How do primary pupils give written arguments in a conflicting mathematical situation[1]

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This paper deals with the written arguments primary pupils (N=201) gave when they faced a conflicting situation with confusing mathematical information. The theoretical framework is derived from the idea that mathematical argumentation is regulated by normative aspects. Both quantitative and qualitative methods were used in the analysing of the data. The results of this study suggest that although most pupils are able to give correct answers, many of them use mathematically irrelevant norms in their argumentation.

How do primary pupils give arguments in a mathematical situation? Argumentation is understood here as a chain of ideas proceeding from the premises to a conclusion. In a classroom, a common possibility for argumentation is, to show the rationality of one's own action when explaining a solution to a problem. Argumentation is related to the concept of reflection, since reflective thinking can be understood as a way to seek grounded perspectives. In education reflection is often understood as a process of social interaction, in which a participant tries to make his or her actions understandable and accountable. According to Bauersfeld:

> *«every kind of reflective process proceeds within a different 'language game' - in Wittgenstein's sense - than the language game used for the description of the direct action that is under consideration.»*
> *Bauersfeld (1996)*

Accordingly, the skill to solve problems does not as such produce the skill to explain the rationality of one's own actions to other people. The skill to use new language games for reflection develops through training only.

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Theoretical background and data collection

In the following, the theoretical framework is derived from the idea that mathematical argumentation is regulated by normative aspects (Yackel & Cobb 1996): What kinds of arguments are justified, acceptable and valid? Thus the communicative nature of the argumentation is emphasized. It is assumed that the norms are produced in the classroom interaction processes. Krummheuer (1995) writes of collective argumentation, which refers to the fact that in this interaction argumentation emerges in co-operation with several participants in face-to-face -situations. *Social norms* are general norms, which regulate social activity, and they can be applied to any subject matter. By *sociomathematical norm* Yackel and Cobb refer to the normative aspects of mathematical discussions which are specific to mathematical activity. A sociomathematical norm includes a shared understanding of what constitutes an acceptable and justified mathematical explanation in each community. An example of a classroom's sociomathematical norm is the understanding of what constitutes mathematically elegant or mathematically efficient solutions (Yackel & Cobb 1996).

This paper deals with the written arguments primary school pupils gave when they were faced with conflicting or confusing mathematical information. In this situation pupils had to make a decision and give arguments for it. The data was collected by student teachers in ten teaching groups in nine different schools. These schools were situated in different parts of Finland, either in rural areas or in small towns. The number of pupils was 201, half of whom (48%) were fifth-graders, i.e. 11 years old, but fourth and sixth-graders (38% and 14%) also participated. The following task was posed to pupils:

Pens 7.50
Pencils, 2 in a package 4.50
Glue 35 ml 9.30
Sharpener 5.80

Accompanied by the illustration shown here, the following task was given to pupils in one classroom: Jack buys six pencils. How much must he pay for them? Arthur says that Jack must pay 27 marks. Lisa thinks that the correct answer is 10 marks and 50 pennies. With which of the children could you agree? Give arguments for your answer.

The pupils were asked to write their answer and argumentation on a sheet of paper. Those written answers became the data for this analysis. The student's teachers read the task aloud as many times as desired to ensure that a pupil's poor reading skills would not prevent him or her from accomplishing the task. The student teachers were also asked to make sure that all pupils understood what they were expected to do. Recording classroom discussions could have collected a richer and more relevant data, but this was not possible in this setting. However «writing mathematics» is also important. Writing is one way to communicate. During the process of writing a person elaborates and clarifies his / her thoughts and tries to make them understandable for others (Applebee 1984, Borasi & Siegel 1994). It has been shown that writing can provide opportunities for students to construct their knowledge of mathematics by constructing arguments. (Countryman 1992). As the data is pupil's written works it is not possible to consider the development of argumentation, but nevertheless, it is possible to see what kind of explanations were used and acceptable in the classrooms' mathematical situations. Careful reading allows us to make considerations about the normative aspects behind the written products. However, it was not assumed beforehand, that this kind of data would reveal the social norms of teaching groups. In fact, the theoretical framework was adopted after the qualitative analysis in order to understand findings: Why arguments in one teaching group were so similar with each other but so different from the arguments in some other group?

Results and typical argumentations

Altogether 71% (= 142 pupils) understood the task and gave, at least to some extent, reasonable arguments. *Understanding* has here a two-fold meaning. First, the pupils were supposed to answer the question posed (*«With which of the children could you agree?»*) and give reasonable arguments and, second, they were supposed to show that they could properly carry out the calculations. It was not expected that pupils should write their calculation on the work sheet - they were allowed to do the calculation mentally if they preferred. Minor errors in calculation did not matter. The main issue was the argumentation.

In the course of analysing it turned out that if pupils had made many errors in calculations, in most cases it was impossible to follow their reasoning or at least the interpretations had been very coincidental. So I decided to skip over those pupils. The total number of excluded pupils was 59 (= 29% of all pupils). Half of these pupils were fourth-graders and the other half was fifth- and sixth-graders. Some of them gave no answers at all, some gave answers and arguments without any reasonable logic, and some of students had made so many errors in calculations that it was impossible for them to draw any reasonable conclusions regarding the task.

Table 1 shows the proportions of «understood» and «not-understood» pupils over grades. Statistical analyses show that grade (or the age of pupil) and understanding are independent on each other (c^2 = 4,89; p>0,10; df=2).

Grade	Number of pupils	"Understood"
4th	77	48 (62%)
5th	96	72 (75%)
6th	28	22 (78%)
Total	201	142 (71%)

Table 1 Proportions of "understood" pupils across grades

When analysing pupils' arguments I applied the method developed by Stephen Toulmin (1974), in which an argument is seen to consist of different elements. Data (D) provide the starting point on which the conclusion (C) is grounded. According to Toulmin, a mere conclusion without data is not an argumentation. The relationship between data and conclusion can be expressed as follows: *«D so C»* or *«C because D»*. This process - from data to conclusion - is legitimised with the help of facts, which are called warrants (W). The argumentation can so be expressed *«D since W, so C»*. Data, warrants and conclusion form the core of argument (Fig. 1). The warrants can be supported by some generally known facts, called backing (B) in Toulmin's model. Krummheuer (1995) used the same method when he investigated the development of argumentation in a classroom.

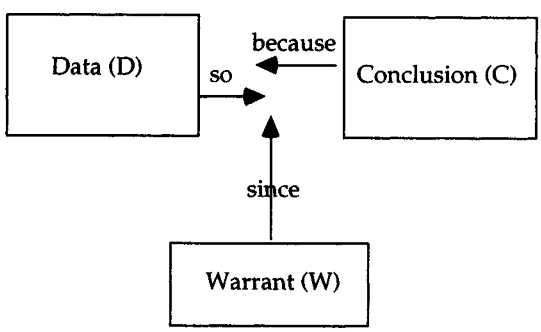

Fig. 1 The core of argumentation according to Toulmin

In the following I will discuss only the responses of those pupils who understood the task as defined above (N=142). Most of them (66%) stated they would disagree with both Lisa and Arthur but the quality of arguments varied a lot. Many pupils used arguments having the nature of mathematical reasoning:

> *«I disagree with both of them, because if the price for two pencils is 4,50 then for six pencils Jack must pay 3 x 4,50 (Jack needs 3 packages). So altogether 13,50.»* (Fig. 2).

They started from the mathematical information and facts and proceeded step-by-step. Combining the data known from the solutions given by Lisa and Arthur with their own mathematical reasoning drew the final conclusion.

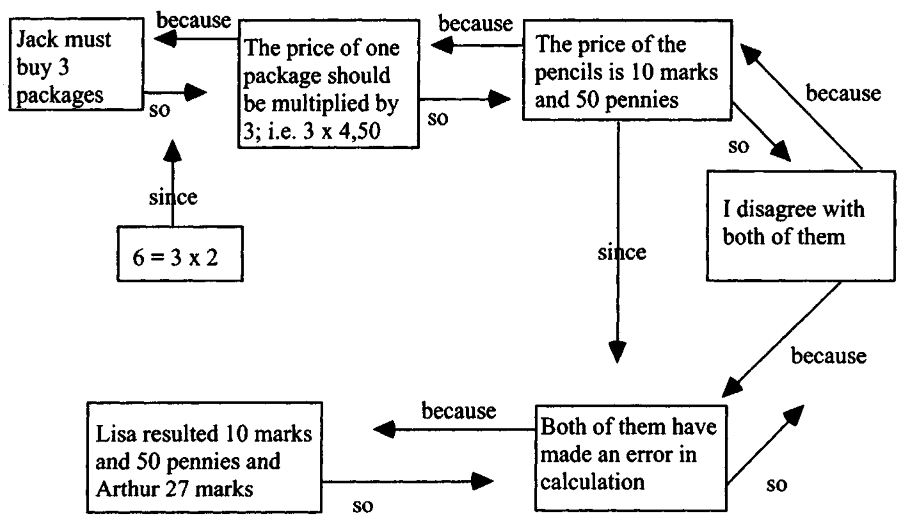

Fig. 2. A mathematically qualified argument

The (mathematically) less qualified arguments were based on pupil's own performances. A typical argumentation was:

> *«I cannot agree with either of them because I got a different answer.»*

Some of them stated:

> *«... because both of them are wrong and the right answer is 13,50 mk.»*

Figure 3 displays an example of this kind of argumentation.

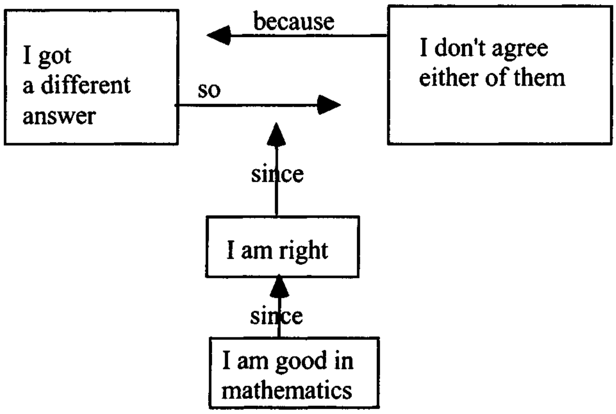

Fig. 3. An example of argumentation based on pupil's own performance

32% (=46 students) wrote they could agree with Lisa. However, majority of them stated that neither Lisa nor Arthur was right, but if they had to choose, they would side with Lisa, because her answer was nearer to the right one. This was the most typical chain of argumentation (Fig. 4).

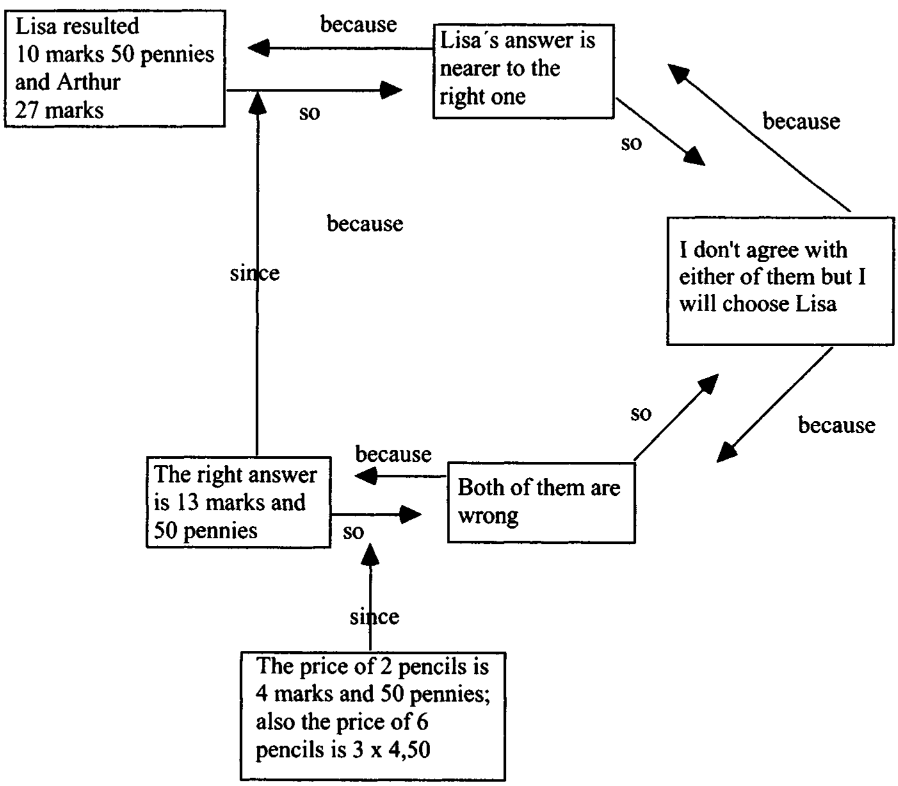

Fig. 4. The most typical chain of argumentation

Only a few pupils gave their support to Arthur, and the arguments given by them were very reasonable. They were sympathetic to Arthur and the error he had made:

> *«He did not notice that there were two pencils in the box. He has calculated the price for six boxes correctly.»*

Some pupils stated that when you are estimating the price it is better to estimate too much than too less, because so you can avoid the embarrassment of not having enough money for payment

The normative aspects of argumentation

I distinguished between two sociomathematical norms. The first sociomathematical norm (type i), which is based on the arguments of those students who agreed with Lisa, can be formulated as follows:

> *«The result which is closest to the right answer, is a better one."*

One third of the pupils used this explanation and regarded it valid and justified. Examples of this norm are:

> *« Lisa's answer is nearer to the right one. That's why I will side with Lisa, if I had to choose.» (Carolyn 5.)*

> *«I could agree with Lisa, because she is nearer to the right answer. The right answer is 13 mk 50 p» (Susan 5.)*

> *«With Lisa. Lisa is nearer, because 13 mk 50 p is nearer to 10 mk 50 p than it is to 27 mk.» (Martin 4.)*

> *«With Lisa, because her estimation was nearer than Arthur's. The price was 13 mk 50 p.» (William 5.)*

This norm is interesting: Although Lisa's process was not in any way reasonable, pupils - however - though that her answer is better than Arthur's reasonable process but wrong answer. This gives us reason to believe that in classrooms' mathematical discussion products are regarded more valuable than processes. Thus an answer that is nearer to the right answer, but is obtained through a wrong process is more appreciated than the mistaken answer obtained through a more proper process. However, every pupil who applied this explanation carried out the calculations correctly and stated that, in fact, neither Lisa nor Arthur was right, but they were ready to give their support to Lisa.

The second sociomathematical norm, mathematical reasoning, (type ii) appears in some of those explanations where pupils stated they would disagree with both Lisa and Arthur. These pupils did not accept either of the given alternatives, but generated their own opinion.

> *« I think the price is 13 mk 50, because if Jack buys six pencils he has to buy three packages. I don't agree with Arthur, and not with Lisa, because to my mind Lisa's answer is wrong and so is Arthur's.» (Tina 4.)*

> *«If Jack buys 6 pencils the price of two pencils must be multiplied by 3*
>
> *4,50*
>
> *3*
>
> *13,50 so the result is 13,50 mk and that's why I don't agree with Jack or Lisa.» (Sam 5.)*

> *«Neither of them! Because Jack's pencils are 13,50 mk! And Arthur says 27 mk and Lisa 10 mk 50 p. Arthur mistakes 13 mk 50 p and Lisa 3 mks.» (Heidi 6.)*

In these the arguments were based in the first place on mathematical reasoning. One third of the pupils applied this norm. This kind of sociomathematical norm gives us reason to believe that mathematical argumentation is practiced in some classrooms.

My hypothesis was that social norms could not be seen in pupils, written products. However this belief turned out to be wrong. I could find two social norms that regulated the nature of argumentation. There were five (out of ten) teaching groups with a great number of pupils who justified their decisions primarily on the basis of their own expertise or the lack of expertise of others (type I). A typical argument was:

> *«I disagree with both Lisa and Arthur because **I** got a different answer.»*

or

> *« ... because both of them have calculated incorrectly»*

(but in Finnish the meaning of "incorrectly" is stronger):

> *«I think that both answers are wrong, because I got a different answer.» (Julie 4.)*

> *«I don't agree with either of them, because I know the right answer. 3 x 4,50 = 13,50. The result is 13,50.» (Bill 5.)*

> *« 3 x 4,50 = 13,50. I don't agree with either of them, because my answer is 13 mk 50 p. Both of them have wrong answers.» (Beth 6.)*

In these teaching groups a valid argument seems to be the fact that the pupil has arrived at a different solution. Some of them have secured their backing by the fact, obviously well known in their classroom:

> *«I am right, since I am good in mathematics.»*

An example of this:

> *«I don't agree with either of them, because they have incorrect estimations. My estimation is correct, since I can calculate well.» (Tom 5.)*

We could say this norm is social, rather than sociomathematical, because it is grounded more on status or supposed expertise than on the efforts to show the mathematical basis. Furthermore, these explanations are often related to efforts to convince the teacher or the reader that the actor is innocent:

> *«I don't want to agree with either of them because both of them are wrong.»*

This kind of explanation has actually nothing to do with mathematics, and it can be applied in any situation to explain the rationality, if it is accepted in the classroom. But it is mathematically irrelevant norm. A mathematician would never accept an argument:

> *«It must be so, because I have calculated it and I know how to do it.»*

The second social norm was encountered in one teaching group. This norm includes the idea that you should understand other people. Regardless of what kind of mathematical arguments the pupils in this group applied, they tried to understand the actions of both Lisa and Arthur:

> *«Arthur has made a little mistake, but it could have happened to anyone. I don't know how Lisa got her answer, but it must be some kind of annoying and human mistake.» (Ann)*

> *«I agree with Arthur, because he calculated correctly his task. He only made an incorrect formula. Lisa calculated the correct task incorrectly.» (Jacob)*

It was interesting to note that the quality of arguments was not so much dependent on students' age. Actually there was more variation between different teaching groups than between grades. There were teaching groups with only one or two pupils who used arguments typical to the sociomathematical norm, type i, and there were other teaching groups that preferred the social norm, type I, to sociomathematical norms. Norms differ from one classroom to another. Similar results have been reported by Yackel and Cobb (1996).

It is obvious that some teachers have trained their pupils to argue and express their opinions. It would also be important that instruction draws attention to some conflicts that confuse students. Irwin (1995) for example noticed that such an instruction helped students to gain better understanding of mathematics.

Concluding remarks

In our schools children are not very well acquainted to write or talk mathematics. Thus the possibilities to develop «new language games» remain minimal. Because language is a mean of thinking, we cannot argue that learning of language games were not important. The teacher has a crucial role in the process of evolving the classroom mathematical practices and traditions (McNeal & Simon 1994). This includes the ways to use language.

As an experienced teacher at primary level I would make a couple of remarks on the findings. It is delighting that more than seventy per cents of the pupils have understood what was asked and were able to calculate quite well. But it is a bit annoying that only one third of pupils applied mathematically valid arguments. The rest of the pupils based their decision on an irrelevant sociomathematical norm or on social norms. Most pupils can calculate quite well, but who could help them to learn new language games and gain a higher level in thinking and reflection.

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Abstract (in Swedish)

Avsikten med detta arbete är att undersöka hur lågstadieelever (N=201) argumenterar inför en inkonsekvent uppgift med motstriding matematisk information. Den teoretiska referensramen baserar sig på tanken att matematisk argumentaion regleras av normative aspekter. Data har analyserats både kvantitativt och kvatitativt. Resultaten visar att trots att de flesta elever kan ge korrekt svar, så använder många av dem matematiskt irrelevanta normer I sin argumentation.

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Reserche Interest

Her research interests include mathematical argumentation, gifted education in mathematics and science, and project work at school.

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