# Going across the grain: mathematical generalisations in a group of low attainers 


#### Abstract

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This paper is a report of a classroom research project whose initial aim was to find out how low attaining students in mathematics would respond to some cognitively challenging prompts usually reserved for higher attaining students. Analysis of classroom incidents revealed the power of a type of mathematical prompt, which can be generalised as "going across the grain of the work". The metaphors of working against and across the grain are also used to describe a particular perception of mathematical structure, the use of unfamiliar methods of interaction with students, and some unorthodox features of the research method. In this study it was found that deep mathematical structure could be encountered by looking across the grain of the work. This finding, although from a very small specific study, is rooted in the characteristics of mathematics. The importance of principles manifested in small classroom studies is discussed briefly.


## Introduction

The purpose of this classroom study was to learn more about how low attainers can be helped to think mathematically through the use of specific kinds of interaction.

The title of this paper uses "grain" to describe a direction of growth and alignment of elements, as in a tree trunk. "Going across the grain" uses the metaphor of working across the direction of the wood, such as one does when sawing, to reveal a cross-section which displays different patterns to those which can be perceived on the surface. «Going against the grain» describes friction caused by using an approach that opposes smoothness.

These metaphors are used to describe three aspects of the research study. Firstly, the giving of challenging prompts, unfamiliar questions and abstract tasks to low attainers goes against the usual grain of providing them with structured, step-by-step, concretised mathematics - designed to help them believe mathematics is easy. It opposes the

[^0]normal expectations, which such students may have of their lessons and their achievements. Secondly, the practicalities of the researched environment led to the use of ad hoc strategies, which cut across the grain of the underlying "teaching experiment" plan. Thirdly, and most importantly, it was found that the prompts and questions which led to interesting mathematical incidents were those which urged students to look at their work from a fresh viewpoint, cutting across the grain of the original work. This exposed cross-sections of mathematical structures, which were not immediately obvious.

## Background

The school selected for the study has an intake skewed towards belowaverage achievement, with a high proportion of socially-disadvantaged students. The group was normally taught with a mixture of practical activities and practice of number-skills. Factors leading to underachievement in national tests had been identified through scrutiny of the school's test papers and some clinical interviews (Watson, 1999). Many of these were the «misconceptions» recorded in the CSMS study (Hart, 1981). Others were related to how the students responded to test questions, such as:

- difficulties in "reading" question layout;
- discrimination between uses of diagrams and pictures;
- deciding if it was appropriate to use everyday knowledge.

Success in these aspects of test-taking requires flexible use of knowledge and some meta-cognitive awareness. The school had restructured its curriculum to raise standards through providing more advanced content to all students, but I felt, following the findings of Boaler (1997), that different teaching approaches might also be of use. I arranged to work with the lowest-attaining group of $13 / 14$ year olds to research the hypothesis, developed in the Low Attainers in Mathematics Project (LAMP, 1987), that students who are normally expected to perform routine mathematics could respond to challenging forms of questioning which require complex, adaptive and reflective mathematical thought. The group was small, with a regular attendance of about 11 students, and contained students with self-esteem, behavioural and attendance problems alongside their very low achievement.

## Mathematical thinking

Trickett and Sulke (1988) refer to general characteristics of gifted children and describe how these were also characteristics of low attainers in challenging classrooms. Rather than focusing on generic learning skills, as they did, I considered the mathematical abilities of gifted mathematicians, described by Krutetskii (1976), to indicate mathematical thought. He was concerned with students' ability to grasp formal structures through an overall sense of pattern; to develop spatial and numerical arguments; to generalise globally; to devise and use valid shortcuts; to be flexible with mental processes. Other aspects found important by Krutetskii, such as memory for mathematical objects, were not suitable foci for a one-term study. Other descriptions of thinking skills and heuristics for mathematical problem-solving, such as those of Mason et al (1982), Schoenfeld (1985), Feuerstein (1980), Romberg (1993), agree the importance of recognition of useful patterns in order to devise shortcuts; generalisation; engagement with underlying structures; use and generation of appropriate examples; development of arguments and flexible thought. These components of mathematical thinking are used to frame my approach.

The LAMP team point out that reports of teaching projects commonly focus on outcomes, but rarely disseminate enough information about how these are attained so that other teachers may use the methods. Their approach was to work with teachers to develop ways of structuring lessons to produce rich activities and discussion. Feuerstein's approach was to teach thinking skills directly. Adhami et al (1998) offer structured planning procedures to promote mathematical thinking. However, the success of any curriculum, teaching approach or task is dependent on the way the individual teacher interprets it in her classroom. Rather than needing special tasks, special knowledge of how to plan, and prior metacognitive training, my approach was to focus on possible teacherstudent interactions in the context of the scheme of work which already existed in the researched classroom. In other words, I explored how students' mathematical thinking might be scaffolded to a higher order, through interactions with me as teacher-researcher, during their ordinary classroom tasks. I wanted to see if these students, who were believed to have low mathematical ability (and believed that of themselves), could act in a cognitively sophisticated way with mathematics, without special metacognitive training, if asked to respond to certain kinds of prompts. In comparison with their past experiences in mathematics this approach would certainly be «going against the grain».

The prompts I used had been developed previously from observations and experiences in a wide variety of classrooms (Watson \& Mason, 1998) and relate to the components of mathematical thinking already described. Students were asked if they could see and generalise patterns, use and generate examples, describe underlying structures, reverse a line of reasoning and so on. Some examples are: If this is the answer to a question, what could the question be?; What is the same or different about .....?; What happens in general?; Give an example which fulfils certain conditions

## Method

I intended to team-teach in two lessons a week with the usual teacher, developing a relationship with her and the students over time, so they would become used to our joint teaching. Lessons would be videotaped so that teacher-student interactions could be analysed. The video-operator would be introduced early in the term. It was important to be sensitive about the inclusion of new adults into the classroom as several of the students had emotional difficulties. The intended research method was dependent on gradually growing relationships between the teacher and me, the students and me, and the students and other adults. This would run in parallel with a systematic introduction of different kinds of prompts whose effect would be monitored with increasingly technical forms of data-collection.

The intended smoothly-grained study had to be radically recast because of pressures within the classroom, and personal pressures on the participants. Small-scale studies are disproportionately affected by such events, and classrooms are particularly prone to such instabilities ${ }^{1}$. Some of the students were more easily disturbed by changes in routines than I anticipated. There was not enough time to introduce the videotechnician gradually into the class and develop a useful recording technique involving specific students. I had hoped to work towards sometimes teaching the whole class for part of the lesson, while being recorded, but staffing pressures within the school led me to take over the role of whole-class teacher sooner and more frequently than I would have chosen. No time could be found to co-plan; tasks were usually set by the normal teacher. In general I would introduce the given tasks to the whole class, encouraging interaction, conjecture and discussion.

[^1]Then, as teacher rather than researcher, it was my responsibility to support students by interacting with individuals or small groups as they did the set work. In the early lessons I had time to make detailed notes about one-to-one interactions, but I could not keep such systematic records while I was teaching.

A low-technology method of data-collection, note-taking, and reflective commentary had to replace the planned approach. I took notes of what I had said and how individual students had responded during, and systematically after, every lesson; with a small class it was possible to think of interactions with each of them. The notes were records of what could be seen, heard and read about the mathematical work of students. Interpretation was avoided as much as possible, but a personal perspective was inevitable, given my involvement as teacher and the inevitable interpretation which has to take place to transform the symbolic manifestations of mathematics into common meanings.

I analysed the notes by identifying students' responses, which appeared to relate to the categories selected from Krutetskii's work. In particular, the questions and prompts, which had been included in these interactions, were noted. Students' unprompted remarks, which I interpreted as showing the desired features of mathematical thinking, were also identified. A record of the incidents and dialogues, which included these features, was prepared after each lesson. The resulting text was also sorted into a separate record of incidents involving each individual student.

As a result of this analysis I planned the prompts I would use in the next lesson, developing the use of those which appeared useful, according to my interpretation, and introducing others not yet used. For example, after one lesson I found that every student except one had given an example, either prompted or unprompted, during the lesson. I decided to ask her directly to give an example, when appropriate, in the next lesson. In this way I systematically explored the ability of all students to display the types of response identified by Krutetskii, and the interactions in which they did so.

The method can be summarised as recording classroom incidents and systematically analysing the record (i) for type of student response; (ii) for the type of teacher utterance related to the desired kind of response; (iii) to monitor the responses of individual students; (iv) to plan future teaching. The whole corpus of data from twenty lessons is too large to deal with in a short paper, so I shall concentrate on one sequence of incidents.

## Five incidents relating to pattern

The following five incidents illustrate a progression in my understanding of how awareness of pattern could be used to prompt these students to engage with structure.

## Incident 1

In a lesson led by the usual teacher, each student had been given a printed $10-\mathrm{by}-10$ grid containing numbers 1 to 100 and asked to shade in all the multiples of $2,3,4$ and so on in colour. The teacher's intention was for them to work on tables and number properties. In practice, students soon realised that spatial patterns emerged which they could exploit to finish the task.

## Comment

Use of pattern led students to transform the task into avoidance of the intention of the lesson. Once a spatial pattern had been spotted, or even assumed on the basis of flimsy evidence, it was used to avoid having to do calculations. The teacher said the relationship between the spatial patterns and the underlying multiplicative patterns was hard for her to grasp - and there was no plan to discuss it with the students. Only two took this task beyond the trivial stages.

## Incident 2

Students had been given a printed blank coordinate grid and a long sequence of coordinate pairs, which they had to plot. The only introduction was a reminder of the order of coordinates. All students except one were plotting points in the order given except one. The exceptional student had restructured the task by picking out all adjacent coordinate pairs, which would give the same vectors when joined ${ }^{2}$. She told me that if the first number went up and the other down by one, she would draw: $\triangle$ if both numbers went up by one she would draw: $\square$ She could predict other vectors similarly.

The next task for all the students was to write their initials on a new blank grid and record the coordinates of each letter. Many letters, such as I, J, E, involved drawing rectangles. When I asked some of them individually about the similarities and differences of the coordinates of the rectangular components of the letters they were able to comment on the relationship, saying that certain numbers had to be equal. I then

[^2]responded by writing the coordinates algebraically: (a,b); (a,c); (d,b); (d,c). Most students were able to decode what I had written and recognise it as the statement they had made.

## Comment

The first part of this incident is another example of the use of pattern to transform the task the teacher has set, but in this case the task becomes more mathematical as a result. The student had created a more abstract task using similarity and classification. This led me to suppose that others in the room would also be able to work in this way, using an abstract overview of the task to simplify the concrete expression of it. The end of the lesson came before I could work with all students, but it was clear that many were able to identify similarities from the coordinate pairs, articulate and recognise generalities, and accept a symbolic representation constructed between us. Yet for most of them the task had originally been a sequence of actions to transfer coordinate pairs one by one onto the grid, joining the dots as a separate sequence of actions afterwards. By asking students to look at similarities in the shapes, students came to look at the task as a different sequence of actions: classifying patterns in the coordinate pairs, selecting and drawing the similar vectors corresponding to the classifications. In other words, actions spanned the breadth of the task rather than repeating constituent elements consecutively; a holistic task rather than a fragmented one. This done, they were able to reverse the process and predict generalities about coordinate pairs which would arise from similar lines on the grid.

## Incident 3

Students were subtracting nine from a collection of two digit numbers they had been given; this was an exercise to generate practice in subtraction by decomposition. I asked two of them to compare the original two digit number to the answer in several cases. I wanted them to see that the units digit increased by one while the tens digit decreased by one. What they said was naive: «they all have two digits» was a typical response.

## Comment

Encouraging students to look for patterns is not automatically useful in getting students to think mathematically. After a while I realised that they were looking for patterns which might connect the answers to consecutive, unrelated, calculations rather than internal patterns which occur in all such examples as if they were following the new, but here inappropriate, practice of looking for patterns across all examples. This
incident led me to discriminate between looking for similarities between elements of all the calculations, as they were doing, and looking for similarities in the relationships within each calculation, as I now hoped they would do.

## Incident 4

I had given them a sheet of questions about the seven times table, similar to previous work on other times tables. There were four columns of calculations to do. Here are two of the rows:

| $3 \times 7=$ | $7 \times 3=$ | $21 / 3=$ | $21 / 7=$ |
| :--- | :--- | :--- | :--- |
| $4 \times 7=$ | $7 \times 4=$ | $28 / 4=$ | $24 / 7=$ |

Students soon realised that the answers to the first and second columns were obtained by successively adding 7 . Answers to the third were all 7, and answers to the last were the natural numbers in order. That is, students had "spotted" and used the patterns, which enabled them to complete the worksheet vertically. They reported that this was how they had filled in similar sheets before. While working with pairs I gave them « $23 \times 7=161 »$ as the start of a row and asked them to finish the rest. All could adapt to this horizontal approach after some thought. Two students made up their own examples to illustrate the structure further, after I had suggested this would be a useful thing to do. In addition most, but not all, students could explain to me why $49 / 7$ was only given once on the sheet.

## Comment

As a result of earlier incidents, 2 and 3, I had deliberately looked for opportunities, within normal tasks, for them to talk about generalisations, which give rise to the patterns inherent in mathematics. That is, I looked for ways to focus on relationships rather than elements. In this task students had not exploited, or even noticed, the relationships horizontally across the page (not all had even seen that the first two columns were the same). The patterns in columns helped students avoid working on multiplication as an operation, apart from as repeated addition; in contrast, the patterns to be found across the page relate directly to multiplication and, as its inverse, division. When I asked them to finish the rest of a row they had initial difficulty recognising that they had enough information to do this. Once they could see the structure in the horizontal lines they had no problems using it; the challenge was to get them to see it.

## Incident 5

Students had been asked to work in pairs using interactive computer software, which prompted them to build a fraction wall. Their previous knowledge of fractions was limited to spatial representations of common fractions, and they knew that, for example, quarters were "four equal parts". The software starts by showing students a collection of blank strips lying horizontally across the screen next to each other. Students insert vertical bars along a unit strip to show halves, quarters, thirds and so on as shown below. When their efforts are correct the program congratulates them and offers another turn.


Rather than allowing them to fill the wall by trial and error, which is how the school usually used this software, I restructured the activity and ask them first to draw halves, quarters and eighths next to each other, followed by other families of fractions. During the lesson I sat with each pair of students in turn and asked them to compare bars across the various strips. Several pairs wanted to delete their diagrams and change task as soon as they had received approval from the program; it was difficult to get them to reflect on their work. On further exploration with me they were able to generate several sets of equivalent fractions by noticing that the bars for one half, two quarters and four eighths matched. They decided which fractions matched and I wrote them down in standard format. I then asked them to suggest relationships between numerators and denominators.

## Comment

My aim was to develop the idea of focusing on comparing relationships by looking at tasks from a different point of view. The ideas initiated by incident 2 and developed in incidents 3 and 4 were used explicitly and deliberately in incident 5, but I had to use persistent intervention to persuade them to shift their perspective. However, once the value of this approach was clear they were happy to proceed. It was not that they could not reflect on their work from a new perspective, but that they did not see this as a desirable practice.

## Prompting mathematical thinking through reflection

In all the reported incidents students pursued tasks set by the usual teacher, all of which had the potential to be treated on a mundane level. Incident 1 illustrates that some students, even in this very weak group, look for pattern spontaneously; incident 2 shows that this search for pattern is not necessarily used to make work easier, but can make the work more interesting and abstract; incident 3 illustrates that the patterns most readily identified are not necessarily useful if they focus on elements instead of relations; incident 4 shows that when attention is forced onto relational features students can recognise of structure; and incident 5 shows, among other things, that encouragement to reflect is important when trying to focus students' attention on particular comparisons.

This work raises interesting questions about the potential of low attaining students, the usual expectations of them and how they might be encouraged to engage more deeply with mathematics. For this paper I am going to focus on a small but important feature of the prompts which contributed to a shift from superficial to structural thinking (Rieman and Shutz, 1996). In incidents 3, 4 and 5 the students were being asked to look at their work again but read it in a different physical direction. In 4, they had worked vertically and now had to compare their work horizontally; in 5 , the reverse was the case. This prompted me to examine other incidents to see if they had within them a sense of "working in a different direction". In incident 2, the student had decided to search along the list of coordinate pairs for similarities, rather than transfer them individually to the diagram. In 3, I had realised that they needed to be reading examples in parallel, not as a series. The focus was not on "what has been done" but "how it compares to what else has been done". In another incident, students had rapidly reached an understanding of prime factorisation by comparing, horizontally, the outcomes of six different vertical factor trees for 48 they had generated on the chalkboard. The students' observations changed from "this one has four twos" to "they are all made up of twos and threes" when given the opportunity to compare, across the grain, in a different direction to the one in which the outcomes were produced.

Bills and Rowland (1999) comment that patterns spotted by students directly from empirical evidence tend to be superficial, focusing on surface features, which may not be related to the concept. Students' responses in this classroom show that empirical, readily-spotted, pattern
highlight features of the way the examples were generated. Structural patterns emerge by looking across examples, thus illuminating relationships and characteristics within the concept. Awareness of structure appears to require reorganising one's initial approach to a concept, by reflecting from another point of view. Sometimes this can be a literal change of viewpoint, such as when integrating from the $y$ axis instead of the x -axis. Other times it might be metaphorical, such as changing representations in order to exploit the potentials offered by other symbolic forms. Dreyfus (1991) sees flexible use of representations as a characteristic of advanced mathematical thinkers. They also see possible generalities even in single examples and global similarities in locally-produced examples. It is expected that successful students will focus on structural relationships, indeed their status may be based on this tendency. My work with this class shows that others, even adolescent low achievers, can be helped to do so by appropriate teacher intervention. This study has identified a teaching strategy which is useful in this respect: prompting students to reflect, describe and use patterns "across the grain" of their work ${ }^{3}$.

The class in this study was small and one-to-one interactions were easy to manage. However, I was also using the prompts with wholeclass work, not reported here because of the difficulty of identifying individual pupils' responses. Regular monitored use of new question styles would have the cumulative effect of introducing a new feature into the common practices of the classroom (a possible influence in incidents 2 and 3 ). With larger classes such practices would take more time to establish.

## Conclusions

In the introduction I suggested that this study went against the grain of simplification by offering complexity. Within a limited situation low attainers showed that they could respond to prompts which expected complex answers and reorganisation of knowledge, although the mathematical content was elementary. I had observed, prompted, expected and achieved mathematical thinking in all the students. I had presented challenges more complex than step-by-step approaches to very low-attaining students. Further, it had been possible to find ways to engage them with mathematical structures.

[^3]A common feature of the prompts which had elicited mathematical thinking became apparent: the intention that students should reflect across the grain of their finished mathematics in order to appreciate structure other than that of the techniques used to generate the initial work. At its simplest, this could be an instruction to students to look for patterns revealed by horizontal or vertical comparisons in their work. More generally, students could compare structures and relationships between several examples, rather than comparing elements or answers. Correct performance of a mathematical task is only an early stage of a learning activity; the next is to reflect on what has been done by deliberately comparing features of the work. Incident 2 suggests that students can be helped to do this even before the task is superficially completed, perhaps by reflecting on the first two examples and generating other ways to approach the task.

The study cut across the grain of traditional teaching experiments: that is, a constructed programme properly observed and recorded, analysis of the students' response to teaching and comparison of before-and-after results. The practicalities of school limited the extent of the research, and constrained the nature and quantity of data, which could be collected. Because it took place within the complexities of a normal classroom it was similar to how teachers can research their own practice. It differed from teachers' research in that it related to an external agenda rooted in literature. The purpose and framing of the study was not specific to that school, teacher and class. Although the data collection was necessarily intermittent, analysis was done systematically within a framework devised to focus on interactions relating to mathematical thinking.

Validation of this kind of research result has to be sought through recognition by professionals of its credibility and value, which is found through usefulness and applicability in other classrooms. Generalisability arises from its acceptability to many practitioners, rather than from a linear study over time in one classroom, with all the attendant situational specifics. However, the outcome is not a practice, which might be picked up and imitated in a superficial manner, but a principle, which was revealed by systematic analysis. The principle that deep structure is to be found by looking across the grain of the work was found with particular students and a particular researcher, but is rooted in the characteristics of mathematics. The articulation of any principle creates possibility. Thus the outcome of the study is generalisable in the sense that any other teacher may recognise possibilities and apply them. The findings of this study therefore contribute to the development of
interactive strategies which enable all students to think mathematically; being explicit about some of the abilities of those who are successful in mathematics.

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[^4]reserveret højt præsterende elever. Analyse af hændelser i klasseværelset viste en styrke ved den type matematiske spørgsmål der kunne generaliseres til at "skære på tværs af åren i arbejdet". Metaforen om at arbejde imod og på tværs af åren bliver også brugt til at beskrive en særlig erkendelse af matematisk struktur, brugen af uvante metoder til interaktion samt nogle utraditionelle træk ved forskningsmetoden.

Der blev fundet, at matematikkens underliggende strukturer kunne mødes ved at se på tværs af åren. Dette resultat er fremkommet $i$ et meget lille og specifikt studium men har rødder i matematikkens strukturelle natur.

Vigtigheden af at formulere principper for undervisningen sådan som disse manifesterer sig i mindre klasserumsstudier diskuteres kort.

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## Research Interests

Her research interests include: teachers' informal assessment of their pupils (the subject of her thesis), mentoring, geometry teaching, teaching low attainers to think mathematically, social justice in mathematics education, and interaction strategies to promote mathematical thinking in classrooms.

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[^1]:    ${ }^{1}$ Acceptance of research methodologies which recognise these possibilities is a topical issue for mathematics education (Valero \& Vithal, 1998)

[^2]:    ${ }^{2}$ She did not yet know about vectors.

[^3]:    ${ }^{3}$ It is not hard to imagine versions of this approach for higher attainers: for example, a vertical sequence of horizontally-worked quadratic factorisations can be compared to see what might be the relationship between the coefficients and the factors.

[^4]:    Abstract (in Danish)
    Denne artikel refererer til et forskningsprojekt i klasseværelset hvis udgangspunkt var at finde ud af hvordan lavt presterende elever reagerer over for nogle kognitivt udfordrende spørgsmål, der sædvanligvis er

