Pupils' views of mathematics teaching in Finland and Tatarstan

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The focus of this comparative survey was the following research questions: What are pupils' views of mathematics teaching in each country? What are the differences and similarities in these views between pupils in Finland and in Tatarstan, Russia?

In this study, data were gathered with the help of a questionnaire. The questionnaire consists of 32 structured statements about mathematics teaching for which the pupils were asked to rate their views on a 5-step scale. The Finnish sample comprised 255 pupils, and the Tatar sample 206 pupils. Our data has been gathered with a non-probabilistic convenience sampling.

The main results of our survey are, as follows: Generally, pupils' views of mathematics teaching and learning in Finland and Tatarstan are rather far from similar. An investigation of the differences between pupils' answers across the two countries also showed views that are characteristic for each country. For pupils in Tatarstan, the most characteristic might be the following views: the importance of exact reasoning and explanations, and the value of making strong demands on pupils. For pupils in Finland, the characteristic views seem to be as follows: a calculation orientation in learning mathematics, and the value of strict discipline and of depending on the teacher.

When discussing problem orientation, we may conclude that in both countries mathematics teaching is striving toward a problem orientation, but with a different emphasis. In the case of independent work, the results seem to indicate that the orientation in question for pupils both in Finland and in Tatarstan is similar, but not very strongly favored. The pupils' views of the demands on them and on their teachers in mathematics seem to be much stronger in Tatarstan than in Finland.

Finland and Tatarstan (in Russia) are two countries within Europe, each having two official languages. In Finland, they are Finnish and Swedish, and in Tatarstan, the Tatar language and Russian. Today's world requires of both countries that their schools have a special language program to enable them to communicate with other countries which might restrict the study of other school subjects, e.g. mathematics, in a similar way.

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1.1 Beliefs and belief systems

Within a constructivist framework (e.g., Ahtee & Pehkonen, 1994; Davis, Maher, & Noddings, 1990) as a base for teaching and learning mathematics, a knowledge of teachers' and pupils' mathematical beliefs is vital if their mathematical behavior is to be understood (e.g. Noddings, 1990, p. 14). Over the last decade, many studies of pupils' belief systems have been undertaken (e.g., Frank, 1985; Pehkonen, 1992; Schoenfeld, 1985, 1989; Zimmermann, 1991). Underhill (1988) has compiled a review of research results on pupils' mathematics-related beliefs. In the reviews by McLeod (1989, 1992) of affect in mathematics education, one can find more information on the research results. An overview of pupils' mathematics-related beliefs, from a European viewpoint, was provided by Pehkonen (1995a, 1995b).

Research has revealed that knowing the right facts, that is, algorithms and procedures, does not necessarily guarantee success in solving mathematical problems. There are other factors–such as decisions the solver makes and the strategies he or she uses, as well as his or her emotional state when solving mathematical tasks–that have a major effect on the solver's performance (Garofalo, 1989; Schoenfeld, 1985). "Purely cognitive" behavior is rare. Belief systems shape cognition, even though some people may not be consciously aware of their beliefs (Schoenfeld, 1985).

The central concept: belief

Interest in beliefs and belief systems started mainly in the 1970s through developments in cognitive science. But the basis of these ideas was first developed in social psychology. Although beliefs are popular as a topic for study, the theoretical concept of *belief* has not been dealt with thoroughly. The main difficulty has been an inability to distinguish between beliefs and knowledge. This difficulty has not yet been clarified (e.g., Abelson, 1979; Thompson, 1992). (For more information on these problems see, e.g., Pehkonen, 1995a, 1995b.) Some researchers have argued that it is not important to distinguish between knowledge and beliefs, but rather to find out how belief/knowledge systems affect teachers' and pupils' behavior in mathematics classes (Thompson, 1992).

Here we understand an individual's *beliefs* as stable subjective knowledge (and feelings) about a certain object or concern for which one cannot find any tenable ground in objective considerations. The notion of a belief system is a metaphor used to describe how one's beliefs are organized (Green, 1971).

Beliefs and belief systems are affected by the way people understand themselves and their surroundings. Belief systems can be seen to be developed from simple perceptual beliefs or beliefs based on authority–via new beliefs, expectations, conceptions, opinions, and convictions–to a general outlook on life (Saari, 1983). Thus, for example, conceptions are higher order beliefs. They are based on reasoning processes for which the premises are conscious. Therefore, conceptions can be seen to have grounds; they are justified and accepted at least for the person him- or herself. One variation of conceptions is *views*. Views are very close to conceptions, but they are more spontaneous and have a stronger affective component. Conceptions are usually more considered than views, and their cognitive component is stronger.

Meaning of beliefs

The central position of beliefs for the successful learning of mathematics has been pointed out by many mathematics educators. Baroody and Ginsburg (1990) state that beliefs can have a powerful impact on how children learn and use mathematics. Both Schoenfeld (1985) and Silver (1985) have pointed out that pupils' beliefs about mathematics may form an obstacle to solving nonroutine problems and to effective mathematics learning. Also, Borasi (1990) stresses that pupils who have rigid and negative beliefs of mathematics and its learning easily become passive learners, who place more emphasis on remembering than on understanding in learning. In her dissertation, Martha Frank (1985) introduced a schematic picture of some factors affecting pupils' problem-solving behavior. Since most of the factors act via pupils' belief systems, we have organized the components of the scheme in another manner (Figure 1.1). This scheme, in fact, shows the regulatory character of a pupil's view of mathematics (i.e., his or her mathematical belief system).

Beliefs play a central role as a background factor for pupils' thought and action. Pupils' mathematical beliefs usually act as a filter that influences almost all their thoughts and actions concerning mathematics. Pupils' prior experiences of mathematics affect their beliefs completely–usually unconsciously. When they use their mathematical knowledge, their beliefs are also highly involved. In contrast, a pupil's motivation and needs as a learner of mathematics are not always connected with his or her mathematical beliefs. Additionally, there are many societal mathematical beliefs, perhaps mythical–for example, that mathematics is merely calculation (for more myths

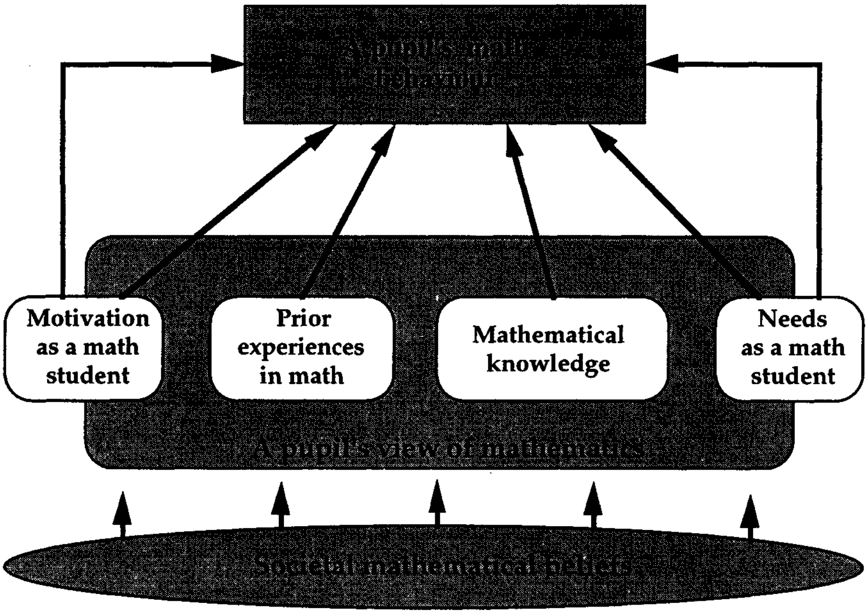

Fig. 1.1. Factors affecting pupils' mathematical behavior.

see, e.g., Paulos, 1992)–that also influence pupils' mathematical behavior via their belief system.

The scheme of Figure 1.1. shows a situation in which a pupil's mathematical performance is influenced by several factors that operate through a system or a network of his or her own beliefs. This is only part of the truth, however; the situation is, in fact, much more complex. Pupils act within a very complex network of influences. Underhill (1990) speaks of a web of beliefs. For example, a pupil's mathematics teacher, classmates, friends, parents, relatives, and teachers of other subjects all have their own views of mathematics and its teaching and learning. These beliefs affect the learners' beliefs more or less, and usually in a contradictory way.

Results of an international comparison

The question of the international comparison of pupils' mathematical beliefs seems to be an almost unexplored field. The main question here is: "Are there essential differences in conceptions of mathematics teaching in different countries?" We know that mathematics can be understood as a universal discipline. So, the question arises as to whether pupils' conceptions of mathematics and of mathematics teaching and learning are also universal, or whether they are perhaps culture bound.

About six years ago, a project entitled International Comparison of Pupils' Mathematics-Related Conceptions was started (Pehkonen, 1995a). Prior to this project, from which some preliminary results have been published (Graumann & Pehkonen, 1993; Pehkonen, 1993b, 1994, 1996; Pehkonen & Tompa, 1994), there was almost no research into variations between pupils' beliefs on an international scale. Only in the International Mathematics Studies (Husén, 1967; Kifer & Robitaille, 1989) were pupils' responses to some questions on the affective domain dealt with in a background questionnaire. For example, the SIMS study indicated that there are large differences between countries on measures of mathematical beliefs and attitudes.

1.2 A look at mathematics teaching in each country

Mathematics teaching in Finland and Tatarstan is dealt with here by exploring some questions. Firstly, the respect of mathematics and mathematics teachers in society is discussed. Secondly, the current mathematics curriculum is dealt with. Special attention is given to the strong and weak points of the curriculum, for example, the number of mathematics lessons and the opportunities offered to pupils with special needs (e.g., talented, low attainers). Thirdly, the realization of teaching in classrooms, including textbooks and other teaching materials (audio-visual equipment, computers, etc.) is discussed. The description given here is due to the authors' personal knowledge from their country.

It is worth noting that mathematics teaching is developing extensively in almost all countries, including Finland and Tatarstan. The situation described here was valid during the administration of the questionnaire, that is, in Finland in 1989-1990 and in Tatarstan in 1994-1995.

Finland

Respect for mathematics is not very high in Finland. For example, teachers do not earn as much as other persons with the same academic degree; one could say that teachers belong to the "academic middle class." In addition, insufficient numbers of students at universities are willing to enroll in the mathematics teacher education program.

The strong points of the curriculum (Anonymous, 1985) are the objectives; they are really good. They include, in addition to numeracy, the promotion of problem-solving skills and the fostering of creativity, as well as applying mathematics in everyday life and developing positive attitudes. Furthermore, the teacher has considerable independence in his or her teaching, for example, in choosing how to teach, what content to emphasize, and in what order. So, an independent teacher with insight who is not merely following the textbook has an opportunity to teach in as "modern" a fashion as he or she likes. But unfortunately, most teachers are very textbook dependent.

The weak points in the mathematics curriculum are as follows: All pupils are to be taught in heterogeneous classes throughout the compulsory grades (Grades 1-9; ages 7-15). In other words, in a ninth-grade mathematics class, you could have, at the same time, a future mathematics professor and some pupils who are not yet able to remember (and use) their multiplication tables. And the teacher is to try to teach them all according to their abilities. The current syllabus is mostly concerned with abstract and theoretical mathematics, and it is the same for all pupils. At the same time, the number of mathematics lessons in the 9-year comprehensive school is the lowest in Europe–perhaps the lowest in the world–according to a Unesco (1986) report, e.g. about half of the number of lessons in Switzerland during the nine first years of school.

The curriculum in force attempts to give equal opportunities to all. In recent years, the question of talented pupils has been raised. They need also some challenges, but in the curriculum there is no extra place for them. Help for the low attainers is arranged through special instruction, for which there are certain resources in the curriculum. Classroom teaching is mostly mechanistic and hurried; teachers work under a continuous press of time. Pupils spend most of their time filling in workbooks or calculating mechanically solvable tasks in their notebooks. The textbooks are published in the pupils' native language (Finnish or Swedish). They emphasize training in performing calculations, and that extends to the upper grades (Grades 7-9; ages 13-15). Hardly any written teaching material is available (in Finnish) other than textbooks. There is plenty of material available in other languages, but very few teachers use these materials–the language is a big obstacle. Most schools have enough overhead projectors, film projectors, television sets, and computers, but there are very few suitable films and very little computer software.

Tatarstan

Since Tatarstan is a part of Russia, it is given a description of mathematics teaching in Russia. Respect for mathematics was very high in the U.S.S.R. during the 1960s when space technologies and cybernetics were developing rapidly. Since the middle of the 1970s, however, respect for mathematics has gradually decreased. A mathematics teacher, like all other school teachers, earns very little: about half the salary of, say, a tram or bus driver.

Up to the beginning of the 1990s, the mathematics curricula in the U.S.S.R. were very extensive. As a result, only mathematically gifted and diligent pupils were able to complete the mathematics course satisfactorily. Even many teachers could not properly orient themselves in the content of the mathematics curricula for upper secondary schools. Today, there is still a single compulsory curriculum in mathematics for all of Russia. In addition, the Ministry of Education demands that a high percentage of pupils get satisfactory marks in the mathematics course. As a result, weak pupils have been promoted to the next grades with unsatisfactory knowledge. No special instruction exists for low attainers.

The curricula are changed rather often. When that happens, teachers all over Russia are supplied with instructions published in the official magazine *Mathematics in School* and with prescriptions for handling new elements or new points of attention. The strength of the curriculum is that most school graduates possess well-developed skills in calculation and in the fundamental rules of mathematics, such as using the Pythagorean theorem and solving quadratic equations. Classroom teaching is generally oriented not toward pupils' independent work but toward their passive listening to explanations. Insufficient attention is given to problem solving. Superfluous requirements are imposed on the accuracy of pupils' written notes and on other formalistic aspects of instruction. Generally, the formal demands on teachers and pupils are very great.

Throughout Russia, teachers are obliged to use the one existing textbook (algebra, geometry, etc.) for teaching each mathematical discipline at each grade level. If a textbook is replaced by a new one, it is changed simultaneously all over the country. Most of the methodological handbooks for teachers are prescriptive. In former times, detailed plans for teaching each theme of the curriculum were regularly published in the magazine *Mathematics in School* and recommended to teachers. On the other hand, many excellent popular mathematical books and the interesting magazine *Kvant* are published in Russia. In most schools, there are overhead projectors, but some schools, especially in rural regions, do not have enough television sets, modern computers, or calculators. Furthermore, there is a shortage of proper computer software.

The conditions of mathematics teaching in Tatarstan are essentially the same as in the rest of Russia (and the entire U.S.S.R. before 1991). The new aspect is that today in some urban schools, teaching is conducted in the Tatar language, which was not possible during the Communist era. At that time, non-Russian pupils could study in their native language only in villages.

The mathematical results of pupils in the former U.S.S.R. seem to have been above the average international level. For example, the Soviet team won about half of all the International Mathematical Olympiads in which it competed, and according to these results was far ahead of the teams from all other countries. Tenth graders from Tatarstan (formerly Kazan) were sometimes part of the Soviet team participating in these olympiads, and individuals won third-place medals.

Who are the seventh graders?

In Finland, there is a comprehensive school system in which pupils move after the primary level (Grades 1-6; ages 7-12) to the upper level (Grades 7-9; ages 13-15) of a comprehensive school. In primary school, the pupils do not have individual subject teachers; their class teacher usually teaches every subject, including mathematics, to the class. But the pupils have individual subject teachers at the higher school levels. The experiences of Finnish seventh graders in this study are mostly based on the impressions they formed at the primary level.

In Tatarstan, the primary school consists of three grades (Grades 1-3, or if pupils enter the school after one year of preschool study in kindergarten, Grades 2-4). Primary school children are 7-10 years old. The lower secondary school (Grades 5-9, pupils aged 11-15 years), where teachers teach separately such subjects as mathematics and literature, begins after the primary school. In most schools, there is no fourth grade. Therefore, the experiences of the seventh graders include more than two years in middle school, where the mathematical curriculum for Grades 5-6 includes natural, rational and integer numbers and some elements of geometry (e.g., symmetry and perpendicular and parallel lines).

Why have we selected just seventh graders? According to educational psychologists (e.g. Good & Brophy, 1977, pp. 309-311), children at this age level are in the transition phase from the (Piagetian) stage of concrete operations to the stage of formal operations. They are already developed critically enough to consider and to perceive the world outside, for example, the way mathematics is taught to them.

2 The Survey

The theoretical framework for the survey is constructivism (e.g., Ahtee & Pehkonen, 1994; Davis et al., 1990). In the understanding of learning that is compatible with constructivism, it is essential that a learner actively works with the new knowledge in order to be able to elaborate his or her knowledge structure. Thus, the meaning of pupils' beliefs (subjective knowledge) concerning mathematics and its learning is emphasized as a regulating system of their knowledge structure. It is most important, therefore, for the teacher to know what his or her pupils think about the subject to be learned and what kind of previous knowledge they have. How learners act and think during the learning process is strongly affected by their belief systems (cf. Fig. 1.1). Hence, a knowledge of pupils' conceptions of teaching and learning forms a necessary base for a teacher's decisions in organizing classroom teaching.

2.1 Research objectives

The present study was an exploratory study aimed, in the first place, at the discovery of interesting research questions. Its purpose was to survey the views of Finnish and Tatar seventh graders concerning mathematics and mathematics teaching. This report details part of a larger study (Pehkonen, 1995a) whose main question is: Are there essential differences in pupils' views of mathematics teaching in different countries?

The focus of the survey was the following research questions, which were derived from the purpose:

1. What are pupils' views of mathematics teaching in each country?

2. What are the differences and similarities in these views between pupils in Finland and in Tatarstan?

2.2 Method

Since this study belongs to the first part of the pilot research within the research project "International Comparison of Pupils' Mathematical Beliefs" (see Pehkonen, 1995a), data were gathered with the help of a questionnaire.

Questionnaire

The questionnaire was developed for another research project, Open Tasks in Mathematics (Pehkonen & Zimmermann, 1989, 1990), and its first version was in German. Its purpose was to clarify pupils' views of mathematics teaching. The questionnaire (see Appendix) consists of 32 structured questions about mathematics teaching, that is, statements for which the pupils were asked to rate their views on a 5-step scale (from 1 = completely agree to 5 = completely disagree).

Administration of the Questionnaire

The questionnaire was translated into Finnish and Tatar by the authors. The Finnish sample comprised 15 seventh-grade classes from Helsinki and Järvenpää (a small town about 40 km north of Helsinki), altogether 255 pupils. The teachers gathered the information in the middle of autumn 1989, having pupils fill in the questionnaire at the end of their mathematics lesson. The pupils' questionnaire in Finland was one of the pretests in the Open Tasks in Mathematics project (Pehkonen & Zimmermann, 1990).

The Tatar sample comprised 8 seventh-grade classes from two schools. One school was the Tatar Gymnasium No. 2 with pupils mostly of rural origin, and the other was the Russian School No. 44 with pupils mostly of urban origin. Altogether, there were 206 pupils. Neither of the schools was a "mathematical" school. The teachers gathered information in the spring of 1995 by having the pupils fill in the questionnaire at the end of their mathematics lessons.

It should be noted that our sample is not trying to be a random sample with which one might generalize the results to the whole population (here, pupils in the same age group in our countries). Using the language of Cohen & Manion (1994, p. 91), one may say that our data has been gathered with a non-probabilistic convenience sampling.

2.3 Data analysis

To analyze the results of the questionnaire, the statistics used were mainly percentages.

The concept of consensus level

People differ in expressing their position regarding a statement: Some like to take an extreme position, whereas others tend to respond carefully. But usually their attitude (positive or negative) is clear. Therefore, for further analysis of the responses, we reduced the original response scale (1-2-3-4-5) by combining the two response values at the extreme ends of the scale, which yields a three-step scale of *agree* (1 or 2), *neutral* (3), and *disagree* (4 or 5). This might dimish some of the tendencies in the data, but on the other hand it offers us a solid base to begin with.

In a previous paper (Pehkonen, 1993a), the concept of *consensus level* was introduced. In the analysis and interpretation of the responses, the terminology for the consensus level was used as follows. We say that the responses to a statement are in

- *complete consensus,* if at least 95% of the test subjects' views were at the same extreme end of the scale;

- *consensus,* if at least 85% but less than 95% of the test subjects' views were at the same extreme end of the scale;
- *almost consensus,* if at least 75% but less than 85% of the test subjects' views were at the same extreme end of the scale;
- *lack of consensus,* if less than 75% of the test subjects' views were at either of the extreme ends of the scale.

The percentage of consensus, that is, the percentage of responses showing agreement (1 = completely agree or 2 = agree) or disagreement (5 = completely disagree or 4 = disagree), is used to describe the consensus level of the test subjects' agreement (or disagreement) with a statement.

Measuring the significance of differences
Since the responses were on an ordinal scale, we used nonparametric statistics to test the statistical significance of differences between the countries. The Mann-Whitney U test we used is equivalent to the ordinary (parametric) *t*-test on an interval scale. The StatView program on the Macintosh computer was used for the analysis. When using Mann-Whithey U test, we have stayed on the unreduced scale (5-point scale). This caused some pecularities, e.g. in item 4 which we will discuss later on.

3 Main analysis of results

Firstly, we give an overview of the results so as to see where we might find interesting comparison situations. Secondly, the consensus percentages of the responses to the statements are analyzed. Finally, we examine the main focus of the study: What differences did we find between the countries?

3.1 Overview of results

In Table 3.1, we present all the consensus percentages in Finland and Tatarstan separately and then together. These are, as a rule, the agreement percentages; if the disagreement percentage is larger, it is also given in parentheses. We can make some observations on the content of Table 3.1, using the language explained in Section 2.3. In Finland, for no single item was there complete consensus. There was consensus for five items: Item 19 (94%), Item 8 (91%), Item 1 (89%), Item 24 (89%), and Item 31 (85%). And there was almost consensus for eight more items (3, 4, 9, 11, 13, 15, 22, 30). In Tatarstan, too, there was not complete consensus for any single item. There was consensus for three items: 16 (89%), 19 (89%), and 5 (88%). Furthermore, almost consensus was found for ten more items(1, 4, 6, 9, 11,15, 18, 25, 29, 30). The largest disagreement percentages were found in Finland for Items 20 (83%), 7 (68%) and 2 (64%); in Tatarstan they were found for Items 2 (71%), 20 (40%), and 17 (36%). Considering the results, we may assume that a high consensus level in the responses to a certain statement indicates some kind of "universality" in the corresponding view. In the case of dual consensus (i.e., consensus in both countries), we say that such a statement represents a universal view for both these countries. For dual consensus, it may be so, but the cases of separate consensus are more complicated. If the consensus percentage for a statement was high in one country, but not in the other, we considered such a statement to be characteristic for the country. For example, in Item 16 (everything reasoned exactly), the Tatars had an agreement percentage of 89% and the Finns only 52%. Thus, we say that Item 16 is characteristic for Tatarstan. On the other hand, if we look at Items 13, 22, and 27, we see the lack of consensus. These items are neither universal nor characteristic. Nevertheless, since levels of agreement both in Finland and Tatarstan were higher than 70%, one might consider these items as candidates for a universal view.

Thus, the thorough observation of the consensus percentages in Table 3.1 allows us to distinguish candidates for universal or characteristic views. For more definite decisions, however, the combining of various approaches to analyzing the results given in Table 3.1 would be useful.

3.2 Analysis by consensus level

Dual consensus

We consider first those statements in which the consensus percentage in both countries was over 75%. In such cases, we refer to *dual consensus* (Table 3.2). Consensus was found in both countries for only one of the items, Item 19 (tasks have practical benefit): 94% in Finland and 89% in Tatarstan. For Item 1 (mental calculations), we found consensus in Finland and almost consensus in Tatarstan (89% and 84%). There are five more items where there was almost consensus both in Finland and in Tatarstan. These seven items (Table 3.2) indicate views that are universal for both countries.

Separate consensus

Now we concentrate on such statements in which consensus could be found only in one country. In such a case, we say there was *separate consensus.*

Table 3.1. Agreement percentages of the statements. (If the disagreement percentage is larger, it is given in parentheses.)

No.	Statement	Finland	Tatarstan
1	mental calculations	89	84
2	right answer more important than way	16 (64)	2 (71)
3	mechanical calculations	77	47
4	pupil can guess and ponder	79	78
5	everything expressed exactly	30 (48)	88
6	drawing figures	67	81
7	right answer quickly	11 (68)	36
8	strict discipline	91	63
9	word problems	76	80
10	there is procedure to follow exactly	35 (39)	53
11	all pupils understand	80	80
12	learned by heart	29 (50)	49
13	pupils put forward own questions	76	71
14	pocket calculators	72	35
15	teacher helps when difficulties	76	79
16	everything reasoned exactly	53	89
17	different topics taught and learned separately	36	22 (40)
18	as much repetition as possible	61	79
19	tasks have practical benefit	94	89
20	only talented pupils can solve	5 (83)	20 (36)
21	it could not always be fun	60	42
22	calculations of areas and volumes	76	70
23	it demands much effort	35	66
24	there is more than one way	89	53
25	learning games	66	84
26	teacher explains every stage exactly	72	54
27	pupils solve tasks independently	73	71
28	constructing of concrete objects	37	56
29	as much practice as possible	64	81
30	all will be understood	75	84
31	pupils are working in small groups	85	69
32	teacher tells exactly what to do	53	44

Table 3.2. Agreement percentages for statements with dual consensus.

No.	Statement	Finland	Tatarstan
19	tasks have practical benefit	94	89
1	mental calculations	89	84
11	all pupils understand	80	80
30	all will be understood	75	84
4	pupil can guess and ponder	78	78
9	word problems	76	80
15	teacher helps when difficulties	76	79

Finland: In Table 3.3, the statements are given in which the agreement percentage reached the consensus level in Finland, but not in Tatarstan. In the table, we see that the Finnish pupils saw the importance of strict discipline (No. 8) and understood that there is more than one way to solve problems (No. 24). Clearly, these views are characteristic for them. Also, the Finns, more than the Tatars, were in favor of working in small groups (No. 31) and of doing calculations with paper and pencil (No. 3). The latter is in harmony with the generally stronger calculation orientation of the Finnish pupils. These views are characteristic in Finland but not in Tatarstan.

For Items 13 (pupils put forward own questions) and 22 (calculations of areas and volumes) in Table 3.3, as well as in the Appendix, the differences between the two countries are not statistically significant. Thus, the corresponding views are not characteristic of either country. Moreover, these views do not seem to be universal. The Finns disagreed with Item 20 (only talented pupils can solve) much more strongly than the Tatars. Therefore, a characteristic view for Finns is that "mathematics is not only for the talented pupils."

Tatarstan: Here we consider the statements for which the Tatars reached consensus, but not the Finns (Table 3.4). In two items reflecting exact reasoning and explanations (Nos. 16 and 5), the differences between pupils in the two countries were surprisingly large, and the corresponding views are certainly characteristic of pupils in Tatarstan. In Finnish schools, as a rule, teachers stress calculation processes and problem solving, thus neglecting exact reasoning because of time restrictions (there are only 3 mathematics lessons a week).

It is strange that the pupils in Tatarstan were more interested in games than pupils in Finland. This result might be explained by the level of boredom of mathematics lessons for less gifted pupils in Tatarstan. The Tatar pupils, more than the Finnish ones, agreed with statements (Nos. 6, 18, and 29) that may reflect a mechanistic approach to the mathematics teaching cultivated in Soviet schools.

Dual nonconsensus

In the final category, we briefly deal with the statements that did not exceed the consensus levels (Table 3.5). The lowest agreement percentage was found for Item 2 (right answer more important than way). Therefore, its disagreement percentages are also given, but they do not exceed the limit of consensus. Furthermore, a very low agreement in both countries was found for Items 7, 17, 12, 10, 28, and 32. Note that in Items 10 and 12, which are related to mechanistic learning, the level of agreement was clearly higher (the differences are statistically significant) in Tatarstan than in Finland, but not as high as one might

Table 3.3. Agreement percentages for statements with a separate consensus in Finland. (If the disagreement percentage was greater, it is given in parentheses.)

No.	Statement	Finland	Tatarstan
8	strict discipline	90	63
24	there is more than one way	89	53
31	pupils are working in small groups	85	69
20	only talented pupils can solve	5 (83)	21 (36)
3	mechanical calculations	77	48
22	calculations of areas and volumes	76	70
13	pupils put forward own questions	76	70

Table 3.4. Agreement percentages for statements with a separate consensus in Tatarstan.

No.	Statement	Finland	Tatarstan
8	strict discipline	90	63
24	there is more than one way	89	53
31	pupils are working in small groups	85	69
20	only talented pupils can solve	5 (83)	21 (36)
3	mechanical calculations	77	48
22	calculations of areas and volumes	76	70
13	pupils put forward own questions	76	70

Table 3.5. Agreement percentages for statements with a dual nonconsensus. (If the disagreement percentage was greater, it is given in parentheses.)

No.	Statement	Finland	Tatarstan
27	pupils solve tasks independently	73	71
26	teacher explains every stage exactly	72	54
14	pocket calculators	72	35
21	it could not always be fun	61	42
23	it demands much effort	35	66
32	teacher tells exactly what to do	52	44
28	constructing of concrete objects	38	53
10	there is procedure to follow exactly	35	53
12	learned by heart	29	50
17	different topics taught and learned separately	35	21
7	right answer quickly	11	36
2	right answer more important than way	16 (64)	2 (71)

expect. On the other hand, the pupils in Finland showed a slightly more positive attitude on Items 32 and 26, which are related to dependence on the teacher.

In Items 23 (it demands much effort) and 21 (it could not always be fun), the differences between responses in the two countries are statistically significant. These differences might be explained by greater demands by pupils in Tatarstan. The Finnish pupils were twice as favorable toward using calculators (No. 14) as the Tatars. In Item 27 (pupils solve tasks independently), the agreement percentages in both countries were close to the lowest consensus level. Thus, the item might be considered as a candidate for a universal view.

3.3 Focus on Differences Between Finland and Tatarstan

Differences in consensus level

In Table 3.6, we arrange items according to the absolute values of differences in the consensus percentages between pupils in Finland and Tatarstan. The difference in consensus percentages is taken according to the order used in Table 3.1 (Finland — Tatarstan). Thus the sign before a difference shows in which country the percentage was larger.

We may suppose that the items with differences ≥ 25% indicate important specific traits of teaching and learning in each country. Therefore, we discuss only that group.

The greatest difference was found for Item 5 (everything expressed exactly), which indicates that the pupils in Tatarstan believed much more strongly than those in Finland that everything should be explained as exactly as possible. A large difference was also found for Item 16 (everything reasoned exactly). Statements 5 and 16 are very close in meaning, but Item 5 is stated more categorically ("as exactly as possible"). That is probably why the pupils in Finland showed less agreement on this item than on Item 16 (consensus percentages 30% and 52%). Large differences, showing more positive attitudes of pupils in Tatarstan than in Finland, were also found for Items 23 (it demands much effort) and 7 (right answer quickly). These figures might suggest the strength of the demands on pupils in Tatarstan schools.

The greatest difference for which the pupils showed more positive attitude in Finland than in Tatarstan is seen in Item 14, which concerns the use of calculators. The pupils in Tatarstan did not believe that good teaching of mathematics includes the use of calculators, perhaps because their teachers rarely used calculators in the classroom. The responses of the pupils in Tatarstan to Item 24, together with their responses to Item 7, might be explained by the fact that pupils in Tatarstan have to solve a lot of problems in each mathematics lesson. Therefore, they are compelled to solve them very quickly, and have no time to seek alternative solutions. The demands on pupils in Tatarstan are very great, which is reflected in their responses to Items 23 (it demands much effort) and 20 (only talented pupils can solve).

Table 3.6. The geatest differences in agreement percentages in Finland and Tatarstan. (The differences are taken in the order Finland — Tatarstan.)

ITEMS WITH DIFFERENCE OF 25% OR MORE		diff.
5	everything expressed exactly	—58
14	pocket calculators	+37
16	everything reasoned exactly	—37
24	there is more than one way	+36
23	it demands much effort	—31
3	mechanical calculations	+29
8	strict discipline	+27
7	right answers quickly	—25

Table 3.7. Frequency (in 10% intervals) of neutral responses by country and of differences by direction.

	Number		Difference	
Percent	FIN	TAT	FIN > TAT	FIN < TAT
0 to 9	7	2	7	6
10 to 19	9	12	4	11
20 to 29	13	9	—	2
30 to 39	2	5	—	1
40 to 49	1	4	—	—

Neutral responses

An analysis of the undecided (or neutral) responses makes interesting addition to our results. In Table 3.7, we have classified the percentages of the neutral responses into intervals of 10. The frequency data by country show that the Tatarstan pupils seemed to have greater difficulty deciding on their opinion than their Finnish counterparts. Among the Finnish percentages, there are many small numbers, whereas the Tatar percentages are much larger. Table 3.7 also contains the number of differences in percentages for these two countries. For only one item was the difference zero. For two thirds of the items in Tatarstan, the percentage of undecided (neutral) responses was larger than the corresponding Finnish percentage. One can see the same trend in the differences as in the data for each country. Thus, we might conclude that the Finns were more ready than the Tatars to express their opinion.

The pupils in Tatarstan were more restrained in self-expression than their counterparts in Finland. An explanation might be that the Tatar pupils were rather low in self-confidence. One might then ask why their self-confidence was so low. That question could be explained by two possible factors: (a) the large number of difficult problems pupils have to solve during each lesson, and (b) an authoritarian style of teaching that negatively influences the development of pupils' self-confidence. Of course, we might always use an alternative explanation that Russian children have not enough experience to answer such type of questionnaires.

The statements with a very high percentage of neutral responses (i.e., over 40%) are of special interest; therefore, we consider them in Table 3.8. For these statements, almost half of the pupils (in one country) were not able to decide, whether they agreed or disagreed. Could it be that these situations are impossible for pupils to imagine? Do they perhaps belong in pupils' minds to a world other than mathematics teaching? In the case of Item 3 (mechanical calculations), one may imagine that the pupils have had difficulty with the word *real* in the stem of the questionnaire. In the statements where the percentage of neutral responses was especially low–less than 10% (i.e., Items 1, 8, 9, 11, 19, and 24)–the responses reached the consensus level, as a rule, or were at least very near consensus.

3.4 Similarities and differences in pupils' views

The analysis of consensus percentages shows the following results:

1. With regard to similarities, we can conclude that Items 19 and 1 may indicate universal views. And perhaps five more items (4, 9, 11, 15, and 30) reflect universal views in both countries. With the help of these items, we might discern some universal features of mathematics teaching and learning in Finland and Tatarstan from the pupils' viewpoint: In the area of mathematical content, mental calculations and word problems have a regular place. Teaching should strive for understanding, and pupils expect their teachers to help them attain this aim. Pupils should nonetheless be given opportunities to guess and ponder. Moreover, there should be practical benefits to learning mathematics.

In addition, there are some items–for example, Item 27 (pupils solve tasks independently)–that may be considered almost universal. A deeper investigation of differences between the countries, however, would appear to be useful.

2. An investigation of the differences between pupils' answers in the two countries also shows some characteristic views for each country.

Table 3.8. Statements with a high percentage of neutral responses (in at least one country).

No.	Statement	Finland	Tatarstan
3	mechanical calculations	18	**46**
14	pocket calculators	20	**46**
17	different topics taught and learned separately	**42**	39
20	only talented pupils can solve	12	**43**
21	it could not always be fun	19	**40**

For the pupils in Tatarstan, the most characteristic were the following views:

- the importance of exact reasoning and explanations, and
- the value of making strong demands on pupils.

For the pupils in Finland, characteristic views were as follows:

- a calculation orientation in learning mathematics, and
- the value of strict discipline and of depending on the teacher.

Further, the analysis also revealed the influence of the low self-confidence of pupils in Tatarstan on their questionnaire responses.

3. To some extent the results confirmed what could have been expected: Views about mathematics teaching and learning reflect a more mechanistic character in the lessons in Tatarstan and more freedom in classrooms in Finland. The agreement on items related to mechanistic learning in Tatarstan, however, was not as high as one might expect. In solving problems, for example, the pupils in Tatarstan believed that the method of solution is more important than the answer.

It might be of general interest to investigate more thoroughly the items related to the problem orientation of teaching. Two additional features of the pupils' views especially stand out: the strength of demands on pupils and teachers in Tatarstan, and teacher dependence in Finland. It might be interesting, therefore, to investigate more thoroughly two more groups of items: those related to pupils' independent work, and those reflecting the strength of demands.

4 Deeper analysis of some questions

In this section, we attempt to analyze responses to some groups of questions related to three interesting aspects of teaching and learning mathematics: problem orientation, pupils' independent work, and demands on teachers and pupils. These aspects are important for effective teaching and for promoting creativity in pupils. The analysis here is based on the original five-step scale.

4.1 Problem orientation

Perhaps the most discussed aspect of mathematics teaching today is problem solving. The key question here is how could we realize problem solving in teaching practice? One possible solution is so-called problem-oriented teaching, that is, using problem solving as a teaching method.

With this in mind, we chose from the questionnaire (see Appendix) Items 2, 4, 24, 26, and 27 to represent a problem orientation in mathematics teaching. Table 4.1 was constructed to contain agreement percentages (fully agree and agree) in Finland and Tatarstan. Since problem orientation in Items 2 and 26 is characterized by disagreement and not by agreement, those two items were *converted;* that is, we considered disagreement percentages of 4 and 5 responses.

Statistically-significant differences in agreement percentages were found for four of these five items. In three of the items, the pupils in Tatarstan showed a more positive attitude toward a problem orientation (– 2, 4, and – 26). An interesting pecularity was that in item 4 the difference in distribution was statistically signicicant, althrough the consensus percentages in both countries were almost the same (cf. Table 3.1). An explanation for this phenomen can be seen in Table 4.1. Item 24 (there is more than one way) was an exception: The Finns were much more favorable toward the statement than the Tatar pupils were. For Item 27 (pupils solve tasks independently), there was essentially no difference. Since consensus level was reached for only two items, we may conclude that in both countries mathematics teaching is striving toward a problem orientation, but with a different emphasis.

4.2 Independent work

When trying to develop pupils' creativity and their ability to use the knowledge they have acquired in school, their independent work plays an integral role. As before, we selected from the questionnaire a group of items (4, 13, 15, 26, 27, and 32) that we thought would represent independent work. In Table 4.2, we have converted the results for Items 15, 26, and 32.

Table 4.1. Agreement percentages (f.a. = fully agree and a. = agree), and significance of difference in distribution for statements reflecting problem orientation.

No.	Statement	Finland	Tatarstan	p
		f.a. – a.	**f.a. – a.**	
– 2	right answer **not** more important than way	19 – 45	30 – 41	**
4	pupil can guess and ponder	29 – 50	50 – 28	**
24	there is more than one way	31 – 58	25 – 28	***
– 26	teacher does **not** explain every stage exactly	2 – 11	4 – 14	**
27	pupils solve tasks independently	32 – 41	41 – 30	

**$p < .01$; ***$p < .001$ (by Mann-Whitney U).

Table 4.2. Agreement percentages (f.a. = fully agree and a. = agree), and significance of difference in distribution for statements reflecting independent work.

No.	Statement	Finland	Tatarstan	p
		f.a. – a.	**f.a. – a.**	
– 2	right answer **not** more important than way	19 – 45	30 – 41	**
4	pupil can guess and ponder	29 – 50	50 – 28	**
24	there is more than one way	31 – 58	25 – 28	***
– 26	teacher does **not** explain every stage exactly	2 – 11	4 – 14	**
27	pupils solve tasks independently	32 – 41	41 – 30	

**$p < .01$; ***$p < .001$ (by Mann-Whitney U).

For two items in Table 4.2 (Items 4 and – 26), there were large differences in the results between Finland and Tatarstan in favor of the Tatar pupils. Since for the four other items the differences were not statistically significant, the results indicate that the orientation to pupils' independent work are similar in Finland and in Tatarstan. But independent work was only moderately favored by the pupils, since only 3 of 12 agreement (or disagreement) percentages exceeded the consensus level.

4.3 Demands on teachers and pupils

Finally, we wanted to compare the pupils' views of demands on mathematics teaching. A proper balance between demands and freedom is one of the characteristics of effective teaching. In the questionnaire, we found five statements that represent demands on teachers (Items 5, 11, 16, 18, and 29) and two statements that are representative of demands on pupils (Items 7 and 23).

The seven items in Table 4.3 are related to the strength of the demands on teachers and pupils. For six of these items, the pupils in Tatarstan agreed significantly more with the statements than those in Finland. The results for Item 11 (all pupils understand) were almost identical in the two countries. Generally, the responses to these statements by the pupils in Tatarstan showed significantly stronger agreement than those of the pupils in Finland. In five items out of seven, the Tatars exceeded the consensus level, whereas the Finns exceeded it in only one. Thus, the pupils' views of the demands on them and on their teachers in mathematics teaching seemed to be much stronger in Tatarstan than in Finland.

5 Discussion

Since the questionnaire is our only indicator, one should take a careful position toward our results. As a matter of fact, they are the results of a pilot study where the data was gathered with a convenience sampling (Cohen & Manion, 1994), and not aiming to give any generalizable results. The questionnaire aims only at revealing interesting problems in an international comparison project (Pehkonen, 1995a). And we did find some critical points which will open up new research questions.

5.1 Summary of Results

Generally, pupils' views of mathematics teaching and learning in Finland and Tatarstan are rather far from similar. But we did find seven items that had common responses in both countries. The Mann-Whitney U test showed that for most items, the reactions of pupils in Finland and in Tatarstan were not correlated. An investigation of the differences between pupils' answers across the two countries also showed views that are characteristic for each country. For pupils in Tatarstan, the most characteristic might be the following views: the importance of exact reasoning and explanations, and the value of making strong demands on pupils. For pupils in Finland, the characteristic views seem to be as follows: a calculation orientation in learning mathematics, and the value of strict discipline and of depending on the teacher.

In addition, the research revealed some peculiarities in pupils' reactions in the two countries to the questions posed. For example, pupils in Tatarstan were strongly undecided about many items. This fact might indicate low self-confidence of the Tatar pupils, which could be caused by an authoritarian style of teaching.

Table 4.3. Agreement percentages (f.a. = fully agree and a. = agree), and significance of difference in distribution for statements reflecting demands on teachers and pupils.

No.	Statement	Finland	Tatarstan	p
		f.a. – a.	f.a. – a.	
	DEMANDS ON TEACHERS			
5	everything expressed exactly	10 – 20	54 – 34	***
11	all pupils understand	49 – 31	50 – 30	
16	everything reasoned exactly	18 – 35	47 – 42	***
18	as much repetition as possible	15 – 46	39 – 40	***
29	as much practice as possible	17 – 47	38 – 43	***
	DEMANDS ON PUPILS			
7	right answer quickly	1 – 10	11 – 25	***
23	it demands much effort	6 – 29	30 – 36	***

***$p < .001$ (by Mann-Whitney U).

When discussing problem orientation, we may conclude that in both countries mathematics teaching is striving toward a problem orientation, but with a different emphasis. In the case of independent work, the results seem to indicate that the orientation in question for pupils both in Finland and in Tatarstan is similar, but not very strongly favored. The results were that pupils' views of the demands on them and on their teachers in mathematics teaching seem to be much stronger in Tatarstan than in Finland.

5.2 Conclusion

The differences between Finland and Tatarstan in pupils' views of mathematics teaching and learning are large. For only 6 items out of the 32 did the differences fail to reach statistical significance. In comparison, when the differences between boys and girls in both countries were tested, only seven items had a statistically significant difference. Thus, the differences between countries are much larger than within countries (e.g., between boys and girls). This state of gender differences might suggest an interesting possibility for further investigation.

Results showing low self-confidence indicate differences in math-anxiety. This might also be an interesting way to follow in further studies.

We found that pupils' responses, both in Finland and in Tatarstan, reflect a rather high orientation toward problem-solving and a moderate orientation toward independent work. In demands on teachers and pupils, in contrast, there are strong differences between the countries. These questions are of interest for more thorough study, for example, by interviewing pupils and observe their classroom performance. It would be especially interesting to compare the results, particularly for the three groups of questions explored in depth, across a larger number of countries.

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References

Ahtee, M., & Pehkonen, E. (Eds.) (1994). *Constructivist viewpoints for school learning and teaching in mathematics and science* (Research Report No. 131). Helsinki: University of Helsinki, Department of Teacher Education.

Anonymous. (1985). *Peruskoulun opetussuunnitelman perusteet* [Curriculum for comprehensive school] Kouluhallitus. Helsinki: Valtion painatuskeskus.

Abelson, R. (1979). Differences between belief systems and knowledge systems. *Cognitive Science, 3*, 355-366.

Baroody, A. J., & Ginsburg, H. P. (1990). Children's mathematical learning: A cognitive view. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (JRME Monograph No. 4, pp. 51-64). Reston, VA: National Council of Teachers of Mathematics.

Borasi, R. (1990). The invisible hand operating in mathematics instruction: Students conceptions and expectations. In T. J. Cooney (Ed.), *Teaching and learning mathematics in the 1990s* (1990 Yearbook, pp. 174-182). Reston, VA: National Council of Teachers of Mathematics.

Cohen, L. & Manion, L. 1994. *Research Methods in Education*. London: Routledge (4th edition).

Davis, R. B., Maher, C. A., & Noddings, N. (Eds.) (1990). *Constructivist views on the teaching and learning of mathematics* (JRME Monograph No. 4). Reston, VA: National Council of Teachers of Mathematics.

Frank, M.L. 1985. *Mathematical Beliefs and Problem Solving*. Doctoral dissertation. Purdue University. University Microfilms International.

Garofalo, J. (1989). Beliefs and their influence on mathematical performance. *Mathematics Teacher, 82*(7), 502-505.

Good, T. L., & Brophy, J. E. (1977). *Educational psychology. A realistic approach.* New York: Holt, Rinehart & Winston.

Graumann, G., & Pehkonen, E. (1993). Schülerauffassungen über Mathematikunterricht in Finnland und Deutschland im Vergleich. In K. P. Müller (Ed.), *Beiträge zum Mathematikunterricht 1993* [Contributions to mathematics teaching 1993] (pp. 144-147). Hildesheim: Verlag Franzbecker.

Green, T. F. (1971). *The activities of teaching*. Tokyo: McGraw-Hill Kogakusha.

Grouws, D. A. (Ed.). (1992). *Handbook of research on mathematics learning and teaching*. New York: Macmillan.

Husén, T. (1967). *International Study of Achievement in Mathematics: A comparison of Twelve Countries*. Stockholm: Almqvist & Wiksell.

Kaplan, R. G. (1991). Teacher beliefs and practices: A square peg in a square hole. In R. G. Underhill (Ed.), *Proceedings of PME-NA 13* (Vol. 2, pp. 119-125). Blacksburg, VA: Virginia Tech.

Kifer, E., & Robitaille, D. F. (1989). Attitudes, preferences and opinions. In D. F. Robitaille & R. A. Garden (Eds.), *The IEA Study of Mathematics II: Contexts and outcomes of school mathematics* (pp. 178-208). Oxford: Pergamon Press.

McLeod D.B. 1989. Beliefs, Attitudes, and Emotions: New Views of Affect in Mathematics Education. In D.B. McLeod & V.M. Adams (Eds.), *Affect and Mathematical Problem Solving* (pp. 245-258). New York: Springer-Verlag.

McLeod, D.B. 1992. Research on Affect in Mathematics Education: A Reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 575-596). New York: Macmillan.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (JRME Monograph No. 4, pp. 7-18). Reston, VA: National Council of Teachers of Mathematics.

Paulos, J. A. (1992). Math-moron myths. *Mathematics Teacher, 85*(5), 335.

Pehkonen, E. (1992). *Problem fields in mathematics teaching. Part 3: Views of seventh grades about mathematics teaching* (Research Report No. 108). Helsinki: University of Helsinki, Department of Teacher Education.

Pehkonen, E. (1993a). What are Finnish teachers' conceptions about the teaching of problem solving in mathematics? *European Journal for Teacher Education, 16*(3), 237-256.

Pehkonen, E. (1993b). Auffassungen von Schülern über den Mathematikunterricht in vier europäischen Ländern. In H. Schumann (Ed.), *Beiträge zum Mathematikunterricht 1992* [Contributions to mathematics teaching 1992] (pp. 343-346). Hildesheim: Verlag Franzbecker.

Pehkonen, E. (1994). On differences in pupils' conceptions about mathematics teaching. *The Mathematics Educator, 5*(1), 3-10.

Pehkonen, E. (1995a). *Pupils' view of mathematics: Initial report for an international comparison project* (Research Report No. 152). Helsinki: University of Helsinki, Department of Teacher Education.

Pehkonen, E. (1995b). Vorstellungen von Schülern zur Mathematik: Begriff und Forschungsresultate [Students' beliefs about mathematics: Concept and research results]. *Mathematica Didactica, 18*(2), 35-65.

Pehkonen, E. (1996). Some findings in the international comparison of pupils' mathematical views. In E. Pehkonen (Ed.), *Proceedings of the MAVI-3 workshop in Helsinki 23.-26.8.1996* (Research Report No. 170, pp. 64-68). Helsinki: University of Helsinki, Department of Teacher Education.

Pehkonen, E., & Tompa, K. (1994). Pupils' conceptions about mathematics teaching in Finland and Hungary. *International Journal of Mathematical Education in Science and Technology, 25*(2), 229-238.

Pehkonen, E., & Zimmermann, B. (1989). Offene Probleme im Mathematikunterricht [Open problems in mathematics education]. In P. Kupari (Ed.), *Mathematics education research in Finland: Yearbook 1987-88* (Publication Series B: Theory into Practice No. 39, pp. 55-77). Jyväskylä: University of Jyväskylä, Institute for Educational Research.

Pehkonen, E., & Zimmermann, B. (1990). *Probleemakentät matematiikan opetuksessa ja niiden yhteys opetuksen ja oppilaiden motivaation kehittämiseen. Osa 1: Teoreettinen tausta ja tutkimusasetelma* [Problem fields in mathematics teaching and their connection to the development of teaching and pupils' motivation. Part 1: Theoretical background and research design] (Research Report No. 86). Helsinki: University of Helsinki, Department of Teacher Education.

Saari, H. (1983). *Koulusaavutusten affektiiviset oheissaavutukset* [Affective consequences of school achievement] (Publication No. 348). Jyväskylä: University of Jyväskylä, Institute for Educational Research.

Schoenfeld, A. H. (1985). *Mathematical problem solving.* Orlando, FL: Academic Press.

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education, 20*(4), 338-355.

Silver, E. A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and directions. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 247-266). Hillsdale, NJ: Erlbaum.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 127-146). New York: Macmillan.

Underhill, R. (1988). Mathematics learners' beliefs: A review. *Focus on Learning Problems in Mathematics, 10*(1), 55-69.

Unesco. (1986). *The place of science and technology in school curriculum: A global survey.* Paris: Author.

Zimmermann, B. (1991). *Heuristik als ein Element mathematischer Denk- und Lernprozesse* [Heuristic as an element in the mathematical thinking and learning process] (unpublished Habilitationsschrift). Hamburg: Universität Hamburg.

Appendix

Pupil questionnaire on mathematics teaching

The questionnaire items are presented below, along with response data. The pupils responded on a five-point scale (1 to 5), and the frequencies of their responses are given in percents. In addition, the z-value of the difference (using the Mann-Whitney U test) is given, along with its level of significance (* for $p < .05$; ** for $p < .01$; *** for $p < .001$). The value of p is also included when the difference is significant.

REAL MATHEMATICS TEACHING INCLUDES

	AGREE 1	2	3	4	DISAGREE 5	*z*	*p*
1: mental calculations							
FIN	40	49	6	3	2	2.03*	.043
TAT	54	30	13	2	0		
2: the idea that the right answer is always more important than the way of solving							
FIN	6	10	20	45	19	3.21**	.002
TAT	0	2	27	41	30		
3: mechanical calculations							
FIN	28	49	18	4	1	5.47***	.0001
TAT	18	29	46	3	4		

4: the idea that the pupil can also sometimes make conjectures, guess and ponder							
FIN	29	50	17	4	1	3.14**	.002
TAT	50	28	14	5	3		
5: the idea that everything ought to be expressed always as exactly as possible							
FIN	10	20	22	34	14	13.50***	.0001
TAT	54	34	11	0	0		
6: drawing figures (e.g., triangles)							
FIN	20	47	20	10	4	7.18***	.0001
TAT	57	24	13	3	2		
7: the idea that one ought to get always the right answer very quickly							
FIN	1	10	20	49	19	8.59***	.0001
TAT	11	25	35	22	6		
8: strict discipline							
FIN	62	29	8	0	1	7.75***	.0001
TAT	31	32	27	8	2		
9: word problems							
FIN	27	49	7	9	7	4.32***	.0001
TAT	50	30	18	1	1		
10: the idea that there is always some procedure one ought to follow exactly to get the correct result for sure							
FIN	10	25	26	26	13	6.52***	.0001
TAT	23	30	37	8	1		
11: the idea that all pupils understand							
FIN	49	31	8	8	4	0.22	--
TAT	50	30	14	5	2		
12: the idea that much will be learned by heart							
FIN	7	22	21	34	16	7.51***	.0001
TAT	19	30	36	13	1		
13: the idea that pupils also, during the mathematics lesson, put forward their own questions and problems and that they will be dealt with							
FIN	36	40	14	8	2	1.28	--
TAT	33	38	17	8	5		
14: the use of pocket calculators							
FIN	35	37	20	4	4	7.02***	.0001
TAT	18	17	46	14	6		
15: the idea that the teacher helps as soon as possible when there are difficulties							
FIN	36	40	9	14	1	1.59	--
TAT	42	37	17	4	0		
16: the idea that everything will always be reasoned exactly							
FIN	18	35	23	20	5	9.21***	.0001
TAT	47	42	9	2	0		
17: the idea that different topics, like calculation of percentages, geometry, algebra, will be taught and learned separately; they have nothing to do with each other							
FIN	14	22	42	20	3	4.58***	.0001
TAT	7	15	39	27	13		
18: the idea that there will be as much repetition as possible							
FIN	15	46	21	13	5	5.93***	.0001
TAT	39	40	16	4	2		

	AGREE 1	2	3	DISAGREE 4	5	z	p
19: the idea that tasks that have a practical benefit will be dealt with							
FIN	55	39	6	1	0	2.22*	.027
TAT	46	43	9	1	0		
20: the idea that only the mathematically talented pupils can solve most of the tasks							
FIN	2	3	12	33	50	11.20***	.0001
TAT	5	15	43	27	9		
21: the idea that it could not always be fun							
FIN	16	44	19	17	4	2.74**	.006
TAT	11	31	40	14	4		
22: calculations of areas and volumes (e.g., the area of a rectangle and the volume of a cube)							
FIN	26	50	15	5	3	1.01	--
TAT	39	31	21	7	2		
23: the idea that it demands much effort from pupils							
FIN	6	29	38	24	2	7.85***	.0001
TAT	30	36	27	5	2		
24: the idea that there is usually more than one way to solve tasks							
FIN	31	58	7	3	1	5.96***	.0001
TAT	25	28	25	15	7		
25: learning games							
FIN	30	36	24	7	3	5.28***	.0001
TAT	50	33	13	2	1		
26: the idea that when solving tasks, the teacher explains every stage exactly							
FIN	35	37	16	11	2	2.59**	.010
TAT	31	23	28	14	4		
27: the idea that pupils are allowed to solve tasks as independently as possible							
FIN	32	41	20	6	2	1.11	--
TAT	41	30	20	6	2		
28: the constructing of different concrete objects (e.g., a box) and working with them							
FIN	9	28	34	18	10	4.89***	.0001
TAT	22	34	30	10	4		
29: the idea that there will be as much practice as possible							
FIN	17	47	25	9	2	5.81***	.0001
TAT	38	43	18	1	0		
30: the idea that all or as much as the pupil is capable of will be understood							
FIN	27	48	21	4	0	3.79***	.0002
TAT	44	40	14	1	1		
31: the idea that also sometimes pupils work in small groups							
FIN	35	50	10	4	1	2.84**	.005
TAT	30	39	22	6	2		
32: the idea that the teacher always tells the pupils exactly what they ought to do							
FIN	26	27	19	22	7	0.62	--
TAT	20	24	35	13	8		

Elevers syn på matematikundervisning i Finland och Tatarstan

Fokus i denna jämförande studie är: Vad är elevers syn på matematikundervisning i respektive land. Vilka skillnader och likheter är det i elevers uppfattningar i Finland och Tatarstan, Ryssland.

Vi har samlat data med hjälp av en enkät, som innehöll 32 strukturerade påståenden om matematikundervisning som eleverna fick ta ställning till i en 5-gradig skala. Urvalen bestod av 255 elever från Finland och 206 elever från Tatarstan. Våra data har samlats in med hjälp "non-probabilistic convenience sampling" (man tar ett stickprov som är lätt att komma åt, men vars representativitet man vet lite om).

Huvudresultaten är som följer: Elevernas syn på matematikundervisning är i allmänhet ganska olika. En undersökning av skillnaderna mellan elevernas svar i de två länderna visar på karakteristiska drag i varje land. Karakteristiskt för elever i Tatarstan är följande uppfattningar: Det är betydelsefullt med exakta resonemang och förklaringar och att ställa höga krav på eleverna. För elever i Finland är det typiskt att orientera sig mot beräkningar vid inlärningen, strikt disciplin värderas och man är beroende av läraren.

I båda länderna strävar man mot problemorientering men med olika betoning. Utifrån enkäten förefaller självständigt arbete värderas ganska lika utan att favoriseras. Eleverna ser kraven på sig själva och på lärarna i matematikundervisningen som mycket större i Tatarstan än i Finland.

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