Discourses in school algebra: the textbooks' different views on algebra and the positioning of students

KRISTINA PALM KAPLAN

The purpose of this study is to understand the school algebra offered in Swedish mathematic textbooks for grade 8. Using a social semiotic perspective, textbook tasks are analysed with a method inspired by Systemic Functional Linguistics. Five school algebra discourses are identified: symbolic discourse, geometrical discourse, arithmetical discourse, (un)realistic discourse and the scientific discourse. It is argued that these offer different views on the nature of algebra and the positioning of students.

How we write mathematics in school matter. Student tasks and especially word problems have been analysed from sociological, ideological and critical perspectives (cf. Dowling, 1996; Herbel-Eisenmann, 2007; Le Roux, 2008) to highlight how different tasks position students either as subjects who already have access to mathematics and mathematical language, or as scribblers[1] rather learning about themselves than learning mathematics. Different word problems may offer students different access to the mathematics presented (Dowling, 1996) and even the view of mathematics may be different. Similar issues have been addressed in investigations of how the mathematical discourse in high-stakes mathematics examinations changes over time (Morgan, 2016; Morgan & Sfard, 2016; Morgan & Tang, 2016). Morgan (2016, p. 124) claims "As mathematical discourse construes ways for human beings to relate to mathematical objects and processes, students may find it more or less possible to accommodate themselves to these relationships". Discourse is then relevant to study as a prerequisite to equal access to education.

Discourse research in mathematics education has expanded during the last 20 years, one of its strands using social semiotics and *Systemic functional linguistics*, SFL (Sfard, 2013). Analytical tools developed within

Kristina Palm Kaplan
Uppsala University

Palm Kaplan, K. (2018). Discourses in school algebra: the textbooks' different views on algebra and the positioning of students. *Nordic Studies in Mathematics Education*, 23 (1), 47–65.

SFL open up the possibility of studying features of subject-specific language (Bergvall, 2016; Schleppegrell, 2004) as well as ideological and epistemological aspects (Herbel-Eisenmann, 2007; Morgan, 2006).

In this study, the textbook and the meanings it offers are focused, as a starting point for further studies of how textbooks may influence student's views on and achievements in algebra. The study seeks to deepen our understanding of school algebra by exploring *school algebra discourse(s)*. Through these discourse(s), human agents are defined, constructed and positioned (Luke, 1995) in relation to algebra. It is also part of its purpose to demonstrate how SFL can be used as a tool to investigate this. Therefore, textbook tasks are analysed to answer the research questions:

- What school algebra discourse(s) can be identified in Swedish textbook tasks?
- How is the nature of algebra construed in school algebra discourse(s)?
- What kinds of activities and positions do school algebra discourse(s) invite to?

Textbook research from a Nordic and an international perspective

This study can fill a gap in the Nordic and particularly the Swedish context, since neither discourses in algebra or in textbooks have been studied here. Within the Nordic context, research on the textbook mainly uses content analysis and mostly concern highly limited mathematical content areas or didactical aspects of the contents such as cognitive demands, according to an overview (Rezat & Strässer, 2015). This general description also applies to Nordic studies on algebra and textbooks. Kongelf (2015) characterises critical aspects and deficiencies of algebra introductions with a qualitative content analysis in a Norwegian study. School algebra has also been studied in textbooks for Swedish upper secondary school by Jakobsson-Åhl (2006) with respect to content variation in a historical perspective.

Though this study mainly contributes to a Nordic context, it may be considered as a methodological contribution to international research by studying secondary school textbooks and using analytical tools developed within SFL. An overview (Fan, Zhu & Miao, 2013) on textbook research states that many studies are conducted in books for primary school. Internationally, textbook research is dominated by document analysis and methodological questions have not been widely addressed (Fan, 2013). This claim is supported by the Proceedings of the International Conference on Mathematics Textbook Research and Development (Jones,

Bokhove, Howson & Fan, 2014) since most of its long papers concern the textbook itself and focus on content analysis, cognitive or technological aspects. Only one of the contributions uses SFL: Alshwaikh (2016; Alshwaikh & Morgan, 2014) shows how the nature of mathematics is construed as timeless in an English and a Palestinian geometry textbook, but the English book uses less subject-specific language and instead links to everyday objects and activities. While the role of the learner is construed either as a "thinker" or a "scribbler", the latter is the most prominent in the English book.

There is a lack of studies which analyse discourse in textbooks (Ryve, 2011) but mathematical discourse has been analysed with tools from SFL earlier. The project The Evolution of the Discourse of School Mathematics (EDSM) (Morgan, 2016; Morgan & Tang, 2016) reports that the mathematical discourse may both help students in participating and hinder them from preparation for higher studies in mathematics. In avoiding linguistically challenging language, the nature of the mathematics has been changed in ways which lower the expectations on students and the tasks demand students to perform isolated manipulations instead of using algebra to solve problems. The EDSM studies show how SFL has the potential to clarify how choices in language are never neutral. Instead, these choices are manifestations of discourse and power in "quotidian aspects of text in use" (Luke, 1995, p. 11), pointing to patterns of social reproduction and cultural representation. Further, according to a quantitative SFL analysis of TIMSS tasks (Bergvall, 2016) the content areas of arithmetic, algebra, geometry and statistics have different profiles when it comes to linguistic features. This indicates that there may be several discourses at play in school mathematics, which makes the distinction by the EDSM project between mathematical and non-mathematical discourse insufficient.

Discourse and textbook tasks

To study how students are invited to engage in algebra, discourses are identified in textbook tasks. Textbook tasks are texts read by students, ideally with the purpose of performing some activity to solve the task and learn mathematics. Luke (1995, p. 15) states that discourses can be seen as "recurrent statements and wordings across texts" which together establish systems of meaning connected to different ideas and versions of the world. Thus discourses are built up by patterns of linguistic resources and at the same time they steer the possibilities of choices for expressing different ideas. Hence, discourses are possible to identify through the choice of linguistic resources "specialized to construct meanings for the particular field of relevant knowledge and belief" (ibid, p. 15). This makes

the question of which discourse(s) dominate a particular field interesting. Therefore, the *school algebra discourse(s)* are defined as one or several discourses which construct systems of meanings for the field of algebra through text and thus position students in relation to algebra.

Methods

Analytic framework and process of analysis

The study uses *Systemic functional linguistics*, SFL (Halliday & Hasan, 1989; Morgan, 2006) to identify the school algebra discourse(s). Here text is studied from a functional perspective. For example, the use of passive voice and nominalisations function to obscure the human agency in mathematical action (Morgan, 2006). If human agency is obscured in the text, the meaning offered is that mathematical activity does not depend on humans (ibid.).

According to SFL, language fulfills three different functions as texts draw on different linguistic resources (Halliday & Hasan, 1989). Through the *ideational function*, experiential meanings are realised by the choices of what participants figure in the text, and by the choices of processes, e.g. doings and events. Through the *interpersonal function* the kind of relation which is established to the reader and between reader and content, is realised by choices of mood. These can be either declarative statements or demanding *questions* and *requests*, e.g. imperatives. Further, this function involves degrees of modality, e.g. modifying expressions of *probability*, *usuality*, *obligation* or *ability*; and how the reader is addressed, e.g. pronouns. Lastly, the *textual function* realises the organisation of language by choices in textual themes and cohesive relations. However, textbook tasks are very short texts and a pilot study of the textual function in the tasks did not contribute further to identify and discern the school algebra discourses. Therefore, the textual function is not analysed in this study.

The analytic framework is captured in table 1 (cf. Alshwaikh, 2016). Examples of how the chosen linguistic resources in the studied textbooks function, follow in the results' section. The findings were discussed at seminars in order to validate the process of analysis as well as the discernment and identification of different discourses. This resulted in a few adjustments. Some previously unclear cases were moved between discourses, a hybrid category of tasks was added and it was decided not to try and fit all of the material in to the discourses but to leave some odd tasks aside as *algebra outside of the discourses*. These will be described in the next section.

Table 1. *The analytic framework*

Field of discourse, realised by the ideational function	
Questions guiding analysis	Indicators in text
How is the nature of algebra construed? What is happening and who are taking part? What processes are human agents engaged in and to what extent do they do algebra?	Types of processes: relational, verbal, material* verbs, e.g. *is*, *explain*, *simplify* Types of participants: personal pronouns or proper names; subject specific or otherwise specialised vocabulary, mathematical symbols; images or tables with information needed to solve the task Presence of human agents, passive verb form Modifiers, e.g. *can*, *will*, *may*
Tenor of discourse, realised by the interpersonal function	
Questions guiding analysis	Indicators in text
What kind of action is the student expected to engage in? How is the student addressed? Are choices or decisions available to the student?	Types of processes in imperatives: relational, verbal, material Questions or requests, personal pronouns, e.g. *you*, *we* Modifiers, e.g. *can*, *will*, *may*

Note. *It is possible to make a more fine-grained distinction of processes within SFL. This would involve deciding whether processes, e.g. *calculate*, *work out* and *solve* are mental, behavioural or material. Since this might open up long epistemological discussions, this possibility is not used in the study.

The textbook materials

In Sweden, mathematics textbooks are primarily used for task-solving (Sidenvall, Lithner & Jäder, 2015). There are four publishers of mathematics textbooks, designed to encompass the whole curriculum, for lower secondary school.[2] For the analysis, the first published textbooks following the curricula of 1994 and of 2011 were chosen from one textbook series each by three of these publishers. One is excluded since their textbook series following the curriculum of 1994 was only published in one edition. To facilitate a comparison between the different editions of the same book series, this publisher's new textbook series following the curriculum of 2011 was also omitted. The chosen textbooks are presented in table 2.

Some of the authors of the Y-series and F-series have published textbooks since the 1970s and the 1980s, respectively. In contrast, the MD series' authors were newcomers by the turn of the century. A qualitative case study like this can show the existence of different discourses. The broad selection of textbooks, reaching over two curricula and different

Table 2. *The textbook material*

Book	Label	Publisher	Curricula	Number of tasks
Matemaikboken Y Röd (1996)	Y96	Liber	1994	338
Matemaikboken Y (2012)	Y12	Liber	2011	455
Matte Direkt år 8 (2002)	MD02	Bonnier/ Sanoma	1994	160
Matte Direkt år 8 (2010)	MD10	Bonnier/ Sanoma	2011	154
Formula 8 Matematik (2007)	F07	Gleerups	1994	196
Formula 8 Matematik (2013)	F13	Gleerups	2011	254

stances in the publishing market, enables looking at the research problem as a more stable phenomenon than if only textbooks from one publisher or one curriculum had been selected.

Textbooks from lower secondary school were chosen to include as much algebra as possible within the obligatory part of the school system. Since the last grade tends to summarise the whole of the curriculum for the compulsory school, it could be expected to have a different character than the rest. Therefore grade 8 (14–15 years) was chosen.

In the selected textbooks, chapters named *Algebra*, *Equations*, *Expressions and equations*, *Algebra and patterns* and similar, are studied. These were chosen since they explicitly focus on algebra[3]. All tasks in the algebra chapters are analysed, including all tasks in all special themes in the chapters such as *Activity, Reflect, Challenges, Work together, Problem solving, True or false, Reason and Develop* et cetera. In total, this amounts to 1557 tasks.

As noted above, not all of the tasks fit into the school algebra discourses identified. 27 tasks (1.7 %) are *hybrids* of more discourses than one and 34 tasks (2.2 %) cannot be identified as belonging to any of the discourses, though they clearly concern algebra. There are two primary tendencies in this *algebra outside the discourses*, but it is questionable to speak of two more discourses since they would include less than 1% each of the material. Finally, 6.3 % of the tasks do not fit into the discourses because they are *not algebraic*[4].

Results – five school algebra discourses

In the analysis, five school algebra discourses are identified. They have been named after the most prominent feature within the material,

which is the difference in participants and processes. These discourses are *symbolic discourse*, *geometrical discourse*, *arithmetical discourse*, *(un)realistic discourse* and *scientific discourse*. They are presented shortly in table 3, where the activities that the students are supposed to engage in are described by examples of prominent and deviant features.

Table 3. *Characteristics of the school algebra discourses*

School algebra discourse	How is the nature of algebra construed? What is happening and who are taking part? What processes are human agents engaged in and to what extent do they do algebra?	What kind of action is the student expected to engage in? How is the student addressed? Are choices or decisions available to the student?
Symbolic discourse	Algebra is depersonalised. No one does algebra except for the student in the demands. In a few tasks with modifiers, the student is explicitly addressed, signalling possibilities to do algebra.	The student is implicitly addressed. *Calculate*, *solve*, *simplify*, *check*, *work out*. Sometimes explicitly addressed, with modifiers: *think*, *explain* and *discuss*. Decisions available in a few tasks, distinguishing true from false.
Geometrical discourse	Algebra is atemporal and depersonalised; relationships between geometrical objects and symbols are in focus. No human agent is present except sometimes when human agency is obscured. A few tasks signal possibilities to do algebra with modifiers.	The student is implicitly addressed. *Simplify*, *calculate*, *express*, *let*; in a few tasks *discuss*. Decisions available in a few tasks, distinguishing true from false.
Arithmetical discourse	Algebra is a human activity, lots of human agents participate, but sometimes human agency is obscured or absent. The activities consists of posing and solving number riddles, using the four rules of arithmetic.	The student is implicitly or explicitly addressed. *Simplify*, *use*, *calculate*, *set up*, *multiply*, *double* and *add;* also *think*, *discuss*, *show* and *explain*.
(Un)realistic discourse	Algebra is an artificial gaze upon the real world. No one does algebra except for the student in the demands. Lots of human agents are participating in doing activities which are not algebraic.	The student is implicitly addressed, sometimes explicitly. *Express*, *simplify*, *solve*, *control*, *and work out*, *interpret*; sometimes *explain* and *let*.
Scientific discourse	Algebra is depersonalised calculating on physical and societal phenomena. Human agency is obscured. In some tasks, modifiers signal possibilities to use algebra in physics and social science. Activity is depersonalised, but the possibility to act is offered.	The student is implicitly addressed. The only processes are *calculate*, *use* and *round off*.

Symbolic discourse

The most prominent feature within the *symbolic discourse* is the extensive use of mathematical symbols and subject-specific words such as *equation*, *expression*, *value*, *parenthesis*, *solution* and *method*. In example 1 the participant *the equations* is specified by the mathematical symbols forming the equations.

Ex. 1.
Solve the equations
a) $3x + 11 = 5x - 3$ b) $7x - 20 = 2x - 10$ (Y96, p. 219)

Generally there are no separate clauses with statements about what is happening and who are taking part, the mathematical symbols are presented in requests. Personal pronouns or proper names are not given, so algebra is construed as depersonalised. The only process is in the request, e.g. *solve* in the above example.

The requests in the *symbolic discourse* demand the student to engage in material processes such as *calculate*, *solve*, *simplify*, *check*, *take away*, *use*, *write* and *arrange*. Thus action is construed as handling and manipulating symbols.

In some tasks, questions function like requests. Since the questions are formulated by an author who, at the same time, provides the answer in a section in the back of the textbook, the function of the question is just as authoritative, as of the requests, but perhaps more polite. Often these tasks include multiple choices with several possible right answers to identify.

Ex. 2.
Which expressions yield the same answer?
P $2a - (2a + 2b) - 2b$
Q $2a - 2a + 2b - 2b$
R $(2a - 2a) + (2b - 2b)$ (MD02, p. 81)

The action here may be less about manipulating and more about thinking, but this is not possible to determine, since what the student is supposed to engage in is hidden in the question.

Beyond this, in the *symbolic discourse* there is, to a smaller extent, a kind of task which actually may include both personal pronouns and modifiers.

Ex. 3.
The equation $5x + 2y = 29$ contains two different unknowns.
a) Can you find any solution to the equation?
b) How many solutions do you think there are? (Y12, p. 238)

The pronoun *you* and the modifier *can* indicate human presence and the possibility of the student engaging in algebra. This kind of task is more frequent in special themes and in textbooks following the latest curriculum, but it may be due to a larger number of special themes in Y12 than in Y96.

Also in these tasks, the student may be demanded to engage in verbal processes, e.g. *explain*, *discuss* and *describe*. This entails that the action is construed in more participatory than manipulating ways in these tasks.

Finally, the discourse encompasses some tasks solely found in one of the textbook series in a recurring special theme labelled *True or false?* Most of this theme's tasks can be identified as belonging in the *symbolic discourse*.

Ex. 4.
$a \cdot a = 2a$ (MD02, p. 85)

The task in example 4 includes no processes in written language, only in symbols. It addresses the student implicitly through the heading *True or false?*, but no other human agents are present. This construes algebra as atemporal; nothing happens. The omitted processes obscure whether the student is requested to distinguish true statements from false, or to motivate this as well. However, indirectly action is construed as expressing an attitude and making decisions. So in these few tasks, decisions are available to the student.

Clearly algebra is construed as depersonalised in the *symbolic discourse* and action as handling and manipulating symbols is dominant. Meanwhile, a small amount of tasks suggest more participatory and decision-making ways for the student to engage.

Geometrical discourse

A typical feature of the *geometrical discourse* is the many relational processes. They are used to express different types of relations between geometrical objects, measurements and symbols. For instance, in example 5, the participant *angle A* is identified as 45°.

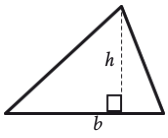
Ex. 5.
In a triangle, angle A is 45°. Angle B is twice as big as angle C.
How big are the angles B and C? (F07, p. 209)

These relationships are described either in written language, in images of geometrical objects or in both combined.[5] Since named persons or pronouns are not present in the relational processes, algebra is construed as both atemporal and depersonalised.

However, in a few tasks with passive verb forms, traces of humans can be found. In example 6, the area *can be calculated*. Calculating is a material process performed by humans, but the passive verb form functions to hide human agency.

Ex. 6.
The area of a triangle can be calculated by the formula $A = \frac{b \cdot h}{2}$ where

A = the area
b = the base
h = the height.



Calculate the area of a triangle with ...
a) the base 4 cm and the height 3 cm
b) the base 12 cm and the height 6 cm (Y96, p. 189)

Example 6 features the modifier *can*, which in this case functions to modify the calculation, stating that it is possible to calculate the area. Thus the modifier opens up a perspective of what can be achieved by algebra.

In this discourse, there are also questions functioning as requests. This is seen in example 5. The requests addressing the student are typically realised by the material processes *simplify*, *calculate*, *write* and *express*, as well as, in a few cases, *perform*, *work out*, *set up*, *draw* and *solve*.

In some exceptional cases, the processes in the requests are verbal, e.g. *discuss*, which is found in a special theme. Other special cases of the discourse are identified in the *True or false*?-theme mentioned earlier.

On the whole, the *geometrical discourse* is characterised by atemporal relations between different geometrical objects where human beings are not present. Action is mainly construed as manipulating symbols and as expressing with symbols, but just as in the *symbolic discourse*, a small amount of tasks suggest participatory or decision-making action.

Arithmetical discourse

Statements about subject-specific participants such as numbers, in combination with subject-specific material processes, characterise the *arithmetical discourse*. It differs from *symbolic discourse* in the respect that the participants are formulated in written language and not in mathematical symbols and they typically include arithmetical operations. This is seen in example 7, where the subject-specific participants *a number* and *the sum* take part in the subject-specific processes *is added* and *is multiplied*; and when *one*[6], an unspecific human participant, *multiplies the number*[7].

There are also relational processes, e.g. *becomes*, which function as the equal sign in an equation.

> Ex. 7.
> A number is added to 5, thereafter the sum is multiplied by 2. The product becomes equally large as when one multiplies the number by 3. Set up an equation and work out the number. (MD10, p. 100)

Another typical feature in this discourse is the many different ways to describe how human agents are taking part. In the above example, the passive verb form *is added* is used as well as the active verb construction *multiplies*, which is used with the pronoun *one*. These linguistic resources function to obscure human agency. In this discourse, human agents are sometimes totally absent, or quite on the contrary, present as a named fictional third person, e.g. *Carl* in the number riddle below, or present as *you*; or even as *we* or *I*.

> Ex. 8.
> Carl is thinking of a number. He multiplies the number by 4 and then adds the product to 8 and gets the result 36. Which number is Carl thinking of? (F07, p. 209)

Thus, algebra within the arithmetical discourse is construed in a variety of ways, either as depersonalised or, more often, as a human activity. Since human agents are engaged in subject-specific material processes, they are doing algebra, and they are engaged in using the four rules of arithmetic. Often this includes thinking of numbers and, implicitly or explicitly, posing number riddles to the student. Typically, a number is known to some human agent but unknown to the student.

As seen in the above examples, the student can be addressed both with requests and questions functioning as requests in the *arithmetical discourse* and the addressing can be either explicit or implicit. The student is supposed to engage in material processes, e.g. *express*, *simplify*, *use*, *calculate* and *write*. There is also another kind of question, asking the student to *discover* and to *think*. This construes action in manipulating, expressing, as well as in more exploratory ways. As in the *symbolic discourse*, when the few tasks in the special theme sections are considered, other processes emerge, e.g. *multiply* (along with the other arithmetical operations), *think*, *choose*, *try*, *show*, *explain* and *create*. Some of these are verbal and some material. They construe action to a greater extent in participatory and exploratory ways, than the previously mentioned processes. These tasks as well as tasks with the more open kind of question sometimes feature tables.

To summarise, algebra in the *arithmetical discourse* mostly deals with number riddles. Obviously, there are human agents posing these riddles, as well as solving them. The student is supposed to engage mainly in solving the riddles posed by fictive first or third persons, but when demanded to pose these riddles or when tables are featured, the student is also offered another range of choices of activities to engage in.

(Un)realistic discourse

Things, phenomena and named third persons engaged in relational or material processes take part in the statements made within the *(un)realistic discourse*. These things are often nutriments or other everyday goods in economic relations to unknown quantities, as in exemaple 9.

> Ex. 9.
> The tickets to a circus cost a SEK per item. Write an expression of how much
> a) 5 tickets cost
> b) Emma gets in change on a 500-SEK banknote when she buys 2 tickets
> c) Sara gets in change when she buys 3 tickets (F13, p. 200)

In this task, the participant *The tickets to a circus* form a relational process, i.e. *cost,* to *a SEK per item*. But when you buy things, they always cost a known amount of money. Hence, the word *(un)realistic* (Verschaffel, Van Dooren, Greer & Mukhopadhyay, 2010; Palm, 2002) alludes to these tasks presenting everyday situations with more or less resemblance of how people act and use algebra in such situations in reality. In tasks concerning e.g. corporate accounting, the algebra used may show more resemblance to corporate accounting in reality, while in example 9 the algebra used shows less resemblance to the situation of buying tickets. Sometimes images or tables connect to these situations.

Emma in example 9 gets and *buys*, i.e. engages in material processes which are not about doing algebra. Neither does *Elias*, in example 10 below, who takes part in the relational process of being *x years old*.

> Ex. 10.
> Elias is x years old.
>
> $\frac{x}{3}$ years $(x+3)$ years x years $(x-3)$ years $3x$ years $5x$ years
>
> What or which of the expressions above can be valid for the age of his
> a) mother b) grandmother c) twin brother d) little sister (F13, p. 200)

So, human agents in this discourse do not typically do algebra. Instead they enter economic transactions, e.g. *selling*, *paying*, *earning*; engage in material processes like *baking*, *swimming*, *driving*; or in relations like *Elias*.

However, algebra cannot be said to be atemporal or depersonalised in this discourse, since it is neither a question of relations between objects existing in themselves, nor of processes which can be described as algebraic. Instead, the nature of algebra is a constrained ontological gaze upon reality which constructs the world as inherently algebraic.

Although no other human agents engage in algebra in this discourse, the student is supposed to. This is achieved in requests with material processes, e.g. to *express*, *simplify*, *solve*, *write*, *control*, *work out*, *set up*, *use*, *let* and the verbal process *explain*. This construes the action as manipulating symbols and as expressing with symbols, but sometimes in this discourse the student is explicitly addressed as *you*, or in questions instead of requests. In special theme tasks there are processes asking the student to *discuss*, *ask*, *exemplify*, *investigate* and *invent*. This contributes to construing action in participatory and exploratory ways as well.

Altogether, the *(un)realistic discourse* construes algebra as a forced ontological perspective of reality, devoid of human agents creating or participating in algebra. At the same time, the student is more often explicitly addressed than in the *symbolic* and *geometrical discourses*. The action is construed in many ways, though mostly as manipulating symbols and as expressing with symbols.

Scientific discourse

Statements in this discourse contain mathematical symbols and concern participants consisting of physical or social phenomena. Most of them form quite long noun phrases, e.g. *The speed of sound at different temperatures* in example 11, and the processes are material and typically something which *is/can be/could be calculated*.

> Ex. 11.
> The speed of sound at different temperatures is calculated with the formula
> $v = 332\,(1 + 0{,}0018t)$ where
> v = the velocity of sound in meters per second
> t = the temperature in degrees Celsius
>
> Calculate the velocity of sound at the temperature …
> a) 0° C b) 10° C c) 20° C (Y96, p. 191)

Only a few other processes are used in this discourse. In addition to this, there are mathematical symbols, expressions or formulas in the tasks.

Compared to the other discourses, the processes here are more often written in the passive verb form and in tasks without the passive verb form, the pronoun *one* is used. This hides human agency, so in the

scientific discourse, algebra is depersonalised. Since depersonalisation and long, abstract noun phrases are common features in scientific writing (Christie, 2012), this is what the name of the discourse indicates.

As described earlier, the verb *can* functions to state that the processes are possible to do. This opens up a perspective of what can be achieved by algebra. Altogether this construes algebra as something which can be used for participating in science, though by whom is obscured.

Just as in the other discourses, there are questions functioning as requests. The student is restricted in the *scientific discourse* to engaging in the material processes to *calculate*, *use* and *round off*[8], and is not addressed explicitly.

To sum up, the *scientific discourse* presents algebra as possible to use in other sciences. At the same time, it is depersonalised and the student's range of action is limited compared with the other discourses.

Extent of discourses, activities and positions offered to the student

The extent to which the school algebra discourses occur in the textbooks is described in table 4. The school algebra discourses are unevenly distributed compared to each other and there are differences between the textbooks as well. This distribution in time, represented by the two curricula periods, and in space, represented by three different textbook series, contributes to an understanding of the school algebra discourses as being established and somewhat resilient to reform.

As summarised by table 5, the discourses mainly invite to activities as material processes, e.g. calculating, manipulating and forming symbolic

Table 4. *Percentage of tasks in respective school algebraic discourse, per textbook*

School algebra discourse	Curriculum 1994 and textbook			Curriculum 2011 and textbook		
	Y96 $n = 338$	MD02 $n = 160$	F07 $n = 196$	Y12 $n = 455$	MD10 $n = 154$	F13 $n = 254$
Symbolic discourse	54%	53%	45%	36%	56%	44%
Geometrical discourse	12%	18%	20%	10%	16%	28%
Arithmetical discourse	9%	9%	16%	8%	7%	12%
(Un)realistic discourse	12%	7%	15%	25%	18%	13%
Scientific discourse	5%	0%	0%	1%	0%	0%
Hybrids	0%	2%	2%	3%	1%	2%
Algebra outside the discourses	0%	0%	1%	7%	0%	0%
Not algebraic	9%	13%	1%	10%	1%	0%

expressions in different situations. Activities as verbal processes, e.g. discussing, explaining and decision making, are available, but less common.

The *symbolic* and *geometrical discourse* together dominates the material. They have an impersonal, more academic tone, they do not refer to anything outside mathematics, and the address is often imperative. These features function to position the student as someone who already can read and use (Le Roux, 2008) algebra. But there is a small amount of participatory and decision-making ways to engage in these discourses, which may instead position the student as an apprentice.

Less prominent are the *arithmetical* and the *(un)realistic discourse*. They amount more often to a personal address, more human agents are present and the student is offered expressing, manipulating and participating actions in algebra. Though these features may be inviting, these discourses are dominated by number riddles and more or less unrealistic everyday situations: 14 to 15 year old students are positioned as children and artificial consumers. Further, the algebra is often presented as processes, e.g. by the use of words for operating arithmetically instead of symbols for arithmetical operations. This is less challenging than mathematical objects, so there are reasons to wonder if tasks like these prepare students for higher studies or not (Morgan, 2016): especially considering that the amount of tasks in the *(un)realistic discourse* has augmented between the two curricula investigated.

By contrast, the *scientific discourse* positions the student as a growing adolescent, since linguistic features as passive verb forms and abstract

Table 5. *Activities which the school algebra discourses invite to*

School algebra discourse	Main activities offered to the student	Less prominent activities offered to the student
Symbolic	Calculating values and manipulating symbols	Discussing and explaining symbolic expressions, or making decisions
Geometrical	Expressing with symbols, manipulating symbols and calculating values in geometrical relations	Discussing geometrical relations, or making decisions
Arithmetical	Expressing relationships in symbols and solving riddles	Posing riddles, explaining or creating with numbers and symbolic expressions
(Un)realistic	Expressing with symbols, manipulating symbols, and calculating, in more or less artificial everyday situations	Discussing and exemplifying symbolic expressions in more or less artificial everyday situations
Scientific	Calculating in scientific situations	–

noun phrases characterise a more complex language (Christie, 2012). However, this discourse only offers the student to engage in calculating.

Conclusions

This study identifies five discourses in Swedish year 8 textbooks, some more dominant than others. These discourses express different views on the nature of algebra, offer mainly material processes, and position the student in different ways. Considering that there are few changes in connection to the curriculum reform, these discourses are quite stable. In the Nordic context, these are new findings. Methodologically, the study exemplifies how tools within SFL can be used to analyse discourses in mathematics texts. It contributes to research on discourse (cf. Ryve, 2011) as well as textbooks (cf. Rezat & Strässer, 2015).

Internationally, it has been argued that some tasks offer students less access to mathematics and the student's access to mathematics is differentiated by the tasks (cf. Dowling, 1996; Le Roux, 2008). While Dowling's study (1996, p. 390) compares textbooks which vary as a consequence of being "differentiated by student 'ability' ", the different school algebra discourses vary between pages in the same book or even tasks on the same page. Further, while Alshwaikh (2016; Alshwaikh & Morgan, 2014) reports different student positions in textbooks from two countries, this study shows that both different views of mathematics and different student positions are constructed even within one and the same textbook. Thus Swedish textbooks present a less coherent picture of mathematics. According to Schleppegrell (2004, p. 141) similar kinds of lack of coherence "provides a poor model" for understanding how information is built up and accumulated. Therefore, how students respond to this incoherent picture of algebra should be further investigated.

Acknowledgements

The author is grateful for valuable comments on the analysis by Caroline Liberg and, on earlier versions of the article, by Johan Prytz.

Sources

Carlsson, S., Hake, K.-B. & Öberg, B. (2002). *Matte Direkt år 8*. Stockholm: Bonnier Utbildning

Carlsson, S., Hake, K.-B. & Öberg, B. (2010) *Matte Direkt år 8*. Stockholm: Sanoma Utbildning

Mårtensson, G., Sjöström, B. & Svensson, P. (2007). *Formula 8 Matematik*. Malmö: Gleerups
Mårtensson, G., Sjöström, B. & Svensson, P. (2013). *Formula 8 Matematik*. Malmö: Gleerups
Undvall, L., Johnsson, K. & Welén, C. (2012) *Matematikboken Y*. Stockholm: Liber.
Undvall, L., Olofsson K. G. & Forsberg, S. (1996). *Matematikboken Y (Röd)*. Stockholm: Liber

References

Alshwaikh, J. (2016). Investigating the geometry curriculum in Palestinian textbooks: towards multimodal analysis of Arabic mathematics discourse. *Research in Mathematics Education*, 18 (2), 165–181. doi: 10.1080/14794802.2016.1177580
Alshwaikh, J. & Morgan, C. (2014). The creation of mathematics in school textbooks: Palestine and England as example. In K. Jones, C. Bokhove, A. G. Howson & L. Fan (Eds.), *Proceedings of the International Conference on Mathematics Textbook Research and Development* (ICMT-2014) (pp. 141–146). School of Education, University of Southampton.
Bergvall, I. (2016). *Bokstavligt, bildligt och symboliskt i skolans matematik: en studie om ämnesspråk i TIMSS*. Uppsala: Acta Universitatis Upsaliensis.
Christie, F. (2012). *Language education throughout the school years: a functional perspective. Language Learning*, 62 (Supplement 1), 1–247.
Dowling, P. (1996). A sociological analysis of school mathematics texts. *Educational Studies in Mathematics*, 31 (4), 389–415.
Fan, L., Zhu, Y. & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *ZDM*, 45 (5), 633–646. doi: 10.1007/s11858-013-0539-x
Halliday, M. A. K. & Hasan, R. (1989). *Language, context, and text: aspects of language in a social-semiotic perspective*. Oxford University Press. Retrieved from https://catalog.hathitrust.org/Record/001833733
Herbel-Eisenmann, B. A. (2007). From intended curriculum to written curriculum: examining the "voice" of a mathematics textbook. *Journal for Research in Mathematics Education*, 38 (4), 344–369. doi: 10.2307/30034878
Jakobsson-Åhl, T. (2006). *Algebra in upper secondary mathematics: a study of a selection of textbooks used in the years 1960–2000 in Sweden*. Luleå Tekniska Universitet. Retrieved from http://epubl.ltu.se/1402-1757/2006/33/
Jones, K., Bokhove, C., Howson, A. G. & Fan, L. (Eds.). (2014). *Proceedings of the International Conference on Mathematics Textbook Research and Development* (ICMT-2014). School of Education, University of Southampton.

Kongelf, T. R. (2015). Introduksjon av algebra i matematikkbøker for ungdomstrinnet i Norge. *Nordic Studies in Mathematics Education*, 20 (3–4), 83–109.

Le Roux, K. (2008). A critical discourse analysis of a real-world problem in mathematics: looking for signs of change. *Language and Education*, 22 (5), 307. doi: 10.2167/le791.0

Luke, A. (1995). Text and discourse in education: an introduction to critical discourse analysis. *Review of Research in Education*, 21, 3–48. doi: 10.2307/1167278

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics*, 61 (1/2), 219–245.

Morgan, C. (2016). Studying the role of human agency in school mathematics. *Research in Mathematics Education*, 18 (2), 120–141. doi: 10.1080/14794802.2016.1176595

Morgan, C. & Sfard, A. (2016). Investigating changes in high-stakes mathematics examinations: a discursive approach. *Research in Mathematics Education*, 18 (2), 92–119. doi: 10.1080/14794802.2016.1176596

Morgan, C. & Tang, S. (2016). To what extent are students expected to participate in specialised mathematical discourse? Change over time in school mathematics in England. *Research in Mathematics Education*, 18 (2), 142–164. doi: 10.1080/14794802.2016.1174145

Palm, T. (2002). *The realism of mathematical school tasks: features and consequences*. Department of Mathematics. University of Umeå.

Rezat, S. & Strässer, R. (2015). Methodological issues and challenges in research on mathematics textbooks. *Nordic Studies in Mathematics Education*, 20 (3–4), 247–266.

Ryve, A. (2011). Discourse research in mathematics education: a critical evaluation of 108 journal articles. *Journal for Research in Mathematics Education*, 42 (2), 167–199.

Schleppegrell, M. J. (2004). *The language of schooling: a functional linguistics perspective*. Mahwah: Lawrence Erlbaum.

Sfard, A. (2013). Almost 20 years after: developments in research on language and mathematics. Review of J. N. Moschkovich (Ed.) (2010) Language and mathematics education: multiple perspectives and directions for research. *Educational Studies in Mathematics*, 82 (2), 331–339. doi: 10.1007/s10649-012-9446-2

Sidenvall, J., Lithner, J. & Jäder, J. (2015). Students' reasoning in mathematics textbook task-solving. *International Journal of Mathematical Education in Science and Technology*, 46 (4), 533–552. doi: 10.1080/0020739X.2014.992986

Verschaffel, L., Van Dooren, W., Greer, B. & Mukhopadhyay, S. (2010). Reconceptualising word problems as exercises in mathematical modelling. *Journal Für Mathematik-Didaktik*, 31 (1), 9–29. doi: 10.1007/s13138-010-0007-x

Notes

1 Herbel-Eisenmann (2007) uses a concept of "scribbler" and "thinker" coined by Rotman, and claims that for the learning of mathematics, both scribbling and thinking is needed.

2 In Sweden, there are no regulations for publishing textbooks. Nor are there public figures concerning the popularity of different textbooks. The National centre for mathematics education, NCM (ncm.gu.se) displays a list of all current Swedish mathematics textbook publishers as well as several international ones.

3 The Swedish course plan discerns algebra from "variation and change" (Skolverket, 2016) so chapters about functions are not studied.

4 These tasks are excepted from the analysis since they only involve arithmetic or statistics. Mathematical symbolic expressions are not given in or needed to solve the tasks. Most of these tasks belong to special themes, e.g. *Tanzania – a country in Africa*, *Problem solving* and *Number sense and mental arithmetic*.

5 In a few tasks, images picture real phenomenon, e.g. a flag. This does not add information to the tasks and pictures are not the most prominent participants, so they function as a backdrop.

6 The Swedish personal pronoun *man* has a generic character and applies to unspecific human beings, like *one*. Though *one* is not really used in this sense in English, it is used here to catch that it functions to hide human agency.

7 As a parallel to the tasks with pictures in *geometrical discourse*, there are a few back drop tasks in the *arithmetical discourse* as well. The participants in these tasks are e.g. years instead of numbers and third or first persons.

8 This may in part be ascribed to the relatively few numbers of tasks in this discourse.

Kristina Palm Kaplan

Kristina Palm Kaplan is a doctoral student at Uppsala University since September 2014. The main research interests are mathematics and language, especially algebra and social semiotics.

kristina.palm.kaplan@edu.uu.se