What may be neglected by an application-centred approach to mathematics education?

A personal view

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It seems that an ongoing discussion of why modelling and applications should be part of mathematics education has mostly centred around utilitarian argument. In my opinion, such an attitude may produce an unbalanced view on the reasons for including modelling and applications in mathematics education, which, in turn, may result in a neglect of the following important issues of mathematical modelling:

- *the complexity of the modelling process;*
- *the distinction between the perspective of model builders and that of model users*
- *the necessity of a demanding interplay among students' cognitive, metacognitive and affective domains in the modelling process.*

The article briefly presents the application-centred approach to mathematics education, examines these neglected issues, and suggests how the teaching of modelling and applications may thus be utilised.

The application-centred approach

Although not all supporters of the application-centred approach to mathematics education share the same perspective, it is my belief that most of them assume that "the ultimate reason for teaching mathematics to all students, at all educational levels, is that mathematics is useful in practical and scientific enterprises in society" (Carss quoted in Ernest, 1991, p.161). In other words, proponents of this approach primarily insist on having mathematics education of non-specialists relevant to their future jobs at all educational levels (see Burghes, 1989). It is clear that such application-centred mathematical educators assume that the educational value of mathematics is mostly achieved through the utility of the mastered topics.

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Wider international activities regarding applied mathematics and mathematical modelling started about twenty years ago. Research has been undertaken in some twenty countries including Australia, Denmark, Germany, the UK and the USA. Researchers have not only examined pragmatic aspects of the approach, but also studied theoretical and philosophical issues. Since 1983 eight biennial International Conferences on the Teaching of Mathematical Modelling and Application (ICTMA) have been arranged. Recent summaries of research findings may be found in Sloyer, Blum & Huntley (1995), Houston *et al.* (1997) and Galbraith *et al.* (1998a). In nearly twenty years, a great number of units supporting the practice of teaching have been developed (see, for example, Burghes, Huntley & McDonald, 1982; Hobbs, 1989; Swetz & Hartzler, 1991; Skovsmose, 1994; Berry & Houston, 1995; and Edusoft, 1996). A reader interested in such units may also examine some articles published an application-centred journal such as "Teaching Mathematics and its Application" (http://www.oup.co.uk/teamat/contents/). Despite encountering difficulties regarding curriculum design, lesson preparation and realization, and assessment implementation (Blum & Niss, 1989; Niss, 1992), the application-centred mathematicians have not only made great efforts to improve problem solving in a real context, but have also received very encouraging feedback from the teaching-learning process (see, for example, Clatworthy & Galbraith, 1991; and Bassanezi, 1994).

It seems that modelling has indeed confirmed itself as a valuable teaching-learning strategy (Bassanezi, 1994). However, my recent study (Kadijevich, 1993) revealed three important issues of the application-centred approach that have been to a certain extent neglected in the teaching of modelling. These issues underline that professional modelling is a complex process involving two kind of actors and that building a model in school is based upon a demanding interaction of the three personal domains. The issues are highly relevant to both teaching modelling and learning through modelling as they reflect the genuine nature of the modelling process that should be realized in a right way by both teachers and their students. Such a realization is still needed since a great number of research studies have underlined that beliefs about mathematics and its learning and teaching strongly influence the mathematical education outcomes (see, for example, Grouws, 1992).

Let us examine the neglected issues in more detail, mostly having in mind cases where a direct and obvious one-to-one correspondence between model and object (reality) being modelled is not present.

THREE NEGLECTED ISSUES OF THE APPROACH

Modelling is a complex process

Professional modelling is a complex process dealing with objects, properties and relations that do develop and change over time as our understanding of the underlying problem progresses. Despite this, it seems that many textbooks offer examples that present modelling statically in the following way: "Supposing this and that, we obtain ... By solving this, we get ... etc.". It is true that some useful remarks regarding an improved model are usually given, but several gradually developed models regarding the same problem are not frequently presented. Let us take an example regarding the break-even analysis (Burghes, Huntley & McDonald, 1982)., recalling that the break-even point informs us about the number of manufactured and sold units for which the total costs of production are equal to the income from the sales. After a relevant real-world problem has been introduced, what is what in terms of variables is listed, and a simple, yet suitable model is then set up and solved. The applied model is then nicely justified by a cutting from *Sunday Times Business News* of July 1975, and the following remark is given: "If the break-even point proves to be unrealistic, then a nonlinear model could be tried or our simplifying assumptions about cashflow amended." (p.56). Readers have probably met similar examples, which, despite skilfully developed content, do not essentially present modelling as a process. It is true that many recent textbooks and research papers contain examples that give high priority to presenting the idea of improving models successively. Such an example can be found in Graham (1991) who describes the modelling process regarding the design of a carton holding 1 litre of juice: an initial solution in the form of a cube has been improved to be that of a rectangular solid in order to keep the cost of the used material at a minimum as well as fit on the door shelf of a fridge. However, my reading suggests that, in general, this process-based approach has not been widely applied in both the literature on and teaching of mathematical modelling.

Professional modelling also often deals with incomplete data and/ or conditions, the treatment of which may require more care and precision than dealing with exact elements (Alsina & Trillas, 1991). This is particularly true for computer based modelling. According to Kovács (1993), it has become evident that such a modelling should not only be based on mathematical programming (optimization/ operations research approach), especially in large scale applications.

This is because the developed program usually cannot cope with a dynamically changing environment and/or handle cases when some of the required data is not available. This shortcoming generates difficulties with modifying the model and/or the view of the applied problem solving, and usually results in an unreasonably long developmental time. To alleviate this, mathematical programming techniques may be combined with the logic programming approach**#1** in the following way: the utilized methods and procedures are written in an imperative programming language such as Pascal or C, whereas the logical structure of the applied model is expressed in logic and implemented in Prolog. This combined approach, which seems suitable for above-average students at the upper secondary (grades 11 and 12) and tertiary level of mathematical education, can easily be utilized in, for example, Amzi! Prolog (http://www.amzi.com) as, among other things, it can efficiently integrate routines written in C and Prolog. The approach, which does allow incremental model development promoting the "process" nature of the modelling activity, can be applied to a number of modelling tasks such as those examined in Burghes, Huntley & McDonald (1982). However, a recent Internet search reveals no evidence that this valuable approach has been widely utilized in the teaching of modelling, especially at the upper secondary level.

Modelling involves two kind of actors

Modelling basically involves two kind of actors: model builders who are building a model and model users who have ordered the model and are applying (or will apply) it.

The fact that model builders and model users may think differently is usually neglected. According to Simon (1981), there are important differences between the natural sciences and the artificial sciences (engineering, medicine, architecture, instruction and the like) regarding the qualities of created objects (things). Do these differences originate from different modes of thinking applied within each of the two scientific camps? Most probably. Lawson (1980) found that natural scientists and architects do think differently. In a design-like problem, requiring the arrangement of coloured blocks onto a rectangular pattern showing as much red or blue as possible, natural scientists tried to discover general patterns, whereas architects were concerned with desired solutions. This suggests that in general natural scientists may primarily be concerned with general solutions, whereas design scientists may mostly experiment with particular solutions. This difference may be called the general/particular solution distinction.**#2**

Another frequently neglected fact is that the process of modelling is shaped by the intentions of model builders and model users, which may not always be in agreement (Skovsmose, 1989). Indeed, model builder may be building a model with certain intentions in mind such as the application of a particular theoretical framework assuming this and that. Model users may be using the developed model without such intention, since what has been initially assumed by model builders may be partly or fully absent. For example, consider a developed model assuming normal data distribution, e.g., *t*-test for independent samples, that is used in cases when such a distribution is not present. A disagreement may also occur while a model is being built. For example, model builders are building a model mostly aiming at production cost minimization by applying some previously acquired techniques that may ignore some established societal values. Model users, on the other hand, want to apply the model to find out the production cost as well as potential profit, possible pollution, the number of workers who may be fired, etc. although they have not clearly spelled out all these intentions. Clearly, all intentions should be made explicit since what goes without saying for model builders may not go for model users, and vice versa, most probably because of their different modes of thinking and systems of values.

It should not be forgotten that modelling also involves model victims - those who are exposed to the consequences of the applied model. Although the modelling process is primarily shaped by the intentions of model builders and model users, these intentions should not neglect the interests of model victims, especially in a truly democratized society.

Building a model in school is based upon a demanding interaction of the three personal domains

According to Schoenfeld (1992), mathematical problem solving may be effectively analyzed by using the following five categories: (1) the knowledge base; (2) problem-solving strategies (heuristics); (3) self-regulation, or monitoring and control; (4) beliefs and affect; and (5) classroom practice. His study also highlights that problem solving performance is based upon a demanding interplay among student's cognitive, metacognitive and affective domains dealing with these categories. Despite this, it is generally overlooked that modelling as a specific instance of problem solving is also influenced by such an interplay, probably in a more demanding way than in pure mathematical problem solving.

Does the last statement really hold true? Yes, it does. Let us support it with some evidence.

Modelling is indeed based upon a number of cognitive and metacognitive questions such as: "Why do we build our model this way?", "How can we come up with this or any similar model?", "Which assumptions do I make and how do I start modelling?", "Are there some modelling heuristics like Polya's (1990)?", and "How does the model fit reality?". It is true that the modelling process has been adequately described by using, for example, the following 3-stage framework: (1)*formulation* (formulate the problem; make assumption in the model (set up a model); and formulate the mathematical problem); (2) *solution* (solve the mathematical problem); and (3) *application* (interpret the obtained solution; validate the model; and use the model to explain, predict, decide or design (Burghes & Wood, 1984). However, researchers have not focused on the student's transition from one modelling step to another, especially from the "formulate the problem" step to the "make assumption in the model" step (cf. Burghes, Huntley & McDonald, 1982; pp. 152-156). Furthermore, except for Dunthorne & Le Masurier (1993), researchers have not concentrated on developing instruction that help students to begin modelling by making simple assumptions.

According to Lambert *et al.* (1989), modelling is still influenced by the student's belief systems regarding the mathematical domain and the problem domain. This is because these systems determine a context (positive, negative or neutral) within which his/her cognitive and metacognitive activities are utilized. Despite this, affective issues regarding modelling have not been thoughtfully studied except for Galbraith *et al.* (1998b). It is quite surprising since research on affect in mathematics education (McLeod, 1992) has firmly evidenced the relevance of this domain to problem solving activities**#3.** It seems that in general many researchers in mathematics education tend to follow their narrow agendas, not taking into account what others are doing (Silver, 1988). Undoubtedly, such an attitude, which is particularly negative for this issue compared to the other examined issues, is unlikely to advance our complex research field in any significant way.

CLOSING REMARKS

We have presented the main features of the application-centred approach to mathematics education according to some relevant research studies undertaken around the world. It is clear that the teaching of modelling and applications that respects the neglected issues should be, for example, based upon the regular utilization of the following didactic suggestions:

Examine not only mathematical knowledge, but also the problem domain knowledge. Help students begin modelling my making simple assumptions through individual or team work. Help them move from one modelling step to another. Encourage computer-based modelling in the proposed way.

Present modelling dynamically, by examining at least two models regarding the same problem. Help students realize that: (a) modelling is an open-ended activity through which we simultaneously acquire knowledge about the problem, its model, the applied/required mathematical concepts and techniques, and the model validity; and (b) the developed model is first and foremost based upon a subjective interpretation of the examined reality, the content and subtlety of the interpretation may, and often does, differ from person to person.

Encourage students' metacognitive activities and help them develop their metacognitive abilities. Uncover students' beliefs and try to make them more positive (if need be), especially those relating to self-competence and the value of mathematics in modelling the reality.

Encourage negotiations between model builders and model users (e.g., two groups of students from the same class). Help them externalize, to some extent, their modes of thinking and their intentions. Encourage them to evaluate the applied models from the perspective of each of the modelling actors.

However, these suggestions may not be suitable for many students, even at the tertiary level of mathematics education. Should we then abandon modelling and applications in mathematics curriculum? It is my opinion that this should certainly not be don at all. Some students (perhaps those with above-average mathematical ability) may still be taught by applying the listed suggestions, which should promote the examined nature of modelling. Other students, who cannot successfully apply their knowledge through complex modelling activities, may solve simpler modelling tasks, like the following two types examined in my recent study (Kadijevich, 1998):

• TYPE 1: Find out why the presented ideas work by pointing out to the relevant items of underlying knowledge. For example,

How is a rectangular foundation dug? A rope marks the foundation, and if the diagonals of the rectangle are (almost) equal, its digging is undertaken. Otherwise, the rope stretching and the equality checking are repeated. Which item of knowledge validates this procedure? (In case of complex ideas, the request for disclosing and justifying the underlying knowledge may be omitted.)

• TYPE 2: Come up with suitable applications of some items of mathematical knowledge. For example:

It is known that a transversal intersects two parallel lines so that the alternate-interior angles are equal. Utilise this item of knowledge in order to make an optical instrument.
(The area of application may or may not be given.)

Although through solving such tasks students will not realise the examined nature of modelling, it is certain that mathematical knowledge will become alive for them and that they will begin to perceive mathematics as a human enterprise, which improves our lives. By using this approach respecting students' ability, both kinds of student will be in a position to realise the true nature of the relevant subject (the modelling process or the mathematical knowledge utility), which, in turn, should improve their learning abilities.

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NOTES

#1 Logic programming is a theory, a programming paradigm and a methodology that fully supports declarative programming, incremental system development and representation of procedural and declarative (conceptual) knowledge. (See Sterling & Shapiro, 1986). These features of logic programming enable solving incomplete problems, the descriptions of which develop over time as our understanding progresses. A computer language reflecting and implementing these ideas is called Prolog.

#2 My PhD study on mathematical problem solving dealing with problems on motion revealed the following fact: while students inclined towards studying sciences basically solved meeting, overtaking and timetable problems through setting up and solving equations, students inclined towards studying humanities preferred arithmetical ways of solving them. The study used a sample of 36 ninth-grade Gymnasium (high school) students whose mathematical and non-verbal intellectual abilities were mostly above average. The students were previously familiarised with various ways of solving such problems.

#3 Our recent study relating to pure mathematical problem solving (Opachich & Kadijevich, 1997) evidenced that the student's math-self is an important predictor of his/her success in learning mathematics, which may be a better predictor of this success than non-verbal intelligence (the correlation between the math-self and non-verbal IQ was not significant). This study, which developed a psychometrically valid scale of mathematical self-concept in the Serbian language by using a sample of 123 ninth-grade Gymnasium (high-school) students, assumed that mathematical self-concept represents an organized system of beliefs about mathematics supplemented by behavioural and emotional reactions regarding the value of mathematics and mathematical way of thinking as well as confidence in and motives for learning mathematics.

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Abstract (in Norwegian)

Denne artikkelen tar utgangspunkt i påstanden om at det meste av den diskusjonen som har foregått om hvorfor modellering og anvendelser bør være en del av matematikkundervisningen for det meste har vært sentrert rundt argumenter knyttet til nytteaspektet. I denne artikkelen blir det ført fram argumenterer som peker på at et slikt standpunkt kan føre til en skjev oppfatning av hensikten med å inkludere modellering og anvendelser i matematikkundervisningen. Denne skjeve oppfatningen vil kunne få som resultat at en i undervisningen forsømmer å legge vekt på andre viktige sider ved modellering som for eksempel:

- kompleksiteten i modelleringsprosessen
- forskjellen mellom de perspektiver som modellbyggerne og modellbrukerne vil kunne ha
- nødvendigheten av et krevende samspill mellom elevenes/ studentenes kognitive, metakognitive og affektive domener i modelleringsprosessen.

Artikkelen presenterer kort den anvendelses-sentrerte tilnærmingen til undervisning i matematikk. Deretter gjennomgår den de viktige sidene som blir forsømt og foreslår hvordan disse sidene kan brukes i undervisningen av modellering og anvendelser.

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Research interests

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