# Discussion activities, teacher beliefs, and the learning of mature, low attaining students

Malcolm Swan

This two-year investigation evaluated strategies for learning mathematics with low-attaining students in Further Education (FE) colleges in England. A substantial collection of discussion activities were developed to enhance metacognitive activity where conceptual obstacles were identified and targeted. A pre-test-post-test design was combined with detailed classroom observation. Two 'constructivist' teachers, sympathetic to metacognitive teaching approaches are compared with two' transmission' teachers who preferred exposition and practice. In the first year, without intervention, students showed modest gains with no clear difference between the two pairs of teachers. In the second year, the discussion activities contributed to statistically significant gains among the 'constructivist' teachers, but little difference was observed with the 'transmission' teachers.

## Introduction

Each year in England, one half of the student cohort at the age of 16 fail to attain grade C or better in the GCSE (General Certificate of Secondary Education) examination in mathematics. This grade constitutes a minimum requirement for many careers and for entry into higher education. Many students therefore decide to retake the examination in a Further Education (FE) institution. The retake course is typically one year long, and often involves a rapid 'coverage' of the entire syllabus. Evidence of learning within such an environment is unimpressive (Audit Commission/HMI investigations, 1993). In addition, recent expansions in this sector, coupled with reductions in capital funding have resulted in less time being devoted to such courses and an increasing reliance on part-time teaching staff. This context provides a bleak and challenging environment for educational research.

This project attempted to create and implement a learning programme intended to foster an increased amount of reflective activity among students aged 16-19. I first attempted to develop a

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collection of activities that would encourage students to identify, confront and over come common conceptual obstacles through supportive social discussion. I then sought to measure the impact of these activities on teaching styles and learning outcomes.

## Background

In the early 1980s, following the widespread use of diagnostic assessments, there was a considerable growth in understanding the extent and nature of common conceptual obstacles in learning mathematics and attempts were made to overcome these by recognising and building on children's existing conceptual frameworks (Johnson , 1989). Our own work with secondary school students explored ways of stimulating "diagnostic teaching approaches" in several content areas within mathematics (Swan, 1983; Bell et al, 1985). Comparative experiments demonstrated in several areas of mathematics the greater effectiveness of teaching which: uses pre-testing to clearly identify students' prior states of conceptual understanding; targets specific conceptual obstacles and misunderstandings; employs sharply focused discussion material, intended to provoke "cognitive conflict"; and engages students in cognitive and metacognitive activity aimed at describing and explaining strategies and errors.

The nature of the cognitive conflict may be both intra- and interpersonal in nature. In our experience, both are powerful, though sociocognitive conflict theory suggests that:

> *the contradiction coming from two opposite points of view is more readily perceived and cannot be refuted so easily as the contradiction coming from facts for an individual. The latter may either not perceive the contradiction or not take it into account when wavering between two opposite points of view and finally choosing one of them. In order to master a task, students working jointly are committed to overcoming conflict. When attempting to solve the contradiction, they may manage to coordinate the two points of view into a third one overcoming both initial points of view and corresponding to a higher level of knowledge (Laborde, 1994; p149).*

Our work has confirmed that in adopting such approaches, both teachers and learners have to modify considerably their conceptions of what constitutes appropriate mathematical activity. An orientation towards 'working through exercises' and 'getting answers' has to give way to 'working on ideas' and 'constructing well-knit concepts and methods'. We note with Baird et al (1986, 1992) the resistance to change generated by such conflicts with students' existing conceptions of learning.

In a further research project (Bell & Swan, 1993; Bell et al, 1997) we worked with teachers to develop a number of classroom interventions which involved students in adopting novel classroom roles. These included, for example, students constructing tests and mark schemes, students teaching students, students designing teaching materials and students evaluating the purposes of lessons. Although observational evidence suggested that these approaches were effective in promoting reflection and learning, their overall effects on mathematical attainment were not measured.

In the preliminary work on the current project, it became clear that the need for introducing more reflective approaches to learning was, if anything, greater in FE than in secondary schools. The repeat GCSE lessons that we observed were longer than those found in schools - they lasted from one to three hours - and it was not uncommon to see students spending this time either passively listening to a teacher explanation or silently working through exercises. (During one, not atypical, two hour lesson observed, the teacher spoke 2046 words (average length of utterance: 93 words) while the students spoke a total of 32 words (average length of utterance:1.7 words).

> *Under pressure to stay busy, students who have become accustomed to meaningless tasks are unlikely to see themselves as needing to learn how to guide their own learning. In other words, the cognitively rich get cognitively richer, while the cognitively poor get metacognitively poorer." (Kilpatrick, 1985, p.17).*

In short, the project attempted to explore how 'transmission' methods of teaching which create a passive learner dependency may evolve into 'constructivist' approaches in which students are given opportunities to interpret ideas, negotiate meanings, to be challenged and thereby construct some new understanding of their own. This leads to a view of the syllabus as possible learning outcomes rather than a list of content that must be explained at a predetermined pace.

> *A constructivist view of the syllabus requires a shift from a knowledge-based view of mathematics to an interpretative base of which reflection is a necessary part. (Burton, 1993, p.9)*

## The structure of the project and some difficulties encountered

During the first year of the project, eight teachers were observed working normally, and their classes were pre- and post-tested on five different mathematical topics using diagnostic tests developed from earlier research (e.g. Hart, 1985). In parallel with this, the teachers and I collaborated in developing a collection of reflective classroom activities. During the second year, four teachers implemented the activities with a fresh cohort of students and qualitative and quantitative comparisons were drawn between the experiences and learning of the two cohorts with these four teachers.

The selection of the four teachers proved unexpectedly difficult. Initially, eight FE colleges were contacted. Three colleges were deemed unsuitable because their courses were being restructured, drop out rates among students were too great, and staffing was undergoing change. One further college contained no identifiable 'groups' of students at all. (In this college, students were allocated individual timetables and their work was completed entirely within drop-in workshops.) From the remaining colleges, eight lecturers were selected. These lecturers planned to teach the same, one year course to two similar samples of at least 15 students using a range of approaches, including whole-class textbook-based and more individualised methods. During the two years of the project, three lecturers withdrew from the project because student attendance became so low that their classes had to be cancelled or reorganised and one teacher withdrew with illness. The sporadic attendance of students also meant that the pre- and post-testing data was based on smaller samples than were originally envisaged.

## Tests and observational Data

Five, 30-minute tests were designed, drawing on previous research, notably the work of the CSMS project (Hart, 1981) and the APU surveys (1980-82), as well as tests which had been devised by researchers at the Shell Centre at Nottingham University (Brekke, 1991); Swan, 1983). It was not felt appropriate to use these tests directly, however, as they were too long and the contexts used in many items were not felt to be suitable for Further Education students. Instead, we decided to design new items which would focus on similar key conceptual obstacles to these tests.

The teachers were asked to administer the pre-tests immediately before beginning each topic and the post-tests at least three months after its completion. The intention was to establish some idea of the extent of learning that had taken place in each class during each year, so that comparisons could be made between the two year cohorts.

**Sample test items on decimals.**

D3 Mike's train leaves at 7.51 am
It arrives in London at 9.07 am
Mike works out his journey time like this:

$$\begin{array}{r} {}^{8}\not{9}.{}^{1}07 - \\ \underline{7.51} \\ \underline{1.56} \end{array}$$

Is Mike correct? ................................
If not, explain what he has done wrong.

| | | | |
|---|---|---|---|
| *D3:* | *Facilities*:* | *'No' with correct reason :* | *46%;* |
| | | *'No' with no reason:* | *14%;* |
| | | *'Yes' :* | *19%* |
| | | *Other + omissions:* | *21%* |

D10. **Ring** the number that is nearest in size to 0.16

| | | | |
|---|---|---|---|
| 0.1 | 0.2 | 10 | 20 |
| 0 | 1 | 2 | 0.21 |

| | | | |
|---|---|---|---|
| *D10:* | *Facilities*:* | *0.2* | *53%* |
| | | *0.21* | *38%* |
| | | *Other and Omissions* | *9%* |

** Percentages show proportions of answers given by a sample of 101 students with Grades D or E in GCSE in the Further Education classes involved in the project*

Each college teacher was observed 10 times for each of the two years and sample interactions were audiotaped, transcribed and analysed.

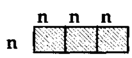
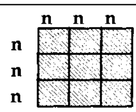
## Learning activities

The mathematical activities designed for this project were intended to encourage students to pause and reflect on the meanings of mathematical concepts, notations and strategies and to distinguish between different modes of learning; notably practising for fluency and sharing meanings through discussion. Five types of reflective activity were developed:

### 1. Collecting together equivalent mathematical representations

Mathematical concepts are inextricably bound up with graphic and symbolic representations. Most concepts have many representations; from conventionally accepted notations to informal mental representations. The type of activity suggested here is intended to allow these representations to be shared, interpreted, compared and classified so that students construct an understanding of the underlying concepts.

One approach involved offering groups of students a number of mathematical representations on cards and asking the students to sort the cards into sets so that each set contains different representations with an equivalent meaning. The focus of the activity was thus on interpretation rather than on the production of representations.

| | | | |
|---|---|---|---|
| **Examples of alternative representations for sorting** |  |  | $3n^2$ |
| $9n^2$ | Square n then multiply the answer by 3 | Multiply n by 3 then square your answer. | $(3n)^2$ |

Teachers initially tried presenting these activities on large printed sheets. This approach did not generate sufficient student discussion, so we decided to use a 'card sorting' approach. This proved more effective because it required a collaborative rather than individual, output, allowed a range of approaches, encouraged translations in multiple directions, allowed teachers to monitor the discussions and adjust the nature of the challenge appropriately (by removing or adding cards).

In general, the students appeared to find sorting activities engaging and enjoyable. Students became aware of each others' difficulties, and they were often observed explaining notation to one another. Disagreements over interpretation cannot always be resolved by rational argument. Notations are, in essence, arbitrary yet convenient conventions. The discussions did, however, raise an awareness of the *need* for agreed interpretations of notational conventions and the teachers were able facilitate the sharing of these interpretations in a meaningful context.

## 2. Evaluating the validity of mathematical statements and generalisations

These activities were intended to encourage students to focus on common convictions concerning mathematical concepts. A number of commonly made statements and generalisations were provided and students were asked to examine each one in turn and decide upon its validity. This typically involved deciding whether a statement was *always, sometimes or never true* , then justifying this decision with examples and explanations. In addition, students were asked to add conditions or otherwise revise generalisations so that they would become 'always true'. In practice, these activities met with mixed success. They resulted in learning only when the students were encouraged to give detailed explanations and generate a range of examples.

| | | | |
|---|---|---|---|
| **Examples of generalisations for evaluation.** | If you multiply 12 by a number, the answer will be greater than 12. | The square root of a number is smaller than the number. | The more sides a shape has, the more right angles it can have. |
| If you double the radius of a circle, you double its area. | The shape with the smaller area must also have the smaller perimeter. | $3 + 2y = 5y$ | $2(x + 3) = 2x + 3$ |

## 3. Correcting and diagnosing common mistakes

Students were presented with common mistakes or misinterpretations which they were asked to correct and comment on. (This type of activity differed from 'evaluating generalisations' because the causes of the mistakes were not explicitly stated - that was left for students to uncover.)

Example of a mistake for discussion:

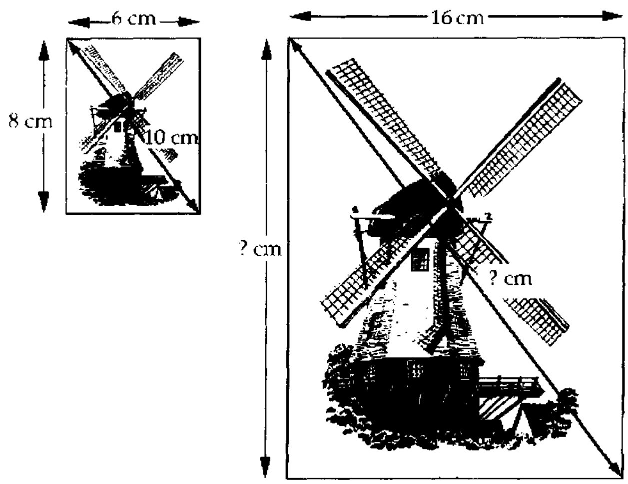
> *The other day I was in a department store, when I saw a shirt that I liked in a sale. It was marked "20% off". I decided to buy two. The person behind the till did some mental calculations. He said, " 20% off each shirt, that will be 40% off the total price." I replied, " I've changed my mind, I think I will take five shirts." Explain the cause of the mistake.*

We observed that mistakes shifted the students' attention from *obtaining* answers to one of finding *reasons* for answers. They provided a *planned agenda* for classroom discussion of common difficulties which otherwise would have only occurred on an 'ad hoc' basis. The approach also permitted the discussion of common difficulties in a non-threatening environment. The main difficulty was in helping students to persist in discussing mistakes until a resolution was obtained.

## 4. Resolving problems which generate cognitive conflict.

In this type of activity, students were asked to tackle problems which were designed to make students aware of their own inconsistencies in understanding. After tackling a problem intuitively, students were invited to revisit the problem using an alternative, given, method. Students were asked to compare the results obtained by the different methods and reflect on the inconsistencies in the answers obtained.

For example, students were invited to solve the following problem intuitively:

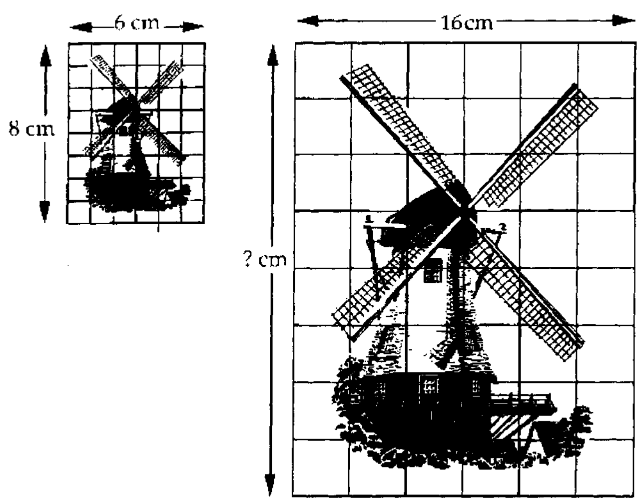


*An artist wants to make an enlargement of a picture 6 cm wide and 8 cm high. She wants the enlarged picture to be 16 cm wide. How high will it need to be? Explain your reasoning.*

*To help check that she has drawn an accurate rectangle, she checks that the diagonal distance between opposite corners are the same. In the small picture, the diagonal distances are both 10 cm. What should the diagonal distances measure on the enlarged picture?*

This task was designed to expose the belief that 'a shape is enlarged in proportion if the same amount is added to each side'. Thus a common response would be to add 10 centimetres to each dimension. When students had committed themselves to such a response, they were asked to draw the enlargement and check that the three dimensions were each correct. The intention was that they would see that their answers were incompatible. This was where the 'cognitive conflict ' was planned to arise, when intuition was plainly in conflict with observation. At this point we suggested that students form small groups to discuss possible reasons for mistakes. This was to further engage interest and create a greater awareness that there was a need for learning.

To help resolve the conflict, the following diagram was supplied to students. This shows one practical way in which drawings are sometimes enlarged, by superimposing a square grid on the drawing and enlarging the content of each square. Students were invited to

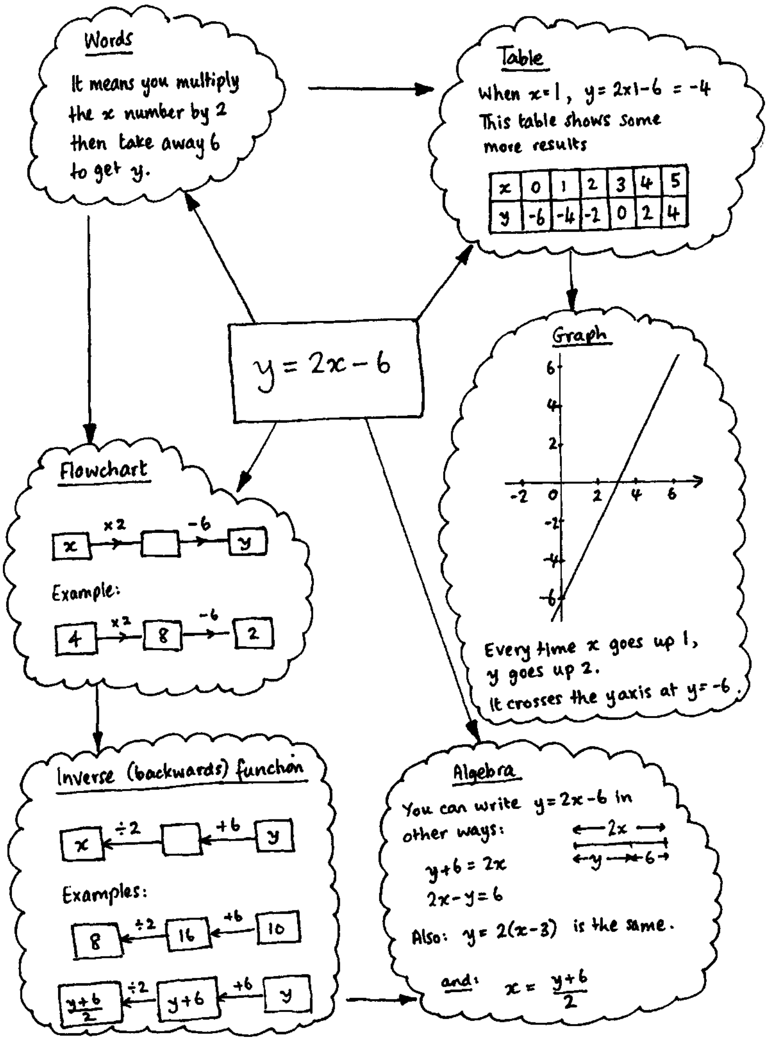


calculate the side length of each enlarged square (16/6) and use this to find the missing height (8x16/6).

In practice conflict problems such as this were successful in exposing the difficulties that students have with understanding mathematical concepts. They were observed discussing common beliefs, such as 'letters in algebra stand for objects' or ' graphs look like pictures of situations' . Some teachers, however, appeared so anxious to 'cover' the syllabus that they were unwilling to generalise and consolidate the learning. They appeared to equate 'obtaining an answer' with 'learning' and, when a correct answer was obtained, they wanted the student to 'move on' rather than spend time redoing the task in a different way, or reflecting on the causes of errors. The result was that students who were profitably engaged in learning were interrupted and moved to a fresh activity before they had been allowed sufficient time to reformulate their understanding.

## 5. Creating problems and connections between concepts and representations.

These activities fell into two categories; students were either asked to construct concept maps to illustrate links within topics or construct fresh examples to illustrate an idea or to satisfy given constraints. Our previous research (Bell & Swan, 1993) showed that lessons involving concept maps can easily become derailed into learning *how to draw* concept maps rather than using them to enhance reflection. We decided therefore to offer students a concept map in each topic to modify and extend, with the encouragement to 'write down all you know about' a given mathematical statement or object.




*Sample concept map for students to use and extend.*
*For example, students might try developing a similar map for a different function.*

Students were also invited to create their own mathematical problems to satisfy given constraints. It was hoped that by engaging with such tasks, students would engage with the structure of problems rather than just their solution.

In one such task, for example, students were given data concerning a car journey and were asked to construct questions that could be answered using part of the data. The statements concerned the fuel capacity of the car (in litres), the rate of petrol consumption on a motorway (miles per gallon), the cost of fuel (pence per litre), the distance (miles) and time (hours) for a given journey and the conversion factor from gallons to litres. Thus the intention was for students to engage in discussion on the structure of rate problems. Most students tended to find the task demanding and 'played safe' by creating straightforward, single-step problems. A few, however, took pleasure in creating more complex multi-step problems which they would then ask their partners to solve. This seemed a useful consolidation activity but we did not see explicit evidence of new learning taking place.

A more successful task involved students in constructing their own equations. They were encouraged to begin by writing down a solution to an equation (e.g. $n = 6$) and then to operate on both sides, preserving the equality ( e.g. Multiply both sides by 3 ). They then repeated this, choosing any operations they wished. In this way, they began to use algebraic symbols meaningfully to construct quite complex equations (such as $(3n-8)/2 + 4 = 9$). After four or five successive operations, they were asked to check that the original value of the variable still satisfied their resulting equation. Students began to read their equations as a series of successive operations and thus began to meaningfully interpret algebraic notation. This activity met with considerable enthusiasm and students generated a range of more complex equations than had been envisaged. For example, here are two with students' own answers:

$$40 - \left(\frac{r^2 - 10}{5}\right) = -3 \qquad (r = 15)$$

$$3\left(\frac{x^2 + 4}{5}\right) - 6 = 18 \qquad (x = 6)$$

It seemed hard to believe that these students, who had previously showed very little facility with algebra were now constructing such complex expressions. Students who had created equations were naturally interested in whether or not their partners could solve them, and would offer help when there was difficulty. This shift of roles from pupil to teacher, or imitator to creator appeared to be a powerful strategy in this context.

The concept mapping activities were not used frequently by teachers. They appeared to see these only as useful 'lesson fillers' or 'lesson enders'. Only once was a teacher observed building a whole lesson around the activity. The value of concept mapping was therefore not fully appreciated or realised. Creating problems was also seen to be something of a 'luxury' activity by some teachers. Where the activity was used, however, there was evidence of reflection and learning. Students appeared to enjoy taking on the role of a teacher as their partners tried to solve the problems they themselves had created.

These creative activities show considerable potential to improve the quality of reflection and analysis of notations, concepts and problem structures, but their 'face validity' is not immediately evident to teachers and students in preparation for an examination which rewards fluency rather than creativity.

## Teacher beliefs and learning outcomes.

The final sample for pre- and post-testing were the classes of four teachers; Alan, Chris, Denise and Ellen. In year 1, each teacher taught in their normal style; in year 2 they used the teaching interventions described above. This design made it possible to compare the effectiveness of the two modes of working.

In the first year of the project, teachers were given questionnaires to elicit some of their beliefs about teaching and learning and informal interviews were held to discuss their responses. In addition, transcriptions of approximately five hours of teaching were made for each teacher. The analysis of this data revealed that the four teachers had different 'belief systems' that affected their normal teaching styles.

Denise and Ellen, both appeared to prefer 'transmission' modes of working, where they emphasised the importance of procedures and routines and saw their task as offering clear explanations followed by intensive practice.

Alan and Chris, however, were clearly more sympathetic to a system of beliefs akin to those described by Askew et al (1997) as 'connectionist'. This is a social constructivist orientation to teaching which encourages students to construct links between different aspects of the mathematics curriculum, and which views learning mathematics as best approached through challenging, interpersonal activity in which students' understandings (including partial- and mis-understandings) are recognised, made explicit and worked on through class and small group discussion.

It is informative to note that Askew used the term 'connectionist' to distinguish this orientation from the 'discovery' learning approaches he observed among primary teachers who, though also constructivist, tended to view learning as being separate to and having priority over teaching. (The contrast between these viewpoints is typified by the contrasting viewpoints of Vygotsky and Piaget ). In the Further Education colleges observed, I did not notice any teachers who could be described as using 'discovery' approaches to learning.

The table below summarises the differences between these contrasting belief systems.

| 'Behaviourist' influences | 'Constructivist' influences | |
|---|---|---|
| Transmission teachers see... | Discovery teachers see... | Connectionist teachers see... |
| • Mathematics asa given body of knowledge and standard procedures to be 'covered' | • Mathematics as a personal construction of the student. | • Mathematics as an interconnected body of ideas and reasoning processes which the teacher and the students construct together |
| • Learning asan individual activity based on listening and imitating until fluency is attained | • Learning as - following development (waiting for the child to reach a state of 'readiness' to learn)- an individual activity based on practical exploration and reflection | • Learning as - leading development- an interpersonal activity in which students are challenged and arrive at understanding through their own articulation |
| • Teaching as- structuring a linear curriculum for the students- giving verbal explanations and checking these have been understood through practice questions- 'correcting' misunderstandings when students fail to 'grasp' what is taught | • Teaching as - assessing when a child is ready to learn- providing a stimulating environment to facilitate exploration - avoiding misunderstandings by careful sequencing of experiences. | • Teaching as- non-linear dialogue between teacher and student in which meanings and connections are explored verbally- making misunderstandings explicit and learning from them |

*Comparison of transmission, discovery and connectionist belief systems (c.f. Ernest, 1991, pp.138-139; Askew et al, 1997, pp.31-32)*

In year 1, Ellen preferred long periods of whole class exposition with shorter periods of practice, while Denise preferred shorter periods of exposition, followed by longer periods of practice. In year

2, though expressing enthusiasm for the project activities, both teachers failed to implement the activities in 'connectionist' ways. Denise tended to issue activities without suggesting why or how students should work on them. Students thus often treated the printed resources as they would a textbook and worked alone or with a partner, often refusing to discuss concepts or review answers. Ellen, however, did modify her approach to encourage more discussion, but she continued to dominate and channel classroom discussion through herself.

In year 1, Alan and Chris felt unable to behave in a way which was fully consistent with their 'connectionist' beliefs because of the lack of time and appropriate resources. In year 2, however, the resources provided by this project allowed both Alan and Chris to implement their preferred teaching approaches more fully.

The pre- and post-test results show that in the first year of the project, there was little difference in the effectiveness of the four teachers. Most of the learning gains were slight (<10%). In the second year, however, the classes belonging to Alan and Chris made greater (>10%) and more consistent gains (across tests) than the classes belonging to Denise and Ellen.

| | **Number (Decimals and Fractios)** | | | **Number (Operations and Rates)** | | | **Algebra (Functions and Graphs)** | | | **Algebra (Expressions and Equations)** | | | **Space & Shape (Length, area, volume +)** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year 1 | Pre | Post | Gain | Pre | Post | Gain | Pre | Post | Gain | Pre | Post | Gain | Pre | Post | Gain |
| Alan & Chris | 42 | 45 | +3 n.s. | 31 | 38 | +7 * | 28 | 33 | +5 ? | 24 | 33 | +9 ** | 29 | 36 | +7 n.s. |
| | n = 19 | | | n = 17 | | | n = 16 | | | n=9† | | | n=11 | | |
| Denise & Ellen | 34 | 40 | +6 ? | 34 | 41 | +7 ? | 21 | 25 | +4 n.s. | 41 | 49 | +8 n.s. | 20 | 33 | +13 ? |
| | n = 13 | | | n = 7 | | | n=11 | | | n=10 | | | n=6 | | |
| Year 2 | Pre | Post | Gain | Pre | Post | Gain | Pre | Post | Gain | Pre | Post | Gain | Pre | Post | Gain |
| Alan & Chris | 46 | 60 | +14 ** | 42 | 56 | +14 ** | 26 | 39 | +13 ** | 29 | 40 | +11 ** | 33 | 46 | +13 ** |
| | n = 24 | | | n = 25 | | | n = 19 | | | n = 22 | | | n = 18 | | |
| Denise & Ellen | 55 | 65 | +10 ** | 55 | 59 | +4 n.s. | 32 | 33 | +1 n.s. | 43 | 45 | +2 n.s. | 30 | 37 | +7 * |
| | n = 15 | | | n = 13 | | | n = 18 | | | n = 19 | | | n = 18 | | |

*Mean percentage pre- and post-test scores for each test during each year of the project for paired teachers.*
*Key: ? represents p<0.1; * represents p<0.05; ** represents p<0.01. (dependent t-test).*
*(† Note that Alan did not take the year 1 algebra test)*

This appears to show that the teaching activities did have some success where the teachers already had a predisposition to work in ways which were sympathetic to the reflective approach; but they did not where the teachers were unable to adopt more connectionist approaches.

The results from the individual tests, aggregated across the four teachers in the final sample, present a sobering view of how difficult it is to achieve substantial learning gains in the FE environment. We have shown that students enter FE with many profound gaps in their understanding of basic mathematical concepts and that lecturers' 'normal' approaches to teaching make little impact on this state of affairs. The reasons for this appear to be due to poor attendance, motivation and passivity among students and an overwhelming emphasis on rapid syllabus 'coverage' using transmission methods of teaching, even among teachers who hold beliefs about learning which conflict with this practice.

Shifting this state of affairs has proved difficult in FE. Significant, (yet modest) gains have only been observed with teachers who are sympathetic with 'connectionist' beliefs about learning when they are also provided with resources consistent with these beliefs. 'Transmission' belief systems appear robust even in the face of evidence of their ineffectiveness. This appears equally true for both teachers' and students' views of learning; many students *expect* to perform imitatively and we have witnessed several negative reactions when teachers attempt to modify their approach. We suspect that more significant gains will only prove possible over a longer time scale, when the belief systems of all participants (teachers and students) are addressed more explicitly. The FE sector has proved a difficult and unfashionable environment in which to conduct research, yet its importance is such that further work is urgently needed, particularly with students who have been failed by our secondary education system.

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## Abstract (in Norwegian)

Prosjektet som er grunnlaget for denne artikkelen prøver å skape og gjennomføre et utdanningsprogram som sikter å framelske og øke mengden av aktiviteter som er velegnet til reflekterende tenkning blant elever i aldersgruppen 16 til 19 år ved Further Education colleges i England. Artikkelen eksemplifiserer dette gjennom å diskutere en rekke diskusjonsaktiviteter. Disse er konstruerte med tanke på få elevene til å fokusere på metacognitive aspekter i tilknytting til de begrepsmessige hindringer som forfatteren har identifisert. En test av elevene er gjennomført før og etter et undervisningssekvens. Resultatene er diskutert med bakgrunn i en klasseromsobservasjon. Fire lærere er studert i deres arbeid med materiellet. To er klassifiserte

som "konstruktivistiske- ", disse er opptatt av metacognitive undervisning og to er klassifiserte som "transmission"-lærere. De siste er mest opptatt av forklaringer og øvelser.
Prosjektet gikk over to år, det første året uten direkte innblanding fra forfatteren mens diskusjonsaktiviteter ble introdusert det andre året. Det var liten mellom elevene til de to lærerene det første året. I det andre året bidro diskusjonsaktivtetene til signifikant framgang blant elevene til konstruktivistlærerene mens minimal framgang ble observert blant elevene til den andre typen lærere.

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***Research interests***

The design of teaching; reflection, metacognition and awareness; assesment.

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