Review

A Multidimensional Approach to learning in Mathematics and Sciences.

Gunnar Gjone

Gagatsis, A. (Ed.) (1999). A Multidimensional Approach to learning in Mathematics and Sciences. Cypros: Intercollege Press. ISBN: 960-312-063-4

The book consists of a number of articles. The articles are divided into five sections (chapters)

Chapter I: Didactics of Mathematics

Chapter II: History and Didactics of Mathematics

Chapter III: Technology in Education

Chapter IV: Psychological Foundations of Science and Mathematics Learning

Chapter V: Mathematics Evaluation – Statistical Methods

The book is part of the Socrates/Erasmus Intensive Programme, and has been published in relation to a conference in Nicosia, Cyprus in the summer of 1999.

The book contains a large number of articles. In this review one article from each chapter has been selected, and given a presentation. So the review will not give justice to the whole publication, but bring a "taste" of what is included.

Chapter I

Curriculum development as Praxis and Differentiation in Practice: The Case of Mathematics (by Athanasios Gagatsis and Mary Koutselini, both from the University of Cyprus).

They cite literature claiming the inadequacy of structural curriculum changes to change classroom practice, hence focusing on curriculum development at the micro level - the classroom. The teacher's role should be emphasized. They state that teachers "nowadays" – when planning classes – takes in a much wider account than was usual earlier, e.g. objectives. They consider questions like the following: What justification is there for choosing this topic as a teaching topic, and why ask the pupils to deal with it?

They argue that curriculum development at micro level is necessary for "good" teaching in a mixed ability class.

Section 2 of this article has the title: Curriculum development at the micro level: some examples from mathematics. The section starts with some general remarks outlining the steps for curriculum development at micro level:

First: Needs assessment

Second: Assessed needs formulated into goals and objectives

(it is not clear what kind of steps these are – if they are necessary, or just happens.)

The objectives are of two kinds, process and product oriented objectives. To distinguish between these two forms they give several examples, but it is not clear to the reviewer what is meant.

Then they go on to describe curriculum organization at macro and micro levels. They outline some choices that have to be made especially for construction of the macro curriculum. They also touch upon *didactic situations* and *action research* but do not give the connection to curriculum development explicitely.

The examples are sketchy and no explicit reference to curriculum development is given. This reviewer also found the description of curriculum development at macro level somewhat one-sided, stating that at this level the student (pupil?) are viewed as an educational product. It is also difficult to see how the curriculum development at micro level differs from what most mathematics educators would call planning. The author's concept of *action research* is also somewhat special:

One student writes on the blackboard 6: ½ = 3

The professor immediately starts an action research with the students who have given the same results. He proposes two series of execises:
...
(a) 6: ½, ...
(b) 6:2, ...
After comparison of the results and a long discussion all the students agree on the cause of the wrong answer.

In the third section it is argued that curriculum development at micro level cannot be realized without differentiated objectives, activities etc.

They claim that there is a growing body of literature on differentiation, but most of the literature referred to is from the beginning of the 1990s. They argue for a differentiated approach, and give a number of references to differences in cognitive styles (e.g. "inchworm" style, "grasshopper" style) and give examples illustrating different cognitive styles, then called "local" and "holistic" styles. They then present different forms of differentiation, without going into detail. They also sketch some categories of learning difficulties in mathematics.

In their conclusion they state that "curriculum development at micro level offers new procedures for answering the old questions on what and how to teach mixed ability classes." For this reviewer it is difficult to see that they offer something radically new more traditional actions e.g. that teachers should participate in the decision making procedure. The article is argumentative and presents a number of claims, some of which are not made plausible.

Chapter II

The design, implementation and evaluation of a preservice teacher program based on the history of mathematics. (By George Philippou and Constantinos Christou, both from the University of Cyprus)

They discuss first the teacher's role with new curricula. They argue that prospective teachers should learn mathematics as well as about mathematics. They briefly discuss the notion of content knowledge and pedagogical knowledge, claiming that pedagogical knowledge might be considered a part of content knowledge, including examples, ideas etc. As a background to their study, they also give a list of myths and misconceptions, as specified by Shoenfeld, that students should overcome – e.g. "Mathematics is a solitary activity, done by individuals in isolation". The main part of the article is "to report on a preservice program, based on the history of mathematics, with the aim to help students "clean" their minds of such myths" and misconceptionsn, and develop a positive view about mathematics and its teaching and learning".

They discuss briefly teacher's beliefs, giving a number of references, as well as the relationship between the history of mathematics and mathematics teaching and learning. They give some of the "usual" arguments about the role of history, e.g. that the study of history can draw attention to possible learning difficulties, provide problems etc.

Content part of the program

The program was built around the historical evolution of fundamental mathematical concepts and methods, based on selected works. They make a number of claims in the presentation, which for this reviewer is not obvious, e.g.

> *"Following the footprints of some big names can free students of misconceptions, fears, and negative attitudes towards mathematics"* - ?

The program consisted of a content course and one method course, at the University of Aegean, with one more content course being developed at the University of Cyprus. Basis for the program was Greek mathematics, with major focus on Euclid and Archimedes. Followed by an overview of the development of mathematics through the history, ending with set theory and Boolean Algebra. From the description it seems that they relate the presentation to early Greek mathematics whenever posssible (In calculus they introduce and discuss Zeno's paradox).

Method parts

The first part focuses on aims and objectives of mathematics education, learning theories etc. The second part relates recent ideas for teaching to historical topics.

The main part of the paper concerns program evaluation. The evaluation is mainly quantitative. They shortly describe the statistical tools used in investigating prospective teachers' attitudes and mathematics teaching efficacy. In the discussion they present the data and discuss the findings, e.g. "that students in general were negative about spending more time in school working with mathematics.

The article at this point is quite technical. Also the role of the different universities involved is discussed. This part is somewhat difficult to follow if one is not too familiar with the different universities.

In the last part of the analysis teachers who had graduated with different programs were interviewed. The authors report significant differences in how the various groups reacted. The authors conclude that the mathematics preservice program was effective in changing teachers' attitudes. They conclude the presentation by outlining some further research questions, e.g. the permanence of the changes observed.

This article gives a strong argument for the introduction of historical topics in mathematics education, which is now the trend in several countries. I miss, however, details, on the content and methods introduced in the program – the information are too sketchy for designing a similar program. It would be very interesting to learn more about the program, so a closer description of the courses would be interesting to see.

Chapter III

Factors of integration of dynamic geometry software in the teaching of mathematics. (By Colette Laborde, University of Grenoble)

The article concerns the use of Cabri geometry, in particular about students solving mathematical tasks in a Cabri –geometry environment using the TI (Texas Instruments) – 92 (which is an "enlarged" graphing calculator). The important factor was that the students had Cabri in a hand held device, to carry with them. They could hence decide when to use the program, and could get homework. The article then goes on to discuss some important questions concerning the use and integration of technology in the teaching and learning of mathematics. Referring to geometry software some elements is presented:

- dynamic geometry software (DGS) makes it possible for students to visualize and manipulate geometrical objects
- students can experience proofs
- DGS can be used for creating intriguing phenomena that are not expected by the students, and hence need some form of explanation. Also predication activities – "what happens if ...?"

In the second part of the article the question of what integration means is discussed. For instance, objects might have been introduced in the usual way (paper and pencil environment) and then the students can get "hands on" experiences with the software.

It is stated, that from the authors' experience, integration takes a long time. Integration is looked upon from the Piagetian metaphor of equilibrium. Following Piaget, three types of reactions to cognitive pertuberations are discussed:

- reaction alpha: ignoring the pertuberation
- reaction beta: integrating the pertuberation into the system by means of partial changes
- reaction gamma: the pertuberation is overcome and loses its pertubing character.

Real integration is then linked to the gamma reaction.

In the section called "The role of the computer environment" it is stated that that the context helps shape the problem itself and the students' problem solving strategies. The example studied is to find the sum of the angles in a convex quadrilateral. The computer is used as a means to study various quadrilaterals. The study of specific quadrilaterals is taken as a starting point for approaching the general property. Some formulations in this section the reviewer found quite interesting (!):

- technology without curriculum is only worth the silicon it is written on
- curriculum with inadequate use of technology is only worth the text processor it is written with

Two possible cases as a consequence of computer use in geometry is outlined:

> *Case of constructivist hypothesis where the teacher withdraws to a large extent, the other is that teachers might view geometry as intrinsically linked to paper and pencil, and hence the teacher will not rely on the use of computers in geometry, and not see paper and pencil as a context.*

A major part of the paper concerns the discussions of geometrical tasks done by using the Cabri program. In this connection several interesting observations are discussed, e.g. the tension between the mathematical and the instrumental – does the student fail because of mathematics or of the technical aspects? This is an important question to discuss concerning all use of advanced technology in mathematics education. The article ends on a note on time management. It is reported – which is no surprise – that the use of computers is "time greedy". However, use of the technology as a personal device, which could then be used for homework, greatly improved the situation.

The article is interesting in that it gives several points for discussing the technology in mathematics. It can be seen as a strong case for a personal technology, e.g. the TI-92 which for this reviewer has a version of Carbri that is quite crude.

Chapter IV

Comprehension and learning from scientific text. (By Irene-Anna Diakidoy, University of Cyprus)

Comprehension and learning from scientific text by Irene-Anna N. Diakidov, focuses on an interesting area in education. The article starts with a historical introduction. It is observed that most of the learning in the higher grades in schools is based on text designed to support and accompany teacher-led instruction. And it is claimed that the 1970s and 1980s are the golden decades of reading comprehension research. The article gives a number of valuable references to the works in this period.

Discussing *Comprehension Processes and Products* highlight the point that

> the product of comprehension – the mental representation – cannot be conceptualized as a simple list of propositions, just as text cannot be conceptualized as a simple list of sentences. Instead, on the basis of textual cues and background knowledge, the reader constructs an organized

> representational structure that reflects a local level of coherence intended by the author or imposed by the reader. This textbase representation, as it is commonly called, is the product of syntactic, semantic and referential processing that has taken place. *(p. 398)*

Further on in the article "story comprehension" and its relation to text comprehension is discussed. The concept of *text model* is introduced as something that integrates text content with existing knowledge structures.

In the section "Comprehension and learning" the distinction between text model and textbase is elaborated and related to theories of learning. In the text a number of references are quoted, and this gives a good background for the issues discussed.

"Learning from scientific text" is the main part of the article. Science learning is related to physics, biology and chemistry, but some of the issues applies to the social sciences as well. Given the nature of the present status of mathematics, it is no too difficult to see the relevance to mathematics. As an example consider the following quote: *reasoning skill and, in particular the ability to coordinate theory with evidence is central in the development of an understanding of science* (read *mathematics*). Even if "evidence" has a different meaning in mathematics and science, it is also possible to consider evidence in mathematics. This has been more evident with the introduction of computers. We can to some extent say that the computer shows us "evidence". A number of cases concerning the logic of a written text are discussed, ending with the following:

> It can be predicted, then, that views of comprehension and learning from text as committing text information to memory will lead to a different set of teaching and learning strategies than views of the same constructs as thinking about text and existing prior knowledge. *(p. 404)*

The claim "the interface between science learning and learning from text remains, to a great extent, an uncharted territory" – is even more true for mathematics. It is also stated that neither textbooks nor teaching practice have kept track with the advances of reading comprehension research.

Learning from texts is an important area, and should be subject to further research, especially in mathematics. This is especially important when we consider the central role of texts in communicating mathematics. On the other hand we might say that there is a development in school mathematics to put less weight on the written texts – much of the modern(!) mathematics teaching and learning is centered around student activity – experimentation and exploration. Hence we might "conclude" as is done in the article:

> From this perspective, then, moving away from text translates into not taking advantage of a medium that is well suited for the instruction and the sharpening of those cognitive skills that render subject-matter learning possible. *(p. 405).*

The article is a good introduction to the field and presents many interesting perspectives on the uses of text in education.

Chapter V

The comprehensive model of educational effectiveness tested by an evaluative research in primary mathematics: implications for research on school effectiveness. (By A. Gagatsis, University of Cyprus, L. Kyriakides, University of Warwick, UK and A. Savva, University of Cyprus)

The article starts by giving background to what is meant by school effectiveness research. Four difficulties with research are presented:

1. the results of school effectiveness studies are heavily dependent of the choice of outcome measures used.
2. test development remains a considerable underdeveloped science.
3. the quality of instruction and schooling particularly in western countries are not independent of the socio-economic status of parents
4. only until the end of 1980s statisticians developed reasonably good methods for analysing hierarchical data derived from studies investigating the school effect (p. 470-471)

Carrolls model of school effectiveness is presented and the extension by Creemer is introduced. The Carroll model states that the degree of mastery is a function of the ratio of amount of time students actually spend on learning tasks to the total amount of time they need. Creemer extended this model in 1994 by adding the concept of opportunity to learn. Creemer introduced more than 30 variables, which might enhance time, spent learning and opportunity used to learn on the part of the pupil. For this study a set of variables was selected, categorised as: context, time, opportunity and quality.

The researchers examined if pupil, classroom and school variables show the expected effect on pupil achievement. This was done by written tests. Thirtytwo Cypriot schools were selected, which were divided into four groups according to rural or urban, small or big.

A fairly detailed description of the variables used is then presented in the article. Such variables were e.g. context, motivation, time factors etc. A number of findings are presented. We give a few of the findings:

- variables associated with the quality of teaching did not have statistically significant effect at classroom level
- the fact that classrooms (and probably teachers) have unique effects on pupil learning can be attributed to the fact that primary teachers have a substantial degree of freedom in their professional practice.

However, they do admit that some of the variables they use are difficult to quantify.

Conclusion

The book covers a wide variety of topics and gives an interesting collection of research topics especially in Greek mathematics education. The articles vary considerable in scope and quality, but as a whole the book leaves a very positive impression.