is not an implied criticism of the teacher. Dichotomies between reform or constructivist classrooms on the one hand, and transmission classrooms on the other are too simplistic and unhelpful, in my view. For instance, the explicitness of criteria of framing offered by the teacher here is supportive of students often thought to need more loose, weaker framing (Cooper & Dunne, 1999). Thus, I have proposed that one can read the following in the teacher's talk: discursive mathematical resources that are provided for the students; differences of ability that are established with differing expectations; and that initiation into esoteric practices (Dowling, 1998) of school mathematics constitutes the orientation of the students' activity by the teacher.

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