Computer support for diagnostic teaching. The case of decimal numbers.

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Computers with different kinds of software were used in mathematics teaching during one school year with Norwegian students of age 10 - 14. The development of students' understanding of decimal numbers was investigated, using computers to support a diagnostic teaching approach. In particular some spreadsheet tasks were used to stimulate mathematical investigations and to generate discussions exhibiting conflict. An item analysis of pre-test data revealed a common pattern of misconceptions on decimal numbers. As the students worked on the spreadsheet tasks, their concepts seemingly were in conflict with what they observed. This led the students into lively discussions. The test results indicated that the computer group improved significantly by comparison with the control group on part of the test, with the greatest improvement from the 'high' spreadsheet users in the research group.

Introduction

The use of computers in mathematics classrooms, have been studied within different settings and teaching approaches. There are claims that the computer has a huge potential for improving students' performance and understanding of mathematics (Bennett, 1991; Watson, 1993; NCET, 1996; Phillips, Pead, & Gillespie, 1995). There is a broad range of ways computers have been used in the classes, including drill and practice, tutorials, problem solving tasks and open ended tasks using the computer as a tool.

Looking at computer software we may be concerned about what view of learning or what kind of learning theories lie behind the way the software presents the tasks to the students. Perhaps, views of learning have not been consciously considered in the software development. Many small programs designed to teach number skills, can be classified as drill and practice software. The behaviourist thinking of stimulus and response seems to have a lot of unconscious

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support in this kind of software (Bigge & Shermis, 1992). From the amount of drill and practice software available in Norway, it seems that this kind of computer use is widespread. I will argue that it is possible to use computers to support a constructivist approach to teaching and learning. According to this view of learning, the students construct their own knowledge. The teaching approach can acknowledge this. According to this view, the classroom should provide a learning environment; offering experiences richly endowed with the concepts and properties to be studied. The students should be challenged by the contradictions of their misconceptions and provoked into further discussions (Davis, Maher & Noddings, 1990). Diagnostic teaching is regarded as a practical implementation of a constructivist view of learning (Bell, 1993a; Bell, Swan, Onslow, Pratt & Purdy, 1985) and has provided a framework for this research. The aim of the research project was to study the development of students' understanding and performance with decimal numbers using the computer as a support for diagnostic teaching.

I will suggest that a spreadsheet can provide an appropriately rich environment for investigations, experiments and problem solving activities. Although different kinds of software were used in the research, the examples given in this paper will be of students using spreadsheet tasks involving decimal numbers.

Previous research

Students' understanding and misconceptions of decimal numbers have been investigated in some large scale projects in England (Foxman et al., 1984; Brown, 1981), and further studies have been done to study in detail students' incorrect patterns of thinking (Bell, Fischbein, & Greer, 1984; Bell, Greer, Grimson, & Mangan, 1989; Sackur-Grisvard & Lèonard, 1985). The students often make mistakes by generalising too much from the rules they know for whole numbers (Resnick, Leonard, Omanson, & Peled, 1989). Similar results have been reported in other recent Norwegian research (Brekke, 1996a; Brekke, 1996b). The students need more experience of exploring and discovering for themselves the new properties of decimal numbers, to accommodate and extend their concept of number (Bell, Greer, Grimson, & Mangan, 1989).

Studies comparing diagnostic teaching with a traditional 'positive only' teaching approach have shown that the students seem to benefit from the discussion in which conflicts are faced. In positive only teaching the students are only given positive responses, trying to avoid misconceptions (Bell, 1993a, 1993b). In particular, the diagnostic teaching approach was also used in a study utilising calculator-based material for teaching decimal numbers (Swan, 1983a).

The use of spreadsheets in the teaching of mathematics has also been explored in other contexts (Healy & Sutherland, 1991; Bissell, 1995; Breiteig & Fuglestad, 1997; Green, Armstrong & Bridges, 1993). But so far, only a few experimental research studies have been reported, e.g. on the understanding of elementary algebra (Rojano & Sutherland, 1991; Rojano, 1993; Beare, 1993).

Research methodology

In this study, computers were used regularly through the school year 1994/95, in different topics in mathematics and with a variety of software. In this way the students became used to computers as a natural part of the mathematics classroom, not just in connection to the particular topic of the research. The class teacher was responsible for the teaching and organisation of the work in class, including the use of computers. Most of the 15 teachers taking part in the research, had no experience of using computers in their classes before. The teachers were given an introductory course lasting three days before the start of the experiment. Different kinds of software were used. Some gave drill and practice in numbers; some provided games for problem solving, and there were tasks to solve on a spreadsheet.

From the outset there were two sets of control classes. One set did not use computers at all. Another used computers, but had no information about the software and worksheets prepared for the research classes. The purpose of this was to monitor the effect of the computer use in the class. The research was performed in Norwegian schools, at year levels 5, 6 and 7, that are students of age 10 - 14. The sample consisted of 242 students in research classes, 297 in non-computer control classes and 97 in computer-control classes.

All classes were given the same pre-test at the beginning of the school year, the same post-test near the end of the school year, and the same delayed post-test after the summer vacation. Some of the students' work at the computers was observed, and information was also collected by questionnaires and by interviewing students and teachers. This article reports on results from the pre-test, some observations from the classroom on students work using spreadsheet tasks, and a review of the results from the post-tests of the study.

The problem of decimal numbers. Results from the pre-test.

The aim of the test, administered as pre-, post-, and delayed post-test, was partly to provide an instrument for measuring the improvement in the students' performance and partly to give information and reveal possible misconceptions in the students' understanding of decimal numbers. For this purpose the test, consisting of 74 single test items, was designed as a diagnostic test, partly using test items from other research in this area (Hart, Brown, Kerslake, Kuchemann & Ruddock, 1985; Swan, 1983b). It covered a broad area of decimal numbers, e.g. ordering, density of numbers, and simple calculations both with pure number work and in context.[1]

The students revealed a broad lack of understanding in the test, with mean scores of 26%, 39% and 51% correct answers in the year levels 5, 6 and 7 respectively. The most difficult area of the test seemed to be the use of multiplication and division. A closer look at some of the test items reveals typical mistakes.

5. a Draw a ring around the biggest of the three numbers:

0,62 0,236 0,4

b How do you know it is the biggest? ....

A common mistake was to give 0,236 as bigger than both 0,62 and 0,4 - explaining this as having *most digits behind the comma.* The answer 0,4 and an explanation of the kind *most decimal places giving the least number*, was less common than found in other research (Swan, 1983b).

Task 5a	Year 5	Year 6	Year 7
Correct 0,62	31	50	77
0,236	60	42	19
0,4	7	6	4
No answer	3	2	1

Table 1: *Answers to Task 5a. Frequences given in percentages*

[1] In what follows I use the Norwegian convention of using a decimal comma as decimal point, a dot for multiplication and a colon for division.

Task 5b			
Correct	20	33	60
Most digits behind comma	27	22	12
Most decimal places. Least number	1	0	1
Other/No answer	52	45	28

Table 2: *Answers to Task 5b. Frequences given in percentages*

A similar pattern occurred in task 6 where the students were asked to order four given decimal numbers in increasing order.

6. Write the numbers in order, the smallest first:

a	0,3	0,7	0,6	0,1	Answer: ....
b	0,07	0,23	0,1	1,01	Answer: ....
c	0,62	0,25	0,5	0,375	Answer: ....

More than 90% solved 6a correctly, but table 3 reveals difficulties in ordering numbers with different lengths of the decimal place.

Task 6c	Year 5	Year 6	Year 7
Correct: 0,25 0,375 0,5 0,62	20	35	62
0,5 0,25 0,62 0,375	56	41	21
Others/No answer	23	24	16

Table 3: *Answers to Task 6c: Ordering numbers*

Results from Task 5 and Task 6 in the pre-test revealed that students often think of the digits behind the comma as a separate number and compare without taking account of the decimal comma.

Cross tabulating 5a by 6c reveales a consistent pattern in the students' thinking. A total of 37% of the students gave correct answer to the tasks, and 34% gave the corresponding wrong answers of 0,236 to 5a and 0,5 0,25 0,62 and 0,375 to 6c. Among students who answered 5a correctly, 82% also gave the corresponding wrong answer to 6c.

Students' explanations on these tasks confirmed their pattern of thinking. In the cases where students gave the correct answer 0,62 to Task 5b, 69% also gave a correct explanation. Of those students who answered Task 5a correctly, gave 49% a corresponding explanation categorised as *'most digits behind the comma'*.

A few test items investigated the understanding of the infiniteness and density of decimal numbers.

7. Write **none** if you think there is no answer to this task.
   Write a number that is:

   a bigger than 3,9 but smaller than 4: ....

   d bigger than 0,63 but smaller than 0,64: ....

   e How many different numbers can
   you write that lie between 0,63 and 0,64? ....

Most students could not give any numbers between 3,9 and 4 or between 0,63 and 0,64 and answered that there are none, or a finite number like 8 or 10 numbers between.

Task 7d	Year 5	Year 6	Year 7
Correct	16	28	45
None	45	49	34
Other/No answer	40	23	21
Task 7e			
Correct: infinite	4	7	20
None	42	37	32
One number	9	15	8
Definite number, e.g. 8 or 10	16	21	22
Others/No answer	30	20	18

Table 4: *Answers to Task 7d and 7e. Frequencies given in percentages*

Task 10 investigates understanding of closeness.

10.
a Ring the number nearest in size to 0,16

0,1 0,2 15 0,21 10

b Ring the number nearest in size to 2,08

209 2,9 2,05 2,1 20,9

Table 5 shows the most common wrong answers.

Task 10a	Year5	Year 6	Year 7
Correct: 0,2	16	37	38
0,21	65	54	53
15	12	5	3
Others/No answer	7	5	6
Task 10b			
Correct: 2,1	7	20	32
2,9	16	6	11
2,05	67	69	52
Others/No answer	12	5	6

Table 5: *Answers to task 10. Frequences given in percentages*

The understanding of place value and meaning of neighbouring digits is crucial for answering these tasks correctly. The students giving the typical wrong answer, seem to think that the closest number should have the same number of decimal places as the one given.

In many cases the mistakes can be explained by the students just ignoring the decimal comma, or looking at the decimal number as consisting of two separate numbers, one before and one after the decimal comma. This was also revealed in extending number sequences in Task 11 and in tasks of simple adding and subtraction of decimal numbers.

11. Write the next two numbers in each sequence:

a	0,3	0,6	0,9	....
b	0,92	0,94	0,96	....
c	1,13	1,12	1,11	....

At first the results from the three items in Task 11 were surprising. Task 11a appeared to be more difficult than the two next. 35%, 36% and 40% of the students gave the most typical wrong answer of 0,12 and 0,15 as the next two numbers, and only 17%, 34% and 36% answered correctly in the years five, six and seven respectively. Task 11a looks easy and the next two more difficult. A possible explanation could be that the students can easily count in threes, giving 3, 6, 9, 12, 15 and so on. Therefore the result here by using similar routine becomes 0,12 and 0,15 for the next two numbers. Some of the incorrect responses to 11a could therefore be viewed as a slip rather than a misconception.

Task 11a	Year5	Year 6	Year7
Correct; 1,2 and 1,5	17	34	36
0,12 and 0,15	35	36	40
Others/Noanswe	49	30	24
Task 11b			
Correct: 0,98 and 1,00	45	63	72
0,98 and 0,100	8	9	7
Others/No answer	47	28	21
Task 11c			
Correct: 1,1 and 1,09	31	50	48
1,10 and 1,9	25	22	27
Others/No answer	44	28	25

Table 6: *Answers to Task 11 a-c. Frequences given, in percentages*

Looking at the next two items, the corresponding routine for counting is weaker; the students seemingly had to concentrate more. This explanation was also confirmed later by looking at students working on this kind of task in short interviews. When prompted to have a closer look at the task, the students in most cases discovered the error and corrected it.

Considering decimals as number pairs was also revealed in Task 13, adding 0,1 to 4,256 and 6,98 giving the most common incorrect answers of 4,257 and 6,99 instead of the correct answers 4,356 and 7,08 and similar answers occurred in the subtraction problems:

13.	Add 0,1 to:		**14.**	Take away 0,1 from:	
a	4,256	Answer:	**a**	15,863	Answer: ....
b	6,98	Answer:	**b**	1,06	Answer: ....

Task 13a	Year 5	Year 6	Year 7
Correct: 4,356	40	56	70
4,257	44	34	19
Others/No answer	16	10	11
Task 13b			
Correct: 7,08	24	41	59
6,99	44	35	21
Others/No answer	32	24	20
Task 14a			
Correct: 15,763	35	55	71
15,862	41	32	18
Others/No answer	24	13	11

Task 14b			
Correct: 0,96	18	35	50
1,05	47	39	26
Others/No answer	35	26	24

***Table** 7: Answes to Tasks 13 and 14. Frequences are given in percentages*

The belief that multiplication always makes the answer bigger, and that division makes it smaller, was revealed both in some pure numerical test items, in word problems and in the explanations given by the students. For example, comparing three calculations, the students belive that 5 · 0,9 gives a larger answer than either 5 : 0,9 or 5 · 0,89, giving the explanation that it has the biggest numbers or it is a multiplication. Another task asked students to give the correct expression to calculate the price of pork chops: "*1 kg pork chops costs 65,50 kr. What will 0,76 kg cost?*" Of the year 6 and 7 students, only 10% and 20% answered this task correctly, with 65,50 : 0,76 as the most common wrong answer. 26% and 40% respectively of the students gave this answer. A similar task, buying 1,7 kg of sausages, was answered correctly by 54% and 61% respectively of year 6 and year 7 students. On challenging students who were discussing a similar problem, the students answered: *"It has to be smaller, so we must divide"*. The view of multiplication as making numbers bigger and of division making numbers smaller was also revealed in the observations, and is well known from other research (Bell, Fischbein, & Greer, 1984; Swan, 1983b; Bell, Greer, Grimson, & Mangan, 1989).

The overall results from the pre-test confirmed results from other studies in this area, with the same misconceptions revealed, only with some differences in the frequencies that could possibly be explained by differences in age levels or curriculum plans (Brown, 1981; Foxman et al., 1984). Studies in the KIM project[2] in Norway recently gave similar results (Brekke, 1995; Brekke, 1996a; 1996b). Further details of the test results can be found in my PhD thesis (Fuglestad, 1996).

The test results reveal that students generally have consistent patterns of thinking, but with some misconceptions, possibly arising from generalising too far from well known properties of the whole numbers. How can a computer help to improve the teaching of

[2] KIM projct web pages: http://www.ils.uio.no/kim/kim.htm

decimal numbers? As the students appear to use their rules in a consistent way, they need experience to reveal where their incorrect rules fail on decimal numbers and to construct their new knowledge. The use of computers may give instant feedback to help the students discover their mistakes, provide a conflict situation and provoke discussion to support a diagnostic teaching method. Some spreadsheet tasks were designed with this approach to teaching in mind.

The spreadsheet as a tool for teaching - examples from the classroom

A collection of worksheets was produced for the research, some providing tasks on pure number work and some using decimal numbers in context, e.g. a shopping list, calculating interest for money savings. The idea was that students should meet decimal numbers in several lessons, not just in a few lessons particularly aimed at teaching this topic, and get used to interpreting decimal numbers in various contexts and with different numbers of decimal places. Decimal numbers should not be avoided, but utilised whenever appropriate to challenge the students' understanding. With a spreadsheet to perform calculations this is easily accessible, and more emphasis can be put on interpretation and meaning of the results. The worksheets prepared are available in a Norwegian version, together with other teaching material for computers (Breiteig & Fuglestad, 1997).

NUMBER SEQUENCES WITH DECIMAL NUMBERS

Work in small groups of 2 or 3:

	A	B	C	D
1	1	0,3		
2	=A1+1			
3	=A2+1			
4				

a Write the number 1 in cell **A1** and the formula =A1 + 1 in **A2**. The next formula in **A3** is going to be A2 + 1. Copy the last formula down. Look at the result. Then put another number in A1 and look at the result.

Then put the number 0,3 in **B1** and write a formula in **B2** so that the number in that cell is 0,5. Copy the formula down and study the result.

b Study these number sequences and write in the next four numbers in each column:

0,1	0,2	0,01	0,12	7,6	1,17
0,2	0,4	0,03	0,135	6,3	1,15
0,3	0,6	0,05	0,15	5,0	1,13
0,4					

Make the same number sequences on a spreadsheet and compare your results.

c Here you can see some more number sequences. Find the pattern and write down some more numbers in each column. Then make the sequences on a spreadsheet:

1	1	4	2	4	24	7
2	3	10	3	2	6	2,8
4	9	25	4,5	1	1,5	1,12
8	27	62,5				

Compare your formulas with those of your neighbouring groups. Are your formulas the same? Could these number sequences be made in different ways?

***Figure 1:** Worksheet 5.*

Some tasks for the spreadsheet were pure number work, planned in particular to challenge the students' incorrect conceptions of decimal numbers and provoke discussions and further investigation. For example, after writing on a piece of paper what the students expected the answer to be, they used the spreadsheet to perform calculations and compare the result. Worksheet 5 (figure 1), exploring number sequences, gives an example of this. Other tasks stimulate estimation and mental calculations as students experiment with numbers to hit a given target.

Before using this worksheet the students had used the computers for a few weeks, and used the spreadsheet in a few lessons before. They knew how to express a simple formula on the spreadsheet and how to copy it to generate a number sequence using only whole numbers. The challenge in Worksheet 5 was the use of decimal numbers.

At first the students (of year seven) worked away from the computers, writing the next numbers in the first three or four sequences and suggested a formula to use on the spreadsheet to make the same number sequence. Then they turned to the computers to try out their suggestions and compare the results. By doing this, the students discovered mistakes and explored further connections between numbers. I observed several groups of students making similar mistakes to those Lisa and Mary did in this example:

0,1	0,2	0,01		7,6	1,17
AC1+0,1	D1+0,2	E1+0,01		F1-1,3	G1-0,2
0,1	0,2	0,01	0,12	7,6	1,17
0,2	0,4	0,03	0,135	6,3	1,15
0,3	0,6	0,05	0,15	5,0	1,13
0,4	0,8	0,07		[illegible] 4,7	1,11
0,5	0,10	0,09		3,4	1,9
0,6	0,12	0,011		2,1	1,07
0,7	0,14	0,013		1,8	1,05
0,8	0,16	0,015		0,5	1,03

Figure 2: Lisa and Mary's work. Showing mistakes that were later corrected by crossing out in columns D and E (column two and three in the table).

Their error is similar to the common mistake in the pre-test Task 11. The students were asked not to wipe out their errors, but just to cross out and write their corrections.

The students gave the correct formula in D2, = D1 + 0,2 but wrote =E1 + 0,01 for the sequence in the E column. Then, at the computers they discovered their errors and corrected them. I observed what they were doing, and asked why this was wrong, referring to their numbers 0,6, 0,8, 0,10. *"Oh yes, 0,10 is less than 0,8 and it has to be bigger."* the students replied and got on with their work correcting their mistakes.

Other students extended the first two number sequences without conflict, but met a more challenging situation when they arrived at the sequence starting 0,12, 0,135, 0,15 ... so they just dropped it. Two boys tried to add 122, but then - *"it should be nought comma ...something"* they said. They tried to add 0,112 and they tried 0,12 but it did not work. Later the one boy found he could add 0,015 but his partner still felt unsure about this and they got into a lot of discussion trying different solutions. Similar observations were made in other groups. In most cases the students had no problem writing the correct spreadsheet formula for their trials, but interpreting and comparing decimals with different numbers of decimal places appeared to be quite difficult. In this context it is important to use the standard format, not a fixed number of decimal places, in the spreadsheet set-up, in order to display different numbers of decimal places.

Next week the class moved on to work on *Worksheet 8* on more number sequences dealing with addition, multiplication and division. Using a fixed reference, by naming a cell, it is possible to explore many number sequences by changing just the starting number and the number added or multiplied in the named cell.

The teacher instructed the students to give the cell B1 a name, *diff,* using the appropriate spreadsheet menus. Then he put a number in B3, the formula =B3 + *diff* in B4 and copied to get the number sequence. This worked out fine for the students and they were then challenged to make other sequences by just changing the numbers in B1 and B3.

	A	B	C
1		0,5	
2			
3	Numbers:	2	
4		=B3+diff	
5		=B4+diff	
6		...	

Figure 3: *Showing the set-up used in Worksheet 8 part c, adding sequence in column B*

Two girls, Sue and Ingrid, were working on part c of this worksheet, making the same mistakes as observed with other students in writing down number sequences, discussing and correcting their work. They needed some hints from the teacher. I observed them several times during the lesson. They worked well and were quite engaged in their work. After some hesitation, following the instructions from the teacher on Worksheet 8, they were able to try out other sequences using the same set-up.

In part d in the worksheet they were challenged to make sequences by multiplication in a similar way. The two girls did this fairly easily by giving cell D1 the name *daff* and using formula =D3**daff* in D4 and copying further down the column.

Then the girls decided they also wanted to make a number sequence using division in a similar way. I observed their work, and they made this also quite easily, only briefly asking if they needed to give cell D1 a name similar to the names used before. They invented new names *duff* and *tuff* for the next cells they needed as divisors in the next two number sequences. After they had been playing around a little with their new number sequences using multiplication and division, I challenged them to try using decimal numbers as the number factor and divisor in the cells C1 and D1. At this stage I also said I wanted to listen to them discussing, put on the tape recorder and left it there for the rest of the lesson.

	A	B	C	D	E	F
1		0,5		2		5
2						
3	Number sequence:	2	Number sequence:	1	Number sequence:	5
4		=B3 + diff		=D3*daff		=F3/duff
5		=B4 + diff		=D4*daff		=F4/duff
6		=B5 + diff		=D5*daff		=F5/duff
7		...		...		...

Figure 4: *The set-up Sue and Ingrid made on the spreadsheet so far.*

AB (the observer): *What happens now, if you divide by a decimal number? Have you tried it at all?*
Sue: *Dividing by a decimal number? No, we have not tried that.*

They put in 4,0 in cell F1 and looked at the result. They tried 4 just before.

Sue: *No, that is the same, it was not a decimal number.*
AB: *Yes, 4,0 is just the same as 4. But what then if you try to divide by ...?*

The students tried a decimal number bigger than one, probably 1,2.

AB: *If you divide by 0,5 then, how will it go?*

The student using the keyboard, put in the number 0,5 in F1

Sue: *Oh???*
Ingrid: *5, 10, but it is ..*
Sue: *Oh!. But it is ... but this is not divided by.*
AB: *Is it not divided by?*
Ingrid: *Oh, yes ...*
AB: *Is it not divided by? You made the formula yourself so that is should be dividing.*
Sue: *Yes, yes it has to be that.*
Sue: *It became just bigger! This is quite funny, it is going bigger, 5, 10, ... but, ... but it should be OK.*

The bell rang for break, but the teacher delayed the break till later. He was aware of interesting discussions going on in the class amoung several groups and wanted them to continue.

Sue: *but, yes it has to be right ...* (she is trusting the computer)
Ingrid: *It is really quite funny that it is going higher.*
Sue: *Why, is it like that?*
AB: *Yes, - why is it like that?*

AB pointed at the number sequence using multiplication in column C, and suggested they should try 0,3. The students put in 0,3 and looked at the result a little hesitating:

Sue: *Hm...*
AB: *How was that?*
Sue: *First .... it goes down ...*
AB: *Have you not seen this before??*
Ingrid: *And so it goes down ....* (sounds a little doubting)
Sue: *And this one goes upwards ...*
Ingrid: *It should really been the opposite way ...*
Sue: *Yes.*
AB: *Should it been the opposite way? Are you sure of that?*
Sue & Ingrid (simultaneously): *It should not ...*
Sue: *It should not, but we feel it should have been the opposite way.*
Ingrid: *Funny this!*
Sue: *Yes, it really is. It is going down ....*
Ingrid: *It really was funny this.*
Sue: *Yeah, but you times, and then it must go bigger.*

The students were looking at the result, murmuring a bit were still thinking of what they had seen, commenting a little. They got so excited they called their teacher to tell him about this.

Ingrid: *Please, Mr. ST, come here a little, here you can see something quite funny, as you probably have seen before ...*

The teacher arrived, and the students pointed at the screen:

Sue: *This is times - and this is dividing ... So, this goes, it goes down and this goes upwards. Why does it do this?*
ST: *Yes, why does it go like this?*
Sue: (laughing) *Yes, that's what we asked you about ...*
ST: *Does it go down when you times?*
Sue: *Yes, can't you see it?*

The teacher challenged the students further, trying to provoke the students on this point by asking again:

ST: *Is it going less.... But it will be more when you times .... something must be wrong here, it cannot possibly be less when you times, can it?*
Sue & Ingrid: *Yes, yes it is doing that!*
ST: *Oh no, it is going to be more .. you can see it* (putting a number bigger than 1 in D1)
E: *Yes, but if we use 0,3 then it goes down.*

ST: *Yes, you can decide that. But I can decide that it is going to be more.* (Again putting in a bigger number as factor).
Sue: *It has nothing to say because ...*
Ingrid: *We take, - we take - like nought point ...*
ST: *Can it probably have anything to do with that, that it will be nought point something? Can it have something to do with that?*
Ingrid: *It **has** to do with that!*
Sue: *Yes, just look, here, this is also going to be less ...*
Ingrid: *Oh, how funny this is!*

While saying this, Sue put in a new number less than one as factor.

Sue: *Yes, but if you times by a number below nought then it is going to be less.*
ST: *Yes, if you times by a number less than 1 then is it just going less? Can't you find a number bigger than one that makes the numbers less?*
Sue: *No, I don't think so.*
ST: *Could it possibly, if it is just a little bit bigger than 1?*

The students moved on trying different numbers slightly bigger than one and commented: *«Look here it is going bigger»*. Then they tried numbers less than one, like 0,9 0,09 0,009 and similar. The students mumbled a little, commenting on what they saw, quietly bursting out with exclamations of surprise.

The students' discussion continued for a while and they also showed their results to the neighbouring groups of students. They discovered numbers with several decimal places, and numbers written in scientific notation, which they had never seen before. The discussion of division continued for a while comparing number sequences made by dividing by whole numbers and decimal numbers. Later they investigated multiplication in the same way - and found similar surprises multiplying with numbers less than one, e.g. 0,2 and 0,5.

The discussions in the class engaged the students. There were a lot of lively arguments going on during the students' work at the computers. The work on the spreadsheet tasks gave the students the opportunity to discover their mistakes, discuss and explain to each other what they did wrong. The students work gave the teacher starting points for further discussion and explanations in the class.

The students seemed to use the spreadsheet with confidence, writing in their numbers and formulas, and copying. Although, there had been a few problems in a previous lesson of the understanding of a formula in the spreadsheet, the work on number sequences moved on without problems connected to the software. Apparently the students were able to utilise the computer in their thinking. The mathematics in itself caused bigger problems and made the students discuss and experiment. The same was seen in other classes, and with use of different spreadsheet packages.

A few more worksheets dealt with multiplication of decimal numbers. The target game of hitting a number by multiplication, appeared to be quite revealing for the students knowledge of estimation using decimals. In the illustration (Figure 4) the target was 100, and starting number was 17. It was therefore necessary to use decimal numbers, also less than one, to hit the target.

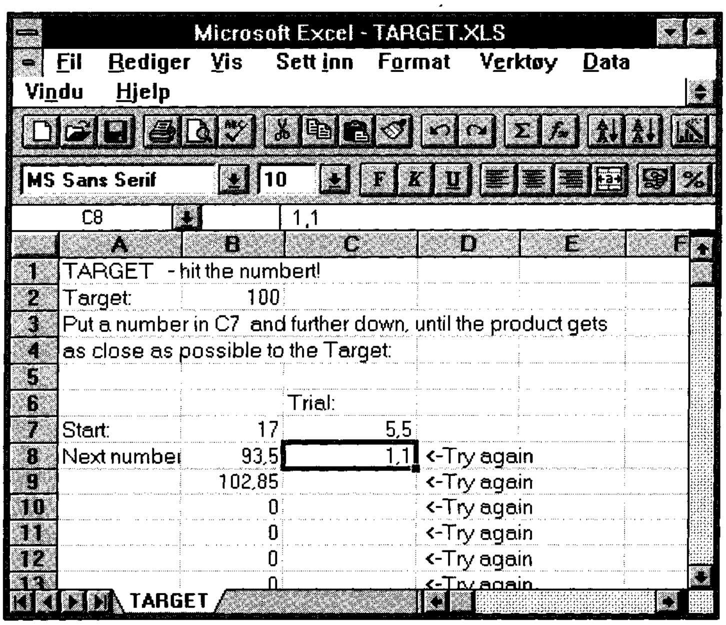


Figure 5: The spreadsheet task TARGET.

The students in year seven worked in small groups on the Target task. In judging what number to try next, they revealed problems estimating results of multiplying by 1,1 or 1,2. In particular, estimates involving 0,9 and similar were difficult. It turned out that to some students, multiplying by a decimal number less than one was a new experience. It turned out they did not discover this for themselves using Worksheet 8, they just heard their peers talk about it. It was apparently necessary to experience this repeatedly, and their own experience counts more than just listening to others.

Steven tried at first to multiply by 0,9, but the result was too small, so he decided to "undo" the result by multiplying. He thought 1,1 would do this. To his surprise this did not work.

Many of the groups did manage after a struggle to get closer to the target, but the problem had to be discussed in the class to help students understand the connections.

Worksheet 12 also dealt with estimating the products of decimal numbers. The problem was to experiment with the sum and product of four numbers, given the sum of 11; can the product of 13,5 be obtained? What is the biggest possible product? What if the sum is one - can the product be bigger than one?

	A	B	C	D
1		6		
2		3		
3		1,5		
4		0,5		
5				
6				
7	Sum	11	Product	13,5
8				

Figure 6: *The set-up in Worksheet 12, showing Fred and Henry's solution*

Fairly soon Fred and Henry found 6, 3, 1,5 and 0,5 would do. Then they tried to find the biggest possible product from four numbers giving the sum 11. Henry. *"Then, we need a small number on the top in B1 and a big one at the bottom in B4"* They discussed whether the position of the number really mattered, Fred thought not, but Henry thought that it did matter. They tried it out by interchanging numbers, and eventually found it did not matter.

Teacher: "*Why is that so?*"
Fred: *"It doesn't matter with the order in sum and times"*.

The teacher discussed with Jane and Ann the effect of increasing one of the numbers. Their spreadsheet showed the four numbers 6,75, 1, 1 and 2 giving the product 13,5. They increased a number from 1 to 1,25 and got very surprised by the effect on the product. But, increasing from 6,75 to 7 gave only a small effect on the product. Why? They expressed their findings: *"Add to the biggest number, then the product doesn't increase that much"*.

Another group tried to make the sum equal to one and the product bigger than one. Sally thought it should be possible, but wanted help to find the 'clever number' for a start. Several groups worked on this, but they could not solve it. All products they found were fairly small. Then Siri found a solution, arguing the question was impossible: *"it has to be one that is bigger than one since we start out by less than one"*. After discovering the product had to be less than one, they started hunting for the biggest product, and several groups ended up with four equal factors of 0,25.

The classroom experience revealed that the worksheet tasks were quite time consuming. Indeed, more than one lesson was needed for this.

Results from the tests

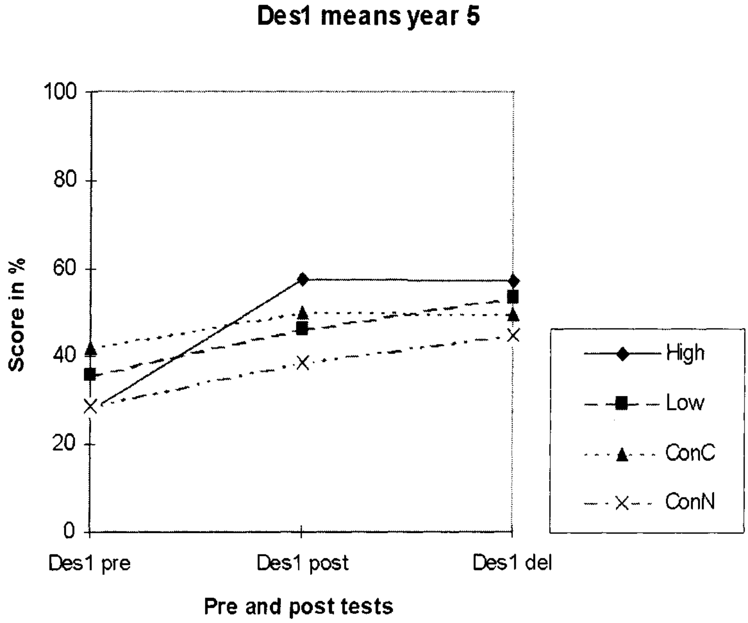
In order to compare results for the research and control groups from the pre-test, post-test and delayed post-test, four variables were created: The test was split into four groups of items and the sum of correct answers calculated. On basis of these variables a regression was performed to calculate residual variables which were then used in the analysis of variance.

Two sets of control classes were assigned from the beginning. The purpose of this was to monitor the effect of the computer use in the classes. At the end, reports from the classes revealed the computer control classes made very little or no use of computers. The test results also revealed only small differences between the two sets of control classes.

Generally, nearly all groups of students improved from the pre-test to the post-test and improved further on the delayed post-test, with the research group showing biggest improvement. The scores on the four different parts of the test showed that the overall improvement was biggest in the first part of the test summarised in the variables Des1 and Des2. The variable Des1 summarises questions about place value, ordering, closeness and density of numbers, and Des2 summarises items of addition and subtraction using decimal numbers.

Statistically significant differences were found in years 5 and 6 on Des1 and in year 6 on Des2, with students in research classes performing better than the corresponding control group. An analysis of variance was used adopting a significance level of $p < 0,05$.

All classes in the experimental group used computers regularly, but observations in the classes and reports from the teachers of what software and what spreadsheet tasks had been used, revealed differences in the way the classes had used computers. For further analysis I therefore found it appropriate to spilt the research group into two parts, based on this information about computer use. The 'High' group were high users of spreadsheets whilst the 'Low' gave little or no attention to the spreadsheet tasks.



***Fiure 7:** Test result - concept of decimal numbers*

This *post hoc* analysis revealed that the differences in performance between research and control classes were due to the 'High' group improving significantly more than the other groups in years 5 and 6 in the first part of the test. Table 8 gives details of the Des1 and Des2 variables. Particularly striking is the strong improvement of the 'High' group of year five as can be seen easily from the diagram Figure 7. Further details of the analysis can be found in (Fuglestad, 1996).

Year	Status4	Des1 pre	Des1 post	Des1 del	Des2 pre	Des2 post	Des2 del	N
5	High	28	57	57	22	41	47	39
5	Low	36	46	53	39	48	55	40
5	ConC	42	50	50	37	49	53	31
5	ConN	29	39	45	29	41	46	110
6	High	56	81	79	62	79	82	35
6	Low	51	62	61	51	61	61	47
6	ConC	31	43	47	37	43	50	11
6	ConN	40	50	53	44	52	55	123
7	High	60	83	83	62	81	84	43
7	Low	54	67	71	62	58	72	38
7	ConC	48	64	67	57	73	68	55
7	ConN	57	72	75	65	78	77	64

Table 8: *Test results, mean score in percent for the variables Des1 and Des2 on the pre-, post-, and delayed post test. The sample in four parts.*

Looking at single test items, I found that the 'High' group showed particularly strong gains in the first part of the test, on tasks involving:

- density and infiniteness of decimal numbers
- closeness or rounding off
- place value
- ordering numbers, biggest and smallest
- simple addition and subtraction involving decimal numbers

All year levels of the 'High' group improved substantially more than the other groups in these areas. Year 5 students in 'High' also improved strongly in questions of biggest number, (Table 9) ordering (Table 10) and some simple multiplication tasks. Year 5 and 6 improved strongly on reading scales. Year 6 and 7 also improved substantially more on the particular tasks involving division by 0,5 and tasks comparing multiplication and division involving 0,9.

The following tables present the test results on Task 5a, biggest number, Task 7e, numbers between 0,63 and 0,64 and Task 10a, nearest number to 0,16.

	Answer	"High"-gr n=39			Others n=181			Total n = 220		
		Pre	Post	Del.	Pre	Post	Del.	Pre	Post	Del.
Year	Correct 0,62	18	69	67	36	48	54	33	51	56
5	0,236	77	23	18	55	35	34	59	33	31
	0,4	5	5	13	7	13	11	7	12	11
	No answer	0	3	3	2	4	2	1	4	2
6	Correct 0,62	69	91	91	49	64	67	52	68	71
	0,236	29	6	3	42	23	19	40	20	16
	0,4	3	3	3	7	11	9	7	10	8
	No answer	0	0	3	2	2	4	2	2	4
7	Correct 0,62	76	94	94	77	87	87	77	88	89
	0,236	18	3	6	19	5	5	19	5	6
	0,4	3	3	0	4	7	7	4	7	6
	No answer	3	0	0	1	1	1	1	1	1

Table 9: *Summary of result of Task 5a: Ring around the biggest number. Comparing 'High', with others and the total sample on pre- and post tests. Frequencies as percentages.*

	Answer	"High"-gr n=39			Others n=181			Total n = 220		
		Pre	Post	Del.	Pre	Post	Del.	Pre	Post	Del.
Year	Correct	3	23	28	4	7	8	4	10	11
5	None	46	21	15	40	34	34	41	32	31
	One number	8	3	10	7	10	5	7	9	6
	Definite n.	18	39	36	16	25	29	16	27	31
	Others/NA	26	15	10	33	24	24	32	22	21
6	Correct	17	74	74	6	16	14	7	25	24
	None	40	6	9	38	35	33	38	31	29
	One number	6	3	0	19	7	7	17	6	6
	Definite n.	20	17	11	19	27	25	19	26	23
	Others/NA	17	0	6	19	15	21	19	13	19
7	Correct	24	61	76	20	40	44	21	43	49
	None	33	6	6	33	19	14	33	17	13
	One number	3	0	0	10	3	2	9	3	2
	Definite n.	24	30	15	20	27	27	21	28	25
	Others/NA	15	3	3	17	12	14	17	11	12

Table 10: *Summary of result of Task 7e. Numbers between 0,63 and 0,64? Comparing the total sample and the 'High' group on pre- and post tests. Frequencies as percentages.*

	Answer	"High"-gr n=39			Others n=181			Total n = 220		
		Pre	Post	Del.	Pre	Post	Del.	Pre	Post	Del.
Year	Correct 0,2	21	51	44	16	24	27	17	29	30
5	0,21	64	36	46	65	61	61	65	56	68
	Others/NA	15	13	10	19	15	12	19	15	12
6	Correct 0,2	51	74	74	36	40	41	38	45	47
	0,21	40	26	23	55	50	51	53	46	46
	Others/NA	9	0	3	9	11	8	9	9	7
7	Correct 0,2	36	73	67	38	58	61	38	60	62
	0,21	52	27	27	53	34	32	53	33	31
	Others/NA	12	0	6	9	8	8	10	7	8

Table 11: *Summary of result of Task 10a Number nearest in size to 0,16 Comparing 'High', Others and the total sample on pre- and post tests. Frequencies as percentages.*

The performance of the 'High' group did not differ very much from the other group in word problems and most problems of multiplication and division.

Combining results from tests and observations

The classes in the 'High' group were the high users of spreadsheets, and in years 6 and 7 these classes used the worksheets on number sequences and the students discussed the problems. In year 5 there was less use of these particular tasks, but they used spreadsheet tasks on shopping lists and similar problems. Can we attribute the differences in test results to the spreadsheet tasks or to use of other software in the 'High' classes?

Looking at the test items and the content of the spreadsheet tasks, there seems to be some connection. The strong results for the 'High' group on the test items dealing with ordering numbers, density, adding and subtracting could be explained by their use of the computers and spreadsheet tasks which dealt with similar tasks. However, other factors may also influence the results, and have to be taken into account in further discussion and investigations.

A constructivist approach?

A constructivist view of learning may be implemented in different ways (Davis, Maher & Noddings, 1990). In this research the intention was to use a diagnostic teaching approach.

Looking back at what happened in the classes, the observations clearly indicate that crucial points in a diagnostic teaching approach were implemented. Students working on the spreadsheet tasks had experiences of cognitive conflict and discussions. The conflicts were resolved, partly during students' group discussions and partly in the class discussions that followed. Further experiments using the spreadsheet helped the students consolidate their knowledge.

The teachers' role seems to be crucial in utilising the computers in the mathematics classroom and in particular with diagnostic teaching. Episodes described here clearly indicate that the teacher's or observer's intervention, asking questions or giving suggestions for further trial, was of major importance for further development. Also, apparently, students need more than one relevant experience in order to change an incorrect pattern of thinking. There was need to follow up, by summarise findings and provide further discussion in the class.

However, some limitations to the research have to be noted: more than half of the teachers in the research had never used computers in their mathematics classroom and had no experience of using a spreadsheet before. Also, it is worth noticing that in interviews at the end of the research the teachers appeared to be less aware of diagnostic teaching than expected. Although they were given an introductory course before the school year, and some support during the year, the teachers will probably benefit from more experience of using computers in the classroom. As the teachers become more conscious of their teaching approach, we may expect stronger results.

Conclusions

An important strategy in diagnostic teaching is to set up teaching activities that force students to confront their own misconceptions. Observations suggest that a very effective way of doing this is to ask students to try a number activity first on paper and then repeat this on a spreadsheet. The students' awareness of their problems and shift in understanding observed in the classrooms, is supported by the test data for a larger sample of students. In this way, the combination of test results and observations confirm the potential of a diagnostic teaching approach for the students' learning. The potential of the computers as support for diagnostic teaching appears to be considerable. Taking the limitations of the research into account we may expect even stronger results from a more conscious implementation of the diagnostic teaching approach supported by suitable computer software.

There is need for further investigations, developments of methods and research to clarify the findings.

References

Beare, R. (1993). How Spreadsheets can aid a variety of Mathematical Learning Activities from Primary to Tertiary level. In B. Jaworski (Ed.). *Proceedings of Technology and Mathematics Teaching.* Birmingham:

Bell, A. (1993a). Principles for the design of teaching. *Educational Studies in Mathematics, 24(1)*, 5-34.

Bell, A. (1993b). Some experiments in diagnostic teaching. *Educational Studies in Mathematics, 24(2)*, 115-137.

Bell, A., Fischbein, E. & Greer, B. (1984). Choice of operation in verbal arithmetic problems. The effects of number size, problem structure and context. *Educational Studies in Mathematics, 15(2)*, 127-147.

Bell, A., Greer, B., Grimson, L. & Mangan, C. (1989). Childrens' performance on multiplicative word problems: Elements of a descriptive theory. *Journal for Research in Mathematics Education, **20(5)***, 434-449.

Bell, A., Swan, M., Onslow, B.A., Pratt, K. & Purdy, D. (1985). *Diagnostic Teaching. Teaching for Long Term Learning.* Nottingham: Shell Centre for Mathematical Education.

Bennett, J.P. (1991). Effectiveness of the Computer in the Teaching of Secondary School Mathematics: Fifteen Years of Research. *Educational Technology, **31(1)***, 44-48.

Bigge, M.L. & Shermis, S.S. (1992). *Learning theories for teachers.* New York: Harper Collins Publishers.

Bissell, C. (1995). Models of Technology. Spreadsheets and the Language of applicable Mathematics. In L. Burton & B. Jaworski (Eds.) *Technology in Mathematics Teaching* (pp. 365-386). Chartwell-Bratt.

Breiteig, T. & Fuglestad, A.B. (1997). *Data i matematikken.* (2nd ed.) Oslo: Aschehoug.

Brekke, G. (1995). Oppfatninger av desimaltall. *Nämnaren*, **4**, 27-34.

Brekke, G. (1996a). A decimal number is a pair of whole numbers. In L. Puig & A. Guiterrez (Eds.) *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education* (pp. 137-144). Vàlencia: Dept. de Didàctica, Universitat de Vàlencia.

Brekke, G. (1996b). Regning med desimaltall. *Nämnaren*, **1**, 17-20.

Brekke, G. & Støren, H. (1995). Kvalitet i matematikundervisningen. *Nämnaren*, **3**, 10-14.

Brown, M. (1981). Place value and decimals. In K. M. Hart (Ed.). *Childrens Understanding of Mathematics: 11 - 16* (pp. 48-65). London: John Murray.

Davis, R.B., Maher, C.A. & Noddings, N. (1990). *Constructivist views on the teaching and learning of mathematics. Journal of Research in Mathematics Education. Monograph number 4.* Reston, VA: National Council of Teachers of Mathematics.

Foxman, D., Graham, R., Joffe, L., Mason, K., Mitchell, P. & Sexon, B. (1984). *A review of monitoring in mathematics 1987 to 1982 Part 1. APU.* National Foundation for Educational Research.

Fuglestad, A.B. (1996). *Computers and the understanding of mathematics. A study of teaching decimal numbers. PhD thesis.* (Forskningsserien nr. 6, 1998. Kristiansand: Høgskolen i Agder.)

Green, D., Armstrong, P. & Bridges, R. (1993). *Spreadsheets: Exploring their Potential in Secondary Mathematics.* The Mathematical Association.

Hart, K., Brown, M., Kerslake, D., Kuchemann, D. & Ruddock, G. (1985). *Chelsea Diagnostic mathematics test..*

Healy, L. & Sutherland, R. (1991). *Exploring mathematics with Spreadsheets.* Simon and Shuster Education.

NCET. (1996). *Integrated Learning Systems. A report of Phase II of the pilot evaluation of ILS in the UK.* Coventry: National Council for Educational Technology.

Phillips, R., Pead, D. & Gillespie, J. (1995). Evolving strategies for using interactive video reources in mathematics classrooms. *Educational Studies in Mathematics, **28(2)***, 133-154.

Resnick, N., Leonard, M., Omanson, S. & Peled. I. (1989). Conceptual basis of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education, 20(1),* 8-27.

Rojano, T. (1993). Towards an algebraic Approach: The role of spreadsheets. In I. Hirabayaski, N. Nohda, K. Shigemats & F. Lin (Eds.). *Proceedings of PME XVII (1)* (pp. 189-196). Tsukuba, Japan: University of Tsukuba.

Rojano, T. & Sutherland, R. (1991). Symbolising and Solving Algebra Word problems: The Potential of a Spreadsheet Environment. In F. Furninghetti (Ed.). *Proceedings of PME 15(3)* (pp. 207-213). Assisi, Italy: University of Genova.

Sackur-Grisvard, C. & Lèonard, F. (1985). Intermediate Cognitive Organizations in the Process of Learning a Mathematical Concept: the Order of positive Decimal Numbers. *Cognition and Instruction, 2 (2),* 157-174.

Swan, M. (1983a). Teaching Decimal Place Value. A comparative study of conflict and positive only approaches. In R. Herschkowitz (Ed.). *Proceedings of PME VII* (pp. 211-216). Haifa, Israel: Department of Science teaching. The Weizman Institute of Science

Swan, M. (1983b). *The Meaning and Use of Decimals. Calculator based tests and teaching materials.* Nottingham: Shell Centre for Mathematical Education.

Watson, D.M. (1993). *The IMPACT Report.* London: Centre for Educational Studies. London: Kings College London.

Abstract (in Norwegian)

Datamaskiner med ulike typer programvare ble brukt i matematikk-undervisning gjennom et skoleår med 10 - 14 år gamle elever i Norge. Dette forsknigsprosjektet hadde som målsetting å studere elevenes forståelse av desimaltall, med en diagnostisk undervisningsmetoder der oppgaver på regneark ble brukt for å stimulere utforsking og konflikt diakusjoner. En analyse av oppgavebesvarelser fra førtesten viste et vanlig mønster av misoppfatninger om desimaltall. Observasjoner i klasserom bekreftet at studentenes misoppfatninger ble avslørt under arbeid med regnearkoppgavene. Test resultatene indikerer at de elevgruppene som bruke regneark hadde signifikant større framgang enn kontollgruppa, med størs effekt fra de 'største' regnearkbrukerne.

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