Disciplinary competence descriptions for external use

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The article addresses the need for competence descriptions of disciplines as a means for fostering more productive communication between different disciplines and between the disciplines and their surroundings. It is argued that the usual competence descriptions devised for use within a discipline itself, e.g. in relation to teaching and learning of the discipline – so-called competence descriptions for internal use – are not the best means to achieve this. The same is true for the general, non-disciplinary competence descriptions. Instead, specially devised disciplinary competence descriptions for external use are called for. Our main illustration is a competence description of mathematics for external use devised so that it can support the dialogue about justification of mathematics education between the discipline's practitioners and its recipients. This description for external use is counterposed with one for internal use i.e. that of the Danish KOM project. It is also counterposed with a competence description for external use for physics, taking into account the different justification problem of physics education. Together these two descriptions showcase how competence descriptions of disciplines for external use may support interdisciplinary collaboration and division of labor in the educational system.

The point of departure for this article is several years of experience with being part of the mathematics and physics "minority culture" at Roskilde University. The university is dominated by much larger institutes of the humanities and social sciences as compared to the natural sciences, of which mathematics and physics are smaller parts. Still, Roskilde University has a declared purpose – with which we strongly agree – of aiming at interdisciplinarity in both research and teaching. This circumstance has forced us, over many years, to consider and (unsystematically) investigate the collaborative roles of the disciplines of mathematics and physics in relation to other disciplines (also across faculties).

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Our accumulated experiences are that disciplines in general, and mathematics and physics in particular, are perceived by their practitioners as certain *ways of perception* (Jensen & Jankvist, 2018) built up over years of training, enabling a practitioner to perceive relationships that could not have been conceived otherwise. At the same time, we – the practitioners – often come to regard other disciplines, i.e. those which are not our own and which we therefore do not comprehend to the same extent, as merely being collections of tool-oriented recipes. For this reason, it is a challenge to find means for communicating meaningfully about disciplines between practitioners of these and recipients. A well-known example of this challenge is that of supporting disciplines at university level, e.g. supporting mathematics classes in a physics program or supporting chemistry classes in a medicine program. Oftentimes it is difficult to reach agreement on the balance between fetishisation of the supporting discipline in its own right, thus making its potentially valuable aspects almost disappear through instrumental integration. One contribution to remedy these difficulties of communication might be to exemplify and name such (only partially recognized) perspective differences.

A particularly and rather severe version of such communication difficulties may, according to our judgement, be found in connection with the teaching of mathematics (at all educational levels). Bringing matters to a head, the practitioners of mathematics often perceive the teaching of this as related to the science of pure mathematics, while the recipients (students, the educational system, and society) rather seem to expect a teaching directed at solving real life problems, i.e. something much closer to mathematical modelling. Since pure mathematics and mathematical modelling are two qualitatively different activities – e.g. consider the crucial diffe-rence between the roles of counterexamples in the two activities – this makes up a severe issue of matching of expectations. In this article, we shall attempt to articulate this issue by means of what we have named *a competence description of mathematics for external use*, and discuss the potential value of one such. It is, we argue, precisely the *competences*, as opposed to the syllabus, that are perceived differently across the gap of communication between the discipline of mathematics' practitioners and its recipients.

As the reader no doubt will know, in recent years the word "competence" (or competency) has become quite a buzzword within educational research (e.g. Han, 2010). Under the OECD auspices, work has been carried out to produce *non-disciplinary competence descriptions* for use in overall assessments of national educational systems, addressing the question of: Which competences, generally speaking, do people and society (the recipients) need the educational systems to deliver? (OECD, 2001).

At the same time, another OECD initiative, PISA, provides competence descriptions for the individual disciplines (OECD, 2013) – competence descriptions which have also been used to develop the teaching in these disciplines (subjects). By describing the subjects in terms of competences, a tighter connection is strived for to the declared goals of the subject as well as better "vertical communication" regarding the subject's place in the educational system – better than that which may usually be reached by means of syllabus descriptions. PISA's framework of mathematical competencies from 2000 to 2018 was essentially a modification of the Danish mathematical competencies framework as described in the so-called KOM-project (Niss & Højgaard, 2011; Niss & Jensen, 2002). Such descriptions may be referred to as *disciplinary competence descriptions for internal use*, since they also serve the purpose of facilitating communication between various practitioners within the discipline. Disciplinary competence descriptions for external use, on the other hand, serve the goal of providing "horizontal communication" between the practitioners of the discipline and the recipients, and thus assist in promoting collaboration.

We are aligned with the KOM-project in its efforts to augment the description of disciplinary content (syllabus) with disciplinary competence descriptions. Nevertheless, in this article we shall argue and substantiate why the KOM-project's competence description for internal use is not sufficient for the purpose of matching of expectation between the discipline of mathematics' practitioners and its recipients. Table 1 below sums up our perception of the relationship between disciplinary competence descriptions for internal and for external use, respectively, in terms of their differences in purpose and characteristics.

Table 1. *Purposes and characteristics of disciplinary competence descriptions for internal and external use, respectively*

	Purposes	Characteristics
For internal use	An *alternative* to *syllabus* descriptions.	Competence descriptions *of* the discipline.
For external use	Positioning the discipline in educational contexts. *Justification* of the discipline.	Description of the discipline's potential *contributions* to people's need for competence development.

The outset for this article is *the justification problem* of mathematics education. Still, we have chosen the title of the article to be "disciplinary competence descriptions for external use" – i.e. what might appear to be a somewhat general title. The reason for this is that in our opinion the justification problem of mathematics, besides being distinctly alarming, constitutes an example of an issue which also exists within other subjects, to a lesser or greater extent.

Besides the competence description of mathematics for external use, we have also constructed a competence description of physics for external use. This partly serves the purpose of illustrating how competence descriptions of disciplines with a different kind of justification problem than mathematics must be devised according to that. At the same time, the comparison of the two descriptions invites to a dialogue between the two disciplines regarding their internal distribution of roles in relation to teaching mathematical modelling as an illustration of how competence descriptions for external use may be utilized within interdisciplinary collaboration in the educational system. We also consider this to be an example of how disciplinary competence descriptions in interdisciplinary contexts may contribute to communication across disciplines at large. It should be mentioned that by counterposing our competence description of mathematics for external use with a corresponding one for physics, the context is enhanced in direction of *utilization*, while the analogous description of physics at the same time provides some backlight. Such backlight could equally well have been provided by counterposing mathematics with, say, the discipline of history, which then might have brought out more cultural aspects of the discipline of mathematics. In this article, however, we have deliberately chosen a more utilitarian focus, since this particular aspect of mathematics is the one often provided when it comes to justifying mathematics education and its relevance in society. In order to situate our pending disciplinary competence descriptions for external use, we now briefly turn our attention to the so-called justification problem of mathematics education.

The justification problem of mathematics education

Blomhøj (2001) points to the fact that the problem of justification of mathematics education has both an objective and a subjective side. The objective side concerns the reasons for mathematics education placed in a societal context. The subjective side, on the other hand, concerns the single individual's (e.g. teacher or student) sense making of participating in mathematics education. Of course, such a distinction between the objective and subjective may be made concerning the justification of any subject. Nevertheless, the situation for mathematics is a special one in terms of objectivity and subjectivity; a situation sometimes referred to as the *relevance paradox* (Niss, 1994). The objective side of the paradox has to do with "the unreasonable effectiveness of mathematics" both in the natural sciences (Wigner, 1960) and in engineering and computer science (Hamming, 1980). That is to say, mathematics applies to – and is applied in – a wide range of extra-mathematical subjects and practice areas, and in particular physics. This use of mathematics permeates society at large, in its past and present functioning and evolution as well as in its future development. The use of mathematics in technology constitutes one example of this, both the material technology (physical objects and systems, e.g. computers and other microelectronic devices) and the immaterial technology (computer software, codes, geographical coordinates, calendars, money transactions, graphical representations, measurements of time, space, weight, currency, etc.) (Niss, 1994). Other examples, also of an immaterial nature, are the various decision-making and controlling processes which take place as part of the infrastructure of a society and various forms of descriptions and predictions, e.g. about the weather and climate. The application of mathematics to these extra-mathematical areas is brought about by mathematical modelling, including the building, usage, and validation of such models. However, the embeddedness of mathematics into mathematical models and (other) immaterial technologies, as well as the further potential embedding of these into material technologies, brings about the subjective side of the paradox, namely that the mathematics in society becomes invisible – hence irrelevant – to us. Or as Philip J. Davis, co-author of *The mathematical experience* (Davis & Hersh, 1981) sees it: "[...] it's invisible to people because it's in programs, it's in chips, it's in laws [...]. So you don't see it, and if you don't see it, you don't think it's there" (Interview with Davis in Jankvist & Toldbod, 2005, p. 17). This is exactly what constitutes the relevance paradox: "the simultaneous objective relevance and subjective irrelevance of mathematics" (Niss, 1994, p. 371).

A common belief – also among the teachers of mathematics – is that society buys the ability to solve problems using mathematics by teaching mathematics in its own right, i.e. pure mathematics. This enforces the relevance paradox by regarding a necessary condition to also be sufficient, i.e. the insufficiency in meeting the relevance paradox is not recognized. At the same time the instrumental perception of mathematics as a discipline paves the road for disciplinary relational understanding to be outsourced to various digital technologies, not least computer algebra systems (CAS) (e.g. Jankvist & Misfeldt, 2015). Collectively speaking, both the mathematics teachers as well as their surroundings need an understanding of the fact that bringing mathematics in play in extra-mathematical problem solving situations demands yet a set of competences besides being able to master the inner aspects of the discipline itself (see also Jensen, Niss & Jankvist, 2017). And it is needed that the surroundings understand that mathematical problem solving and modelling, as something different than actions of routine, in the extra-mathematical domain necessarily presupposes a non-algorithmic and creative relationship to mathematics in its own right. Without such understandings, among the discipline's practitioners as well as its recipients, it does not appear sure that mathematics will continue to be the major subject in school that it is today.

A competence description of mathematics for internal use

In the Danish KOM-project, Niss and Højgaard (2011) generally define a person possessing a competence within a discipline, as taken to be someone who is able to master essential aspects in relation to that discipline effectively, incisively, and with overview and certainty of judgement. They talk about the discipline of mathematics as being associated with two such overall *competences*: (i) the ability to ask and answer questions in and with mathematics, and (ii) the ability to deal with mathematical language and tools. Each of these competences is made up of a set of mathematical *competencies* (notice the distinction used between competenc*e* and competenc*y*). While "mathematical competence comprises having knowledge of, understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role", "a mathematical competency is a well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge" (Niss & Højgaard, p. 49). The first overall competence (i) consists of the four competencies of mathematical thinking, problem tackling, modelling, and reasoning. The second (ii) consists of the four competencies of representing, symbol and formalism, communication, and finally that of aids and tools. Each of the eight competencies has both an analytic side involving understanding and examining mathematics, and a productive side involving carrying it out. The modelling competency, for example, consists of the ability to analyze the foundations and properties of existing models and to assess their range and validity, on the one hand. On the other hand, it involves being able to perform and utilize active modelling, including mathematizing, in given extra-mathematical contexts and situations.

In addition to the eight mathematical competencies, the KOM-project also describes three types of "overview and judgement" (OJ), which are to be thought of as "'active insights' into the nature and role of mathematics in the world" (Niss & Højgaard, 2011, p. 49). Niss and Højgaard state that "these insights enable the person mastering them to have a set of views allowing him or her *overview and judgement of the relations between* *mathematics and* [...] *nature, society and culture*" (p. 73). These are: (OJ1) the actual application of mathematics in other subject and practice areas; (OJ2) the historical development of mathematics, both internally and from a social point of view; and (OJ3) the nature of mathematics as a subject.

The KOM-project's competence description for internal use, with its eight competencies and three types of overview and judgement, makes up a clear invitation to non-mathematicians who are genuinely interested in understanding what mathematical thinking is composed of. But with mathematical modelling as only one of eight competencies, and not as *the* central component corresponding to the expectations of society, the description is not optimal as a basis for dialogue between the discipline's practitioners and its recipients.

A competence description of mathematics for external use

The purpose of competence descriptions for external use is to function as a platform, not for communication between practitioners internally, but for communication between practitioners of the discipline and its surroundings in the form of students, practitioners of other disciplines, authorities, and society in the widest sense. In our formulation of the competence description of mathematics (and later that of physics) for external use, we have confined ourselves to looking at three overall competences; competences which we find may span the potential space of dialogue. Taking into account how easily words may be adjusted to the reader's universe, and hence also unintentionally misinterpreted, we shall provide illustrative examples for each of the competences to brace the formulations.

For mathematics, we have formulated a description directed at the schism between, on the one hand, the kind of focus which mathematicians naturally have on the distinctive characteristics of the discipline, manifested by "pure mathematics", and on the other hand, the surroundings' expectations that teaching actually trains students in tackling extra-mathematical problems by means of mathematics. As is the case with the KOM description of mathematical competencies for internal use (Niss & Højgaard, 2011), competence descriptions of mathematics for external use may not be so sharply defined and described that overlaps will not occur. The competences of the descriptions for internal use and external use, respectively, are bound to be connected, also in terms of overlap, and certainly the competences in the descriptions for external use cannot be developed in isolation from those in the descriptions for internal use. For the description for external use, we have identified the following three mathematical competences:

1. Being able to think as a mathematician
2. Being able to relate and respond to actual applications of mathematics
3. Being able to bring mathematics in play in extra-mathematical settings

The first competence is one that encompasses seven of KOM's mathematical competencies: mathematical thinking, problem tackling, reasoning, representing, symbol and formalism, communication, aids and tools as well as aspects of OJ3 on the nature of the subject of mathematics. (In relation to teaching, it would hopefully also include aspects of OJ2.) The second and third of the mathematical competences in the description for external use are related to the modelling competency in the KOM description. Recall that the modelling competency comprises two parts; one analyzing part and one performing part. Under the assumption that practically all uses of mathematics for extra-mathematical purposes boil down to a use of modelling in some sense, the second competence – "being able to relate and respond to actual uses of mathematics" – then embraces the analyzing part of the modelling competency, whereas the third – "being able to bring mathematics in play in extra-mathematical settings" – corresponds to the performing part of the modelling competency. In addition to being made up of the analyzing part of the modelling competency, the second competence in the description for external use includes also OJ1 on the actual application of mathematics in other subject and practice areas. This being said, the second and third competences above can of course not be practiced without substantial possession of KOM's other seven competencies, i.e. the first competence in our description for external use. The reverse, however, may indeed be the case; students trained in problem tackling within the internal mathematical domain may not be able to model extra-mathematical situations (e.g. Ikeda & Stephens, 1998). We now illustrate the three competences of the competence description for external use with three examples.

Being able to think as a mathematician

As an example, we take the following discrete mathematics problem, which students often find intriguing: *Show that at a party, where there are at least two people, there are (at least) two people who know the same number of other people there.* To solve this problem, we consider an arbitrary person, say p, who is at the party. We define the function $K(p)$, to be the number of other people at the party which person p knows. Now, the possible values for $K(p)$ are 0, 1, ... , $n-1$, where $n \geq 2$ is the number of people at the party. We then observe that it is impossible for both 0 and $n-1$ to be in the range of K, since if one person knows everyone else, then nobody can know no one at the party (we make an assumption here that "knowing" is a symmetrical relation). Therefore, the range of K has at most $n-1$ elements, whereas the domain of K has n elements, so K is not a one-to-one function, which corresponds precisely to what we wanted to show!

As can be seen, if analyzing our reasoning step by step, the majority of KOM's mathematical competencies come into play when providing the above chain of arguments. We boil the situation down so that we become able to apply a well-known principle from mathematics, the *Dirichlet drawer principle* (or the pigeonhole principle), which says that if $k+1$ objects or more are to be placed into k boxes, then there is at least one box containing two or more of the objects. Of course, non-mathematicians would askwhy we bother to name something so obvious as "a principle". But this illustrates exactly one aspect of being able to think as a mathematician; to take a more complex situation, e.g. that of the party, and reduce it to something to which we already know the answer, in this case the Dirichlet drawer principle.

Being able to relate and respond to actual applications of mathematics

To illustrate the second competence we take as our example models for forecasting populations. Many a country maintains a model for forecasting its population development. In such models some basic population dynamical mechanisms are rock solid, while others are subject to assumptions, estimations, etc. The Danish model for forecasting populations, in its present form, has been maintained by *Statistics Denmark* since 1977 (for a discussion, see Niss, 2000), and operates with progression through age classes. The model comprises the following rock solid mechanisms: individuals already born progress in the age classes by becoming one year older per year, provided of course that they survive; individuals may enter or leave an age class during a given year by either immigrating/emigrating or by dying; and new individuals are born by women in the fertile age classes. However, when doing actual forecasting, estimates must be made of death frequencies, net immigration figures, and not least the fertility quotients to enter the model. These are highly dependent on biological, cultural, and socioeconomic factors, making them vary from "almost solid rock" over "soft soil" to mere "mud" (using the terms of Niss, 2000). For example, over time the age classes in which Danish women typically give birth to children have shifted, e.g. due to better family planning or because women wanted to finish their education before having children, which resulted in an unexpected drop in the fertility quotients of young women in the 1980s; something the model failed to predict. Being able to tell the difference between the fixed mechanisms in a given model and mechanisms which are based on loose estimates and assumptions (even if including historical data, as the Danish model of course does), is part of being able to relate and respond to actual applications of mathematics. Besides mastering mathematics, the competence of "being able to relate and respond to actual applications of mathematics" presupposes epistemological judgements based on experience and interactions with the domain mathematics is applied to.

Being able to bring mathematics in play in extra-mathematical settings

As an illustration of the third competence we offer the example of answering the question: *How far away is the horizon?* In order to tackle this question, we first need to conceptualize what is meant by "horizon" – from a modelling perspective, this step corresponds to what Niss (2010) refers to as pre-mathematization, i.e. crucial steps preceding the actual process of mathematization. This is illustrated by means of figure 1.

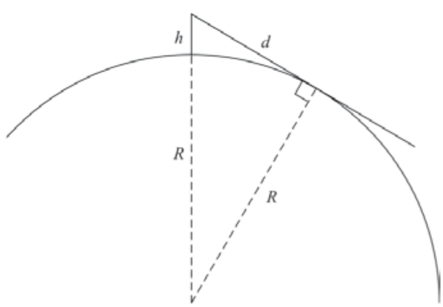

Figure 1. *An illustration of the distance, d, to the horizon, taken from the height of observation, h, to the point where the tangent line touches Earth's surface. R is Earth's radius*

We assume the height of observation, h, as being quite small in comparison to Earth's radius, R. For that reason, we shall calculate the distance, d, from the top point of h, from where we observe, to the point of the horizon as being approximately the same as the distance from the ground point of h along the surface of Earth to the point of the horizon. It has no practical importance if we by the distance to the horizon take one or another. We now see that d is given by the Pythagorean theorem

$$(R+h)^2 = d^2 + R^2, \text{ where } d = \sqrt{\left((R+h)^2 - R^2\right)} \approx \sqrt{2Rh}$$

since h^2 is much smaller than $2Rh$. Now, if $h \approx 2$ meters and $R \approx 6000$ kilometers in the formula above, we get $d \approx 5$ kilometers. If, say, we put our observation post in a tower, so that h is increased by a factor 16 to 32 meters, we can immediately deduce from the formula that d is increased by a factor 4, so that the distance to the horizon now is 20 kilometers.

While the competence of "being able to relate and respond to actual applications of mathematics" requires a knowledge of the disciplinary domain to be modelled, this is not the most important component of the competence of "being able to bring mathematics in play in extra-mathematical settings". The essential thing here is not to model using a lot of problem-specific knowledge, e.g. except knowing that the Earth is spherical as in the example, but to bring mathematics to use by making continuous simplifications to the extra-mathematical problem. In the words ascribed to Albert Einstein, the idea is to make things as simple as possible, but not simpler. This is indeed unlike the competence of "being able to think as a mathematician", where the idea is to make things logically exact.

Discussion – part 1

What happens when shifting the focus from a description for internal use to one for external use is that some of the competencies in the description for internal use are grouped together as competences in the description for external use, while others are split up and differentiated. As the reader will have noticed, the proposed competence description of mathematics for external use may be seen as a merger of seven of the KOM-project's competencies into one competence (i.e. "being able to think as a mathematician"), while taking the last competency on modelling and splitting it up according to its analyzing and its productive sides (i.e. "being able to relate and respond to actual applications of mathematics" and "being able to bring mathematics in play in extra-mathematical settings"). As illustrated by our first example above, being able to think as a mathematician does involve to represent things mathematically, to apply symbols and formalism, conduct mathematical problem tackling and reasoning, communicate the solution and reasoning involved, etc. – and from an internal mathematical point of view it does make sense to distinguish these different competencies involved. But for someone outside the discipline of mathematics, doing so may cause more confusion than clarity, since all of it is being able to do mathematics and some possession of all of these competencies is required to tackle extra-mathematical problems of society. In terms of the mathematical modelling competency, however, being able to analyze already made models is quite different from being able to build mathematical models oneself, which is also illustrated by our two examples above. Being able to relate and respond to actual applications of mathematics requires sound knowledge of the, for example, societal contexts in which these are a part. Being able to bring mathematics in play in extra-mathematical settings, however, requires practice with doing modelling and it requires "implemented anticipation" (Niss, 2010; Stillman & Brown, 2014), which refers to the modeler being able to anticipate forthcoming steps in the modelling process and "to implement this anticipation in terms of decisions and actions that frame the next step to be made" (Niss, 2010, p. 56). Both of these competences are of societal importance. Still, the recipients to a larger degree seem to expect possession of the latter than the former of the two.

The competence descriptions of mathematics for internal and external use, respectively, are not in conflict with one another. They merely stress different aspects of the discipline. The competence description for internal use guides the discussion concerning what characterizes mathematical activity, whereas the competence description for external use opens up for discussion concerning the arguments for including mathematics in a given interdisciplinary setting (in both research and teaching). It all depends on the purpose of the description (and discussion); in general, competence descriptions are directed normatively against problems at hand. The internally oriented one for mathematics is directed against *syllabusitis* (Lewis, 1972) as a problem, i.e. it constitutes an alternative to syllabi descriptions of the discipline. The external oriented one for mathematics has a purpose of bridging a potential conflict between an internal perception of the discipline and the expectations of the recipients of the surrounding world (cf. table 1 in the introduction).

Returning to the problem of justification of mathematics education, Niss and Højgaard offer two reasons why mathematics is such a dominating educational subject. Firstly, because mathematical competencies "often are necessary to be able to utilise technology in the traditional sense, and secondly, because mathematics by means of mathematical models, in itself constitutes such technology." They continue: "This completes the reasoning which from a recipient's point of view justifies the importance of mathematics teaching: An educational policy placing mathematics teaching at the front is, amplified by the labour market policy, essential for the maintenance and development of the welfare state" (Niss & Højgaard, 2011, p. 169).

A competence description of physics for external use

As the discipline of mathematics, mathematically oriented physics is subject to the previously mentioned relevance paradox. However, as can be seen from figure 2, the consequences of the relevance paradox are different for the two subjects within the dominating part of Danish upper secondary school (the classical stream, referred to as stx). The upper secondary school subject of mathematics is, as previously mentioned, threatened by an instrumental perception of the subject resulting in a tendency to outsource it to various digital technologies. But quantitatively speaking, the subject of mathematics is not threatened so far. The subject of physics, on the other hand, is. As illustrated by figure 2, physics has over a long period of time been on retreat in the Danish upper secondary school.[1] In comparison to the subject of chemistry, the retreat of physics is most likely due to its larger content of applied mathematics. Unfortunately, it is precisely this aspect of physics, i.e. the involved application of mathematics, which is perhaps of most objective relevance generally speaking.

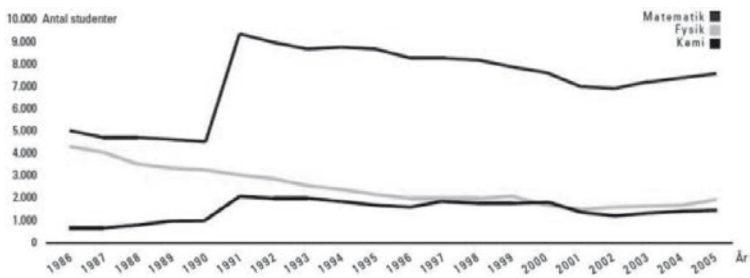

Figure 2. *Number of stx students with A-level in mathematics, physics or chemistry from 1986 to 2005 (UVM, 2006, p. 35). A-level corresponds to having the subject for three consecutive years at upper secondary school*

In order to focus on the justification problem of physics education in such situations, we thus find it relevant to devise a disciplinary competence description of physics[2] for external use as consisting of the following three competences.

1. Being able to think as a physicist
2. Being able to tackle problems in an experimental manner
3. Being able to tackle problems in a formalizing manner

These three competencies correspond to three quite diverse reasons for teaching physics. A trustworthy justification for the subject of physics as part of the educational system presupposes such nuances, i.e. that it is a subject about the nature, an experimental subject, and a mathematical subject – and the justifications for subjects about nature, for experimental subjects, and for mathematical subjects are not the same. In situations such as that of the Danish upper secondary school there is a need for an external competence description of physics, which can be used to foster communication between the practitioners of physics and the outside world regarding what is lost, when the subject of physics is phased out. The present competence description therefore heralds a review of the relative importance of the different kinds of justifications for teaching physics. Is it the character of physics as a subject about nature which makes it hard to do without? Is it physics as an experimental subject which makes it valuable? Or is it physics as a mathematical subject that constitutes the most important justification for the subject within the educational system?

Being able to think as a physicist

The acquisition of physical concepts and knowledge as a potential profit from physics education is of course important. As an example of this, think about the importance of understanding the concepts of heat and temperature. Some years ago in Denmark, measures taken to save energy resulted in a law stating that the common heating bill in apartment buildings should be distributed between apartments by means of radiator gauges. But radiator gauges do not measure heat expenditure. This requires expensive flow measurements of the radiator water. Instead they measure the surface temperature of the radiator, which is much cheaper to measure. Now, if a radiator is in some way covered up or built in, measures of the heat expenditure will quite surpass the actual heat consumption. On the other hand, money can be saved on the expense of the neighbors by ensuring a high degree of air circulation around the radiator. For that reason, radiator gauges only distribute fairly the expenses if all radiators are placed and built in similarly in all apartments – which is not the case in old apartment buildings. However, for the majority of people in Denmark the concepts of heat and temperature are more or less synonymous. Therefore, the installation of radiator gauges has overall led to energy savings. This would not have been possible had people known that they actually paid for something different than what they thought.

Thus, the competence of ”being able to think as a physicist” – here illustrated by the usefulness of knowing and understanding concepts from physics – is of course not an unimportant part of the potential profit of having been taught physics. Nevertheless, this competence, among the three making up our competence description of physics for external use, is the one which is the easiest to communicate externally.

Being able to tackle problems in an experimental manner

Communicating the second competence to people without any experimental experiences is more challenging. For example, how do you explain the crucial differences of experimental evidence for earth rays and radioactive rays, respectively? Both are impalpable and invisible. In order to understand the differences some already established sense of ”scientific method” is needed. To a certain extent the usefulness of the competence is first recognized when you possess the competence. To experimental competence belongs, besides a methodical approach, also skills of measuring techniques, handling of numbers, calculating with units and assessing uncertainty. But whether focus is on the skills or on the overall experimental grip, one has to ask if these matters may not be equally or perhaps even better motivated in chemistry and biology teaching. In England the introduction of ”combined science”, which integrates physics, chemistry and biology, has promoted biology at the expense of physics in upper secondary school (Smithers & Robinson, 2006). In relation to developing the competence of tackling problems in a formalizing manner, this poses a problem. However, in relation to developing the competence of tackling problems in an experimental manner, this need not necessarily be the case.

Being able to tackle problems in a formalizing manner

Besides being a subject about nature and an experimental subject, physics is also a mathematical subject, which from upper secondary school level onwards trains students in being able to tackle problems in a formalizing manner (see also Jensen, Niss & Jankvist, 2017). To illustrate what we mean by this, we offer the following example of answering the question: *How does the thickness of the ice on a lake of water of 0°C grow with time if the air is freezing cold*? In order to tackle this problem, we first represent it by the simple figure 3.

Next, we analyze the physics in the problem. When the water beneath the ice is freezing, energy is released as heat. This heat is transported away through the ice to the air, allowing the freezing to continue. Our third step is to formalize this in a mathematical manner. From an area A of the ice, an amount of heat $LAdx$ is produced, when the ice is increasing its

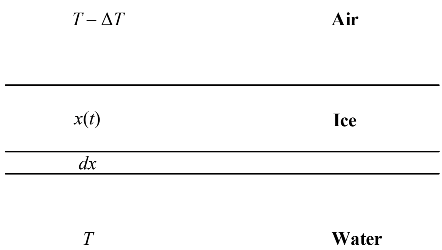

Figure 3. *Growing of ice due to freezing of water from beneath. x(t) is the thickness of the ice, dx an infinitesimal increase of x, T the temperature of the water and ΔT the decrease of temperature from water to air*

thickness with dx, and L is the latent heat of water/ice. If the increase dx of x happens during the time dt, the amount of heat conducted through the area A of the ice into the air at the same time is $(\kappa A\Delta T/x)dt$, where κ is the heat conduction coefficient of the ice. Setting equal the amount of heat produced with the amount of heat conducted away in the time period, dt, we get:

$$LAdx = \frac{\kappa A\Delta T}{x}dt$$

Having mathematized the problem, assuming ΔT constant, it can be solved integrating the above equation. If we measure t from the onset of freezing, we get:

$$x(t) = \left(\frac{2\kappa\Delta Tt}{L}\right)^{\frac{1}{2}}$$

Thus, the thickness of the ice on a lake grows as $t^{1/2}$ from the beginning of the time of freezing. The heat conduction coefficient κ is defined as the proportionality constant between heat flow density and the temperature gradient. In a stationary situation the heat flow in and out of any layer of the ice must be the same. Thus, the temperature gradient has the constant value $\Delta T/x$ throughout the ice.

Discussion part 2

With the above example, we try to insinuate some of the essence in solving problems by means of formalizing. As was the case for the previous problem of the distance to the horizon, the solution to the above problem also makes use of relatively simple mathematics. Still, the problems would be difficult for the majority of upper secondary school students, because the tackling of them presupposes formalizing and mathematization or pre-matematization (notice also that in the horizon problem, a typical perception of the physicist – or the modeler – comes into play, when disregarding the term h^2, such reasoning would not be usual for the pure mathematician). As explained, the third bullet in the competence description for external use of mathematics – being able to bring mathematics in play in extra-mathematical settings – concerns exactly this aspect. Hence, it more or less coincides with the third bullet in the competence description for external use of physics – being able to perform formalizing problem tackling. The outlined competence descriptions for external use therefore herald a dialogue between both mathematics and physics themselves as well as in relation to their surroundings. Does the teaching of mathematics deliver the kind of competence wanted by society? Is the main potential of physics to be a practice ground for bringing mathematics in play in extra-mathematical situations? Is it the teaching of mathematics or the teaching of physics which holds the largest potential to educate students to be able to tackle, for example, problems of infinitesimal calculus as that of the growth of ice on a lake's surface or problems of geometry as that of the distance to the horizon? Competence descriptions for external use can make up the departure point for such interdisciplinary dialogue.

Without dialogue between the mathematics educators and physics educators, both internally and with the surrounding world, there is a risk that the educational system's support to develop students' competences for formalizing problem solving withers away. Who picks up the gauntlet, if mathematics is oriented towards proofs and proving, and physics is oriented towards the laws of nature? Does not the responsibility for teaching this competence then fall between two stools? Who in the surrounding world is to detect such lack of responsibility before it is too late and without being made aware of it? This is to say that it is not sufficient that only a minority of the population become acquainted with physics as a mathematical subject, and that we have to consider the danger of the subject of mathematics being reduced to memorizing recipes and pushing buttons. Despite the different justification problems of mathematics education and physics education, both the quantitative success and qualitative letdown of mathematics, and the subject of physics' retention of qualitative standards with the quantitative letdown as a consequence, is connected to a too weak understanding of mathematical modelling. It is realized as important for society, but not realized as a distinctive competence.

As initially mentioned, many of for example OECD's competence descriptions do not relate to a discipline (subject) at all, but to cross-curricular student activities, teaching methods, school organization, etc. And surely subjects have many common challenges of competence development in common, e.g. dedication, precision, critical sense, etc. Such general competences are not the topic of interest in this article. Rather our focus is on the communication between disciplinary practitioners and their surroundings in relation to the various disciplines' specific potential contributions to students' competence development. From experiences at Roskilde University we know that it is difficult to carry out common discussions about interdisciplinary collaborative teaching (Jensen & Jankvist, 2018). It is simply difficult to have discussions across disciplines between their disciplinary practitioners, because one views his or her own discipline from the inside and other's disciplines from the outside.

The term "interdisciplinarity" is often used within quite different contexts given the term's different meanings (Jensen, 1991, 2012). It is used in the process of establishing new disciplines on the border between already established disciplines. It is used in the endeavors to orient already established disciplines towards various functions and problems in different applicational contexts. Finally, it is used when there are calls to cognitively bridge the divide between disciplines. We imagine that competence descriptions for external use may be of assistance in all such cases, although in particular in situations calling for some kind of "bridge building".

In a sense, the preparation of competence descriptions for external use may be compared to "speaking to blind people about colors". Disciplines may to varying degrees be constituted by specific domains of knowledge, specific methods, or by specific ways of perception (Jensen & Jankvist, 2018). The more a discipline's center of gravity is made up by a specific way of perception, the more difficult external communication becomes; the character of the specific way of perception is difficult to comprehend, if it is not itself mastered to a certain extent. Both mathematics and physics are disciplines which to a very large degree are constituted by their different, fundamental ways of perception. Hence, mathematics and physics are among the disciplines that are particularly challenged when involved in interdisciplinary activities. One of the most challenging aspects is to achieve insightful communication with other disciplines concerning their contributions in planning educational programs. Our competence descriptions of mathematics and physics for external use are thus also meant as a door opener in this situation. Is it, for example, physics, chemistry or biology which first and foremost are to deliver the competence of "being able to tackle problems in an experi-mental manner"? Or is it in the mathematics teaching or the physics teaching that the competence of "being able to bring mathematics in play in extra-mathematical settings"/"being able to tackle problems in a formali-zing manner" is to be developed? Competence descriptions of disciplines for external use may be means for dealing with such questions.

Concluding remarks

In this article, we have augmented the general, non-disciplinary competence descriptions of what purpose the educational programs might possibly serve and the disciplinary competence descriptions for internal use with two examples of different competence descriptions – descriptions for use in the discussion of disciplines' potential contribution to its surroundings, not least in interdisciplinary collaborations for the purpose of fulfillment of the educational programs' overall purpose. We have named these "disciplinary competence descriptions for external use". While the purpose of the disciplinary competence descriptions for internal use is to ease the "vertical communication" within the disciplines, the purpose of disciplinary competency descriptions for external use is to ease the "horizontal communication" between disciplines and their surroundings, i.e. between practitioners of the disciplines and the recipients.

Let us illustrate with a metaphor: As a precondition for well-functioning international cooperation, it is important that the national cultures are open to mutual understanding. And this demands much more than formal tolerance. It is necessary to consider how one declares oneself to the others. This is also the case with interdisciplinary collaboration – in all its different shapes. Competence descriptions for external use may contribute to qualifying considerations regarding how the nature of interdisciplinary collaborations depends on the types of involved disciplines.

There is, we argue, a need for different degrees of detail and accentuation of involved sub-competences depending on whether the target group is made up of the practitioners of the discipline or of collaborators from other disciplinary areas. The most defining needs for externally oriented competence descriptions arise once disciplines are to contribute to complex abstract wholes outside the disciplines themselves. For example, due to a lack of mutual meta-understanding it is difficult to find a balance between elimination and disciplinary fetishism once one discipline is to act as support for another discipline. Or generally speaking, when entire educational programs are planned involving several disciplines. Returning to the metaphor, as in international collaboration this requires a common and mutual understanding of each other's cultures. Disciplinary competence descriptions for external use may assist in providing just that.

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Notes

1 Similar international tendencies are reported by Jørgensen (1998) and Smithers and Robinson (2006).

2 Of course, a competence description of physics for internal use could also be given, but that is not our purpose here.

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