Review

Didactics of mathematics. A scientific discipline

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Biehler, R., Scholz, R.W., Strässer, R. & Winkelmann, B. (eds.) (1994). *Didactics of Mathematics as a Scientific Discipline.* (Mathematics Education Library, Volume 13). Dordrecht, NL: Kluwer Academic Publishers, 467p.

For those working in the field of mathematics education this book has a very interesting and important title: Didactics of Mathematics as a *Scientific Discipline.* Since the first International Congress on Mathematical Education in Lyons in 1969 researchers in the field has been interested in establishing mathematics education as a discipline.

Here there is a question about terminology. *Mathematics education* is the terminology denoting the field, used in most English speaking countries. Mathematics education has, however, a double meaning: (1) Mathematics education as an activity, e.g. the teaching – learning process in the classroom and elsewhere, (2) Mathematics education as a research area. In many other countries some form of the word *«Didaktik»* is being used to denote this second aspect.

In American-English, two terms strongly associated with European educational thought – *didactics* and *pedagogy* - have taken on negative connotations. To call someone a *didact* is to imply not only that the person is someone who teaches but also that her or she is likely to be moralistic about it. A *pedagogue* is not just a teacher, the word implies that the teacher is long-winded and boring, rather like a *pedant.* American use *education* and *educator* to avoid these unpleasant connotations. They view the field of education as one that, though low in status and prestige, has begun to establish its place in academe.

> ...
> In Germany there is a similar usage: The term *Mathematikdidaktik* refers to mathematics education considered as an academic field: Americans, however, resist the term *didactics.* The use *mathematics education* to refer both to the activity and to the field. (Kilpatrick, 1994)

In the Nordic countries we use the word «Matematikkdidaktikk» to denote the science. Perhaps we should now start to use the term *«Didactics of mathematics»* to denote the science in the English language – in spite of the negative connotation – as is implied by the title of this book. In the following presentation I will use this term.

Didactics of Mathematics as a Scientific Discipline – was edited at Institut für Didaktik der Mathematik (IDM) in Bielefeld. IDM was founded in the beginning of the 1970s, and has been one of the leading European research institutions in the didactics of mathematics. The book contains a large number of articles by various authors, and was dedicated to Hans-Georg

Steiner at IDM on his 65th birthday. One interesting feature of the book is the description they give in the preface about the field:

> Didactics of mathematics certainly exists as a discipline, at least in the social sense, as can be seen from journals, research and doctorate programs, scientific organizations, and conferences.

and they continue:

> As a fairly young discipline, its system of objects, methodologies, and criteria for valid knowledge exhibits more variability and less consensus. Its role among other sciences at the university is still disputed (p.2)

They also make a distinction between science and technique:

> Didactics of mathematics is an applied area of activity: As in engineering, (applied) psychology, and medicine, the boundary between scientific work and (constructive) practice is - to say the least - «fuzzy». (p.3)

They try to lay down what they call a «topology» of the field, or research in the field - a state-of-the-art portrait. They identify the chapters of the book as *subdisciplines.* We find the following chapter headings:

1. Preparing Mathematics for Students
2. Teacher Education and Research on Teaching
3. Interaction in the Classroom
4. Technology and Mathematics education
5. Psychology of Mathematical Thinking
6. Differential didactics
7. History and Epistemology of Mathematics and Mathematics Education
8. Cultural framing of Teaching and Learning Mathematics

In characterizing the structure of the book (and the field) the editors state that: «the first five (areas) are structured, the last three not so much. It is also stated that the book tries to give an overview of mathematics education: «chapter introductions to provide a synthesis and an orientation for the research domain represented in the contributions».

The book contains 30 articles written by prominent researchers in the field, as well as 8 chapter introductions written by the editors. The contributions are evenly distributed with 3 or 4 articles in each chapter. The chapter introductions provide in general good overviews of the articles and try to give a short account of the area (subdiscipline). To give a review of all the articles in the book will be beyond the scope of this presentation, so I will consider one article from each of the 8 chapters.

In the article *Mathematical Curricula and the Underlying Goals* Uwe-Peter Tietze looks upon curriculum development in mathematics. He is looking at curriculum development from a historical point of view. The New Math movement and countertendencies provides him with ideas in looking for structures in this otherwise complex field. «Elementarization» and «Fundamental Ideas» are two elements of curriculum development he considers in detail. However, after the discussion his conclusion is somewhat negative concerning (general) didactical curriculum theory:

> New empirical research shows the limits of curriculum development in principle. The teacher alone determines the effectiveness of curriculum by his or her decisions, behavior, attitudes, and cognitive processes, no matter how carefully the curriculum has been developed. The high expectations educators once had about the benefits of scientifically developed curri-

> cula have been supplanted by a more modest assessment. Recent research has placed more emphasis on everyday curriculum in the classroom, on teachers' ideas and subjective theories concerning their quotidian preparation of classes, their subjective learning theories, implicit and explicit objectives, philosophy of mathematics, and the influence of these cognitions on their teaching.

Heinrich Bauersfeld's article: *Theoretical perspectives on interaction inn the mathematics classroom* takes as starting point two traditions in mathematics education – the psychological (individual) and the social (collective). He tries to establish a «mediating» position, which he calls interactionism:

> Teacher and students interactively constitute the culture of the classroom, conventions both for subject matter and social regulations emerge, communication lives from negotiation and taken-as-shared meanings.

The next part of the article concerns the consequences for educational practice. Bauersfeld calls cognitive psychology a «prototype» for the individualistic perspectives, and activity theory a «prototype» for the collectivist perspectives. Paralleling this view, he considers «connectionism» – or neural net models for interactionism. He then comments on various aspects of education in the light of such models. The article gives an interesting theoretical basis for mathematical education, but many will probably find the discussion too short for the importance of the topic.

In section 4, David Tall in his article *Computer environments for the learning of mathematics* seeks to link computer generated environments to knowledge development and learning theories. In referring to Skemp's theory of building and testing conceptual structures, he present an extension. He introduces the mode *cybernetic* to include that the stimuli comes from systems that are set up to act according to pre-ordained rules (computer programs). This is an important article since it suggests a way to introduce the function of computers into existing educational theories.

Efraim Fischbein starts his article *The interaction between the formal, the algorithmic, and the intuitive components in a mathematical activity* by stating that mathematics can be considered from two points of view: (a) mathematics as a formal, deductive rigorous body of knowledge and (b) mathematics as a human activity. For Fischbein, mathematics is a human activity, and in the article he considers the interaction between the three components - the formal, the algorithmic, and the intuitive. He draws examples from the historical development of mathematics – Euclidean geometry, the notion of the infinite, sets and limits. He looks at these examples in teacher training and states many interesting observation. One of his main points is that intuition plays an important role. He ends his article by stating the complexity of the problem and calls for cooperation among disciplines close to the didactics of mathematics.

Gila Hanna's title ends with a question mark: *Should girls and boys be taught differently?* and even if her article also ends with question mark, her opinions on the subject are clear:

> Those who argue for an intrinsically feminine way of understanding mathematics, most of them feminists and all of them well-intentioned, are actually doing a disservice to education and to other women. (...). In reinforcing the traditional view of women as caregivers who are better at personal relations than abstract ideas, they run the risk of portraying women as fundamentally unsuited for science.

The scope of Hanna's article is a presentation and commentary on research on gender differences and on international surveys. In most cases she find there are no gender differences in cognition found in the studies or surveys. Where differences are stated, e.g. in the debate following Carol Gilligans book: *In a Different Voice,* Hanna presents strong arguments against the results:

Gilligan's work may be important as a critique of sexist bias in the literature on human development. Because her study was of very modest proportions and her data open to alternative interpretations, however, her female-male dichotomy remains far from convincing.

Paul Ernest's article has the title *The philosophy of mathematics and the didactics of mathematics.* The conclusion of his article contains a fairly concise description of the content:

> ... a brief review of the philosophy of mathematics and the didactics of mathematics. The treatment of the former is a balanced account of the development in philosophy, albeit from one perspective. However, in reviewing didactical implications, arbitrary choices have been made and personal preferences compressed into a short account.

He observes a shift in academic philosophy of mathematics, from absolutist epistemological to ontological concerns. He is arguing that any philosophy of mathematics has implications for social and educational issues. He presents his philosophy as social constructivist, and he claims it offers a synthesis of the other philosophies. The main body of the article is drawing consequences from his philosophy for education. This part is very concentrated and a bit sketchy. However, it present interesting perspectives in this very important area.

David Robitaille and Cynthia Nicol start with a discussion on comparative research in their article: *Comparative international research in mathematics education.* They argue for the importance of such studies and goes on to discuss the international mathematics studies (IEA studies). They give some results from the studies but the main part of the article concerns organization and scope of the studies.

The book provides an interesting and important collection of articles relating to didactics of mathematics. It contains many important contributions to our field. However, the collection has also some weaknesses when taken to give a representative picture of the field. There is a strong American and German dominance in the selection of contributors. There are for example only two French and one Italian contributor - and none from Eastern Europe, Australia or Japan.. Also the articles show quite a variation in scope and quality.

One also feels when reading some of the articles that it has been important to compress the presentation, some articles hence, are difficult to comprehend for beginners in the field. Mainly due to this observation, there has been mixed experiences by using the book in a course in mathematics education at the University of Oslo. But I should also add that the overall impression has been positive.

As a collection giving information to the more experienced mathematics educator it is very good. It gives an overview of the variety in the field of didactics of mathematics – and is an important contribution to the literature of the discipline.

Reference

Kilpatrick, J. (1994). Vingt ans de didactique francaise depuis les USA. In M. Artigue, R. Gras, C. Laborde, & P. Tavignot (eds.), *Vingt ans de didactique des mathematiques en France.* Paris: La Pensee Sauvage.