A model for the role of the physical environment in mathematics education

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In this paper, we develop an analytical tool for the role of the physical environment in mathematics education. We do this by extending the didactical triangle with the physical environment as a fourth actor and test it in a review of literature concerning the physical environment and mathematics education. We find that one role played by the physical environment, in relation to mathematical content, is to portray the content in focus, such as geometry and scale. When focusing on teachers, students, and the interaction between them, the role of the physical environment appears to be a precondition, either positive (enabling) or negative (hindering). Many of the findings are valid for education in general as well, such as the importance of building status.

Several theories talk about the environment as a frame for where learning can take place. For instance, in mathematics education, one important example is the work about milieu by Chevallard and Brousseau (for a longer discussion, see Schoenfeld, 2012). Their work includes a proposed model of a learning situation, a didactical situation, in mathematics consisting of three actors: the learner, the content (here mathematics) and the teacher. These three actors create a didactical triangle with relations between the different actors. This *milieu* is considered part of the learner's environment for a certain instance of knowledge in Balacheff's (2000) modelling tool for conception. Another example using the educational environment is Gomez's (2002) model of didactical analysis of teachers' work. Here, the educational environment is the part of the context setting the frame in which teachers' deed can take place. Even if theories and theoretical frameworks already exist, there are not many empirical studies in mathematics education that focus on what the environment actually is and what it does in a learning situation.

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Also, theoretical frameworks or models do not provide answers in themselves. One of the few to address this lack of knowledge is Schoenfeld (2012) when he, in his review of research on the didactical triangle, tackles the question about the classroom as a place for doing mathematics. However, the focus in his review is on resources such as technology and the issue of the environment itself is not elaborated. We would like to address this gap by creating an extension of the didactical triangle with a fourth actor, the physical environment, to interact with the three original actors, the student, the content, and the teacher and their interactions. The first aim of this paper is to test the extension of the didactical triangle as an analytical tool for the role of the physical environment in mathematics education. The second aim is to characterize the different roles the physical environment can play in mathematics education.

Background

As stated earlier, there are few empirical studies in mathematics education addressing the physical environment as a factor for learning. One of them is Liljedahl (2016) regarding the work on *Building thinking classrooms*. Here the physical environment is part of two out of nine elements that make a difference to students' mathematical learning: (1) student workspace while working on mathematical tasks; and, (2) room organization, both in a general sense and specific such as when students are working on mathematical tasks. Liljedahl (2016) shows that there is a difference between if students are sitting down working on a task or if they are standing up, where the latter one evoked more activity of the participants. However, although addressing the issue of the physical environment and showing with empirical data that it matters, the focus in the discussion of the results is not on the physical environment, but instead if and in what way the differences are a result of breaking classrooms norms when using the physical environment in a non-traditional way (Liljedahl, 2016). Broadening the scope, we see the same trend in general education, where one rare empirical study is Szczepanski and Andersson (2015), looking at university professors' conceptions of the importance of space in outdoor learning. In the background to their study, they refer to both Maria Montessori and John Dewey when arguing for why the physical environment is important to understand. When trying to link individual's cognitive knowledge, mindscape, and the spatial understanding of the space, landscape, their conclusion is that a different theoretical tool is needed. Szczepanski and Andersson (2015) turn to Tuan (1975) and his theory about centers if meaning. Using a humanist perspective, Tuan (1975) shows how different places are central based on how people experiences them. One study providing arguments why and how the physical environment plays a role is Lindgren (2010), who looked at how differences in physical space may influence education even in a longer perspective such as segregation in further education. The analysis in Lindgren (2010) is based on Soja's theories when discussing how individuals experience the environment and take space personally. Using Soja's three dimensions, Lindgren (2010) show that different aspects of environment comes to play depending on if we talk about the first dimension of landscape called firstspace (physical), the second dimension called secondspace (mental), or the third dimension called thirdspace (an alternative perspective including both the first dimension and the second dimension focusing for instance on the intersection between the first two dimensions). This is linked to social justice since these three dimensions are addressing the critical question: Who takes the power of space? (Soja, 2009). These different theories add a physical dimension to Hiebert's (2003) opportunity to learn. Hence, we operate as educators and learners in a physical space. We can extend this thinking by adopting a perspective based on posthumanist performativity. Barad (2003) has concluded that matter, such as the physical environment, matters; that we need to think about matter as ”an active participant in the world's becoming” (Barad, 2003, p. 803), that discourse has been granted too much power, and that it is important that we understand how matter matters. Results from several studies emphasize that how the physical space is created originally, such as how walls are built, decides how sound travels and this has a substantial impact both on what students can hear (e.g. Klatte, Hellbrück, Seidel & Leistner, 2010) and the physical response such as stress symptoms in the teachers due to the noise resulting for instance in sleeping problems (Enmarker & Boman, 2004). In these situations, the power is located outside the didactical situation, for example, with the architects who designed the building. Yet, the physical space plays a role in the educational outcome and therefore it is important to understand how teachers and students experience the physical environment and how they think they can work with the matters. These how questions have been summarised in one: How do the matter influence teaching and learning? It has been considered as one of the important didactical questions since it focuses on where teaching is taking place and the role of the space (Szczepanski & Andersson, 2015).

Conceptualizing physical environment

The words physical and environment are somewhat overlapping when standing alone. Most research in the area is focused on a single or a few factors and rarely define physical environment as a whole. In addition, there is no coherence in how physical environment is defined or which factors should be part of it. Some studies solve this by singling out factors and define each factor specifically, for instance, a study of noise (Ising & Kruppa, 2004); lighting (Barnitt, 2003); or the combination of noise, temperature, and indoor lighting (Hygge & Knez, 2001). By doing so, that is, isolating single factors, it is hard to talk about the physical environment on a general level and apply a macro-perspective. Other studies aimed at the bigger picture by including more factors: one study prioritizes 31 criteria that in a way becomes a definition (Earthman, 2004), another one sets out to take a multilevel holistic view by including 37 factors (Barrett, Zhang, Moffat & Kobbacy, 2013). The latter one provides an illustrative picture of their holistic view where environmental factors are distinct from non-ditto (Barrett et al., 2013). However, a list of factors in itself does not provide a definition and the different lists could cause confusions about which factors are important and which are not.

One researcher, who has highlighted the confusion of definitions and meanings is Gump (1978) when asking: "When you speak of a school or classroom environment, what are you talking about?" (Gump, 1978, p. 131). He continues to state that several phenomena can be the answer to the question, for instance, physical context, social climate, and life space. Instead of providing yet another definition, Gump's (1978) solution is to take a real-life classroom example where there is a distinction between objective characteristics of the physical environment such as classroom shape and dimensions and more subjective observations such as interesting objects or a sense of openness (Gump, 1978). Still, an example is not a definition and it is not clear how this particular example could be applied to other situations.

Another way is to focus instead on space and place in time so that the physical environment is defined as the sense of a position in space at a given time span (Tuan, 1979). We will use this definition as a starting point when we define the concept physical environment, used in this paper. We define physical environment as physical objects that we can touch (walls, chairs, etc.) and physical sensations that we can sense (temperature, light, etc.). In this definition learning material and manipulatives are included just as physical objects. Previous research has addressed the different roles manipulatives and learning materials can have in a teaching/learning situation in mathematics (Hunting & Lamon, 1995) and that "physical involvement with learning materials can greatly enhance the understanding and retention of difficult concepts" (Scarlatos, 2006, p. 293). However, here such a situation is considered a result of someone, for example, a teacher, taking control of a part of the physical environment. Lastly, we want to limit this paper to the indoor environment, since, at least in Sweden, most mathematical education takes place in an indoor setting. Also, most schools are designed in similar ways (corridors and classrooms with walls, windows, benches, etc.) whereas the outdoor environment differs.

Extending the didactical triangle

Our standpoint is that matters do matter (Barad, 2003), and the idea of centers of meaning (Tuan, 1975), that space plays a role in meaning-making in learning situations. Using these ideas, we argue that it is possible to identify which role the physical environment play in the interactions between actors. The role here means the purpose or influence of someone or something in a particular situation, here mathematics education. However, compared to impact, we see role as an ongoing dynamic process. In the process of identifying actors, we started with the didactical triangle, as often presented in papers (e.g. Straesser, 2007), with the three actors teacher, student, and content illustrated and the relationships between these actors indicated with double-arrows (see figure 1).

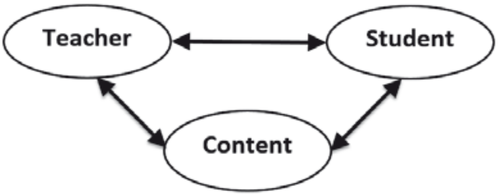

Figure 1. *The didactical triangle*

Inspired by Rezat and Sträßer (2012), who expands the didactical triangle first to a didactical tetrahedron, and then to a socio-didactical tetrahedron using a socio-cultural perspective, we expanded the didactical triangle by adding the physical environment as an actor. A similar idea, to expand the didactical triangle, has been done previously, for instance, ICT and the didactical triangle (Gadanidis & Geiger, 2010; Olive et al., 2010). This expansion would then mean a difference compared to see the physical environment as the milieu or as a frame of the educational activity, the context, as in the model presented by Gomez (2002). In our extension of the didactical triangle, the physical environment is an active participant acting explicitly or implicitly in mathematics education. As an actor, it can be an artefact similar to Rezat and Sträßer (2012) but does not have to be. In addition, in Rezat and Sträßer's (2012) model, discourse could be seen as an environment. Here, we focus only on the physical part and the physical environment becomes an actor in itself whether used purposely or not. From this base, we depicted the extended model to describe the different ways the physical environment can play a role in the didactical situation (see figure 2).

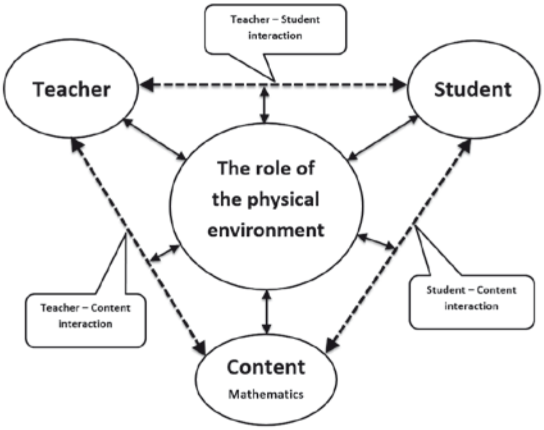

Figure 2. *The physical environment as an actor in the didactical situation*

As seen in figure 2, the physical environment can interact with the teacher, the student and the content separately but also with the different interactions between these elements: student–teacher, teacher–content, and student–content (marked with solid arrows in figure 2). Each relation has a dualism regarding the direction of purpose or influence (hence the double pointed arrows in figure 2). This dualism comes from our definition of role as a dynamic process, where the purpose or influence, has a direction. This view is inspired by the movies or theatre, where there is an ongoing interaction between actors and their co-actors. In this process, there is a directional dualism when one actor acts towards a co-actor (delivering a line in the dialogue) and then the direction switches when the co-actor acts back (delivering the reply-line in the dialogue).

It is not farfetched to expand this two-dimensional model to three dimensions, just as in Rezat and Sträßer (2012), where we also have the interaction teacher-content-student. However, in the present study, the choice is to see the three-part interaction, teacher-content-student, partitioned into disjoint sets of two-part interactions.

Method

A literature search was made in order to collect the literature to review, our data. The data that constitute the basis for the analysis were generated from the ERIC database, May 2015. The search terms were physical, environment, and mathematics (and all combinations using math or maths), in peer-reviewed journal articles. The number of papers resulting from this search was 100. The next step was to read the abstracts and exclude those articles that already in the abstract did not fit our aim or definition of physical environment for this study. Most excluded articles in this step concerned physical education. In this step, all but 14 pieces were excluded. The remaining articles were read in full and another eight articles were removed after this reading since they were focusing on other aspects than what the chosen aim and definition encompasses, such as purely outdoor environment, manipulatives, or social climate and neighbourhood status. This full-text and abstract reading were done individually and the results were compared in order to increase the validity. After removing the eight articles that were not relevant for this study, the data consisted of the remaining six publications. To provide a context regarding the data to be analysed, a short presentation is given for each chosen article (see table 1 on the following page).

Given that our first aim is to test the extension of the didactical triangle we consider the relatively small number of articles sufficient and the context provided in table 1 shows an adequate diversity within the data. The result/findings and discussion/conclusion sections of the six remaining articles were analysed using our developed analytical tool, figure 2. In the tool, the role of the physical environment comes to play in six interactions, with the teacher, the student, and the content separately but also with the different interactions between these actors: student–teacher, teacher–content, and student–content. The articles were analysed using these six interactions. They were marked and coded using the different interactions as a first step starting with the identification of the actor's content, teacher, student, and physical environment. As a second step, the findings were coded regarding the different directions, that means, deciding which actor is acting towards what or who. For instance, if students have a sense of belonging to the school building, this is coded as from student towards the physical environment. An example of coding as from physical environment towards student–teacher interaction is when limited classroom size obstructs student–teacher interaction. In this way, the main analysis was deductive and theory-driven with predefined categories (cf. Braun & Clarke, 2006). The third step was an inductive approach looking for patterns within or between categories and different parts of the results.

Table 1. *Context for the data to be analysed*

Reference	Rationale and design	Findings
Johnston (2012), Maths in the board room	Describes experimenting with having the students working in small groups standing at small wall mounted blackboards during mathematics class. Descriptive, student survey and observations.	Students worked more engaged and together than they did before. It is reasoned that the students learn at a faster rate than before because their learning is socially mediated. The author also mentions a limitation: That he as a teacher has a cognitive limitation in following all groups' calculations simultaneously, which might be a hinder for them to learn even faster.
Soygenis and Erktin (2010), Juxtaposition of architecture and mathematics for elementary school students	Discusses the Archimath program implemented in a few schools in Turkey. The program integrates Architecture and Mathematics for students in grades four through eight. The objective is to develop awareness of the built environment and the possibility to improve it. Descriptive, observations, teacher and student surveys.	The results showed that students increased their concern for the environment, awareness of the physical environment, sense of responsibility, and a more positive attitude towards changing the environment. With regards to the learning in mathematics they found that students' motivation for learning mathematics increased when the subjects from the curriculum could be related to real life matters.
Tanner (2009), Effects of school design on student outcomes	Relates the three school design classification variables: movement and circulation, day lighting, and views to student achievement in mathematics and other subjects. Data from 71 schools in the USA. Quantitative-reduced regression analysis.	Significant effects are found in mathematics among other basic skills test. Interestingly enough daylighting did not have a significant effect on mathematics in this study. Patterns of views did though, implying that students need to be able to rest their eyes by viewing outside occasionally.
Uline and Tschannen-Moran (2008), The walls speak: the interplay of quality facilities, school climate, and student achievement	Examines the mediating role of school climate on students´ performance. In a survey with teachers from 80 middle schools in the USA, school climate index measures were gathered together with student achievement data in mathematics and other subjects. Quantitative.	Their findings leads them to the conclusion that school climate plays a mediating role between school facilities and student achievement. There is also a teacher influence, teachers in poor rated facilities are less likely to show enthusiasm for their job and put in the extra effort in teaching their students.
Uline, Tschannen-Moran, and DeVere Wolsey (2009), The walls still speak: the stories occupants tell	Persuades the intricate relationship between school buildings´ physical properties and teaching and learning in mathematics and other subjects. The two schools used are in the top quartile of facility quality rating from the 2008 study. Case study. Individual, focus group, walk-though and photo interviews.	Some key conclusions stressed in the discussion are that research participants described how the built environment facilitated daily interaction and activities; sometimes they were moved to confront identified problems or missing opportunities and teachers and other leaders constantly challenging spatial routines and improving layouts inside and outside classrooms.
Uline, DeVere Wolsey, Tschannen-Moran, and Lin (2010), Improving the physical and social environment of school: a question of equity	Examines schools scheduled for renovation. Six themes from the 2009 study were related to the three academic press variables identified in the 2008 study. Since these schools were scheduled for renovation the building characteristics represented by the themes were often described as missing features. Mixed method triangulation design.	In their findings a trend of compensation emerged. For instance teachers bring their own fan when air conditioning was absent, students wearing extra clothing if it was too cold, and one teacher of mathematics cramming up the desks a little to add in the aisle enabling her to reach each student.

Results

The results are first presented with the role of the physical environment in relation to the three actors: teachers, students, and content, and then with the role of the physical environment in relation to the three interactions between these actors: student–teacher interaction, student–content interaction, and teacher–content interaction. Each relation is presented in one table, divided into two parts regarding the direction of influence in the relation. First, we focus on the results regarding the role of the physical environment (PE) in relation to teachers (see table 2).

Table 2. *The role of the physical environment (PE) in relation to teachers*

References	Findings – role of PE
Direction from PE towards teachers	
Uline, et al. (2009, p. 409–417)	promotes personal development and professional identity; triggers involvement and expectations; make teachers feel welcomed by, like at home, unfettered by, satisfied by, inspired by, and secure in. PE as the building
Uline, et al. (2010, p. 615–627)	promotes professionalism, triggers compensation and extra effort. PE as poor school building status
Johnston (2012, p. 12)	triggers innovation to improve. PE as standard classroom layout
Uline & Tschannen-Moran (2008, p. 66)	cause flawed attitude and enthusiasm. PE as poor school building status
Direction from teachers towards PE	
Uline, et al. (2010, p. 615–627)	sense of belonging to, concern for, pride for, commitment to, influence over. PE as school building
Uline, et al. (2009, p. 408–417)	responsible for. PE as classroom status
Johnston (2012, p. 12)	command over. PE as classroom layout

As we can see in table 2, a majority of the findings have an emotional connotation such as secure in (a positive feeling) or restricted by (a negative feeling). We also find associations to motivation, some explicit such as being inspired by but also implicit such as influence over the physical environment. If we instead switch to the student perspective, the following table (table 3) shows the role of the physical environment in mathematics education in relation to students.

Most of the findings presented in table 3 also have an emotional implication such as freedom to move about (a positive feeling) or claustrophobic (a negative feeling), similar to the findings for the teachers. The same similarity between these two actors is found in the affective factors motivation and enthusiasm. There are also some differences like accountability for and adapt to the physical environment. The next perspective focus on the role of the physical environment in relation to the content, which is not always specific for mathematics (see table 4).

Table 3. *The role of the physical environment (PE) in relation to students*

References	Findings – role of PE
Direction from PE towards students	
Uline & Tschannen-Moran (2008, p. 60–64)	enhance academic press (construct, describing serious and orderly schools driven by a quest for excellence). PE as good school building status
Uline, et al. (2009, p. 408–417)	form identity; make students positive to changes; provide freedom to move about, visual rest, free sight, ample space, elbow room, and privacy; feel welcomed by, like at home, relief, unfettered, imaginative, satisfied by, important, safe, and motivated. PE as building and classroom
Uline, et al. (2010, p. 623–627)	trigger compensation and extra effort; to be uneased by, hindered by, distracted by; feeling of inequity and to feel claustrophobic. PE as poor school building status
Direction from students towards PE	
Uline, et al. (2009, p. 408–417)	sense of belonging to, pride of, enthusiasm for, anticipation of, curious of, connected to, and involvement with. PE as school building
Uline, et al. (2010, p. 616)	adapt to. PE as poor classroom climate

Table 4. *The role of the physical environment (PE) in relation to content*

References	Findings – role of PE
Direction from PE towards content	
Uline, et al. (2010, p. 624)	room for technical equipment. PE as dedicated classroom space – no specified subject
Uline, et al. (2009, p. 411)	Content-area poster display. PE as space around classroom door indicating specific topic studied in that classroom –mathematics and other subjects
Johnston (2012, p. 13)	vertical writing surfaces. PE as several blackboards in mathematics classroom
Direction from content towards PE	
Soygenis & Erktin (2010, p. 412)	two and three dimensional shapes, distance and peri-meter, scale. PE as geometrical aspects of school building and furniture

The physical environment has different roles regarding the mathematical content depending on the direction, as shown in table 4. When the direction is towards content the physical environment is dedicated to the content in some way such as vertical writing surfaces. In the other direction from the content towards the physical environment some content is to be represented by the physical environment such as scale for instance. In this case, there are no positive or negative attachments to the findings as with the two previous cases. Moving to situations with the interaction between students and teachers in focus, the findings show the following roles of the physical environment (see table 5).

As we can see in table 5, when it comes to the direction towards student–teacher interaction, the role is about setting the frame, mainly

Table 5. *The role of the physical environment (PE) in relation to student–teacher interaction*

References	Findings – role of PE
Direction from PE towards student–teacher interaction	
Uline, et al. (2009, p. 409–410)	shaping, preparing students and teachers for their interaction. PE as good design of school building
Uline, et al. (2010, p. 621)	obstructing or limiting. PE as insufficient classroom size
Direction from student–teacher interaction towards PE	
Uline, et al. (2010, p. 624)	violation of [physical] personal space. PE as crowded space

Table 6. *The role of the physical environment (PE) in relation to student–content interaction*

References	Findings—role of PE
Direction from PE towards student–content interaction	
Uline & Tschannen-Moran (2008, p. 66)	affecting student achievement in English and Mathematics. PE as school building status
Uline, et al. (2009, p. 409–417)	influence learning activities such as work, group work, discussion, reading, researching, and affect time on task. PE as design of school building—no specific subject
Tanner (2009, p. 392)	effect test results in mathematics and other basic skill test. PE as school design
Soygenis & Erktin (2010, p. 412)	serve as content when students are doing mathematical tasks, calculating perimeters, analysing, and differentiating between dimensions. PE as geometrical aspects of school building and furniture
Johnston (2012, p. 15–16)	enable students to stand up when finding solutions, writing, observing, and listening, during mathematical problem solving in groups. PE as several blackboards in math classroom
Direction from student–content interaction towards PE	
Johnston (2012, p. 13)	display of students' written solutions to mathematical problems. PE as several blackboards in math classroom
Soygenis & Erktin (2010, p. 412–413)	present in students' presentations, explanations, questions and drawings; be in focus when students design space with scale and plan drawings. PE as geometrical aspects of school building and furniture
Soygenis & Erktin (2010, p. 412–413)	develop the students' awareness of, attitude towards, and concern for the physical environment. PE as the built neighbourhood and the world we live in

the limitations and/or what is enabling but also preparing the teachers and students for interactions. In the direction from student–teacher interaction, the roles found are more of an outcome from the interaction, such as violation of [physical] personal space. If we shift focus to the interaction between the student and the content, the analysis resulted in the following findings (see table 6).

The findings presented in table 6 show that regarding the role of the physical environment in the direction towards the student–content interaction, the physical environment can become the content, for instance when the students are calculating perimeters. The role can be either enhancing or hindering such as effecting time on task in a positive or negative way. In the direction from student–content interaction, there is a dominance of displaying outcomes from the student–content interaction. The results for the last analysed interaction, the physical environment in relation to the teacher–content interaction, are (see table 7).

Table 7. *The role of the physical environment (PE) in relation to teacher–content interaction*

References	Findings—role of PE
Direction from PE towards teacher–content interaction	
Johnston (2012, p. 14)	cause increased cognitive demand when monitoring the mathematical process of groups of students standing up during problem solving. PE as several blackboards in math classroom
Direction from teacher–content interaction towards PE	
Johnston (2012, p. 12)	be considered when teachers set student goals before teaching mathematics. PE as classroom layout
Soygenis & Erktin (2010, p. 410)	be the focus when teachers prepare for teaching mathematics. PE as geometrical aspects of school building and furniture

The results presented in table 7 imply a direct effect on the teachers' ability to interact with the content when the direction is from the physical environment towards the teacher–content interaction. In the direction from teacher–content interaction, the role of the physical environment relates the teachers' planning before meeting the students.

Summary

This summary and the short sections between the result tables are results of the third inductive step in the analysis where we looked for patterns within or between categories and different parts of the results. From this, we characterize the role of the physical environment in relation to teachers, students, the interaction between them, and their interaction with the content, as affecting the conditions for the actors and their interactions. In most of these interactions, the role is positive or negative and sometimes with links to emotions and/or motivation. When the direction is towards teachers, students, and the interaction between them, the role if the physical environment appears to be a *precondition*, either positive, enabling or negative, hindering. In the other direction, from teachers, students, and their interactions, the role of the physical environment has an *ongoing* or *post* character. When we look at the student–content and teacher–content interaction, the role is active *during* in the direction towards the interactions in both cases. Changing to the direction from teacher–content and student–content interaction, the roles are opposed in relation to the two interactions. For the student–content interaction the role is of post character, mainly in displaying results from the student–content interaction. On the other hand, for the teacher–content interaction, the role has a *pre* character, mainly when teachers prepare for the teaching. Finally, in the role of the physical environment in relation to the content, which is in both directions ongoing or during, it becomes apparent in the findings, that both content and physical environment, in this case, are human constructs or artefacts.

Discussion

The first aim of this paper was to test the extension of the didactical triangle as an analytical tool for the role of the physical environment in mathematics education. The results show findings for all six relational categories of the analytical tool. There is a diversity between the findings in these different categories, which implies that they are relevant, which means the tool serves a purpose. The second aim of this paper was to characterize the different roles the physical environment can play in mathematics education. The analytical tool was applicable to achieving the second aim as well. For instance, the identified role of the physical environment in the direction towards teachers, students, and their interaction, turns out to be a precondition, either positive or negative. This result is similar to the work about milieu (cf. Balacheff, 2000). Another parallel is where the educational environment is the part of the context to be considered as in Gomez's (2002) model of didactical analysis of teachers' work. In this way, the results from the present study incorporate both these stances. When the direction instead turns from the teachers, students and their interactions, the role of the physical environment that was a precondition in the first direction changes to an ongoing influence. This change from precondition to ongoing influence supports the choice to have the dual direction of influence for each relation in our analytical tool. A similar transformation takes place when we look at the student–content and teacher–content interaction. At first, in both interactions, the role is an active participant during mathematics class. This can be compared to the situations where the power of space is located outside the three main actors, but sometimes under control if the teacher as in Liljedahl (2016), or even when the student is aware of the preconditions. The ability to address the question of control is also a benefit from having these dual directions in the tool where it becomes apparent if an actor is in control or controlled by something in the physical environment (cf. Soja, 2009). However, when switching to the direction from teacher–content and student–content interaction, the roles are opposed in relation to the two interactions. The findings for the student–content interaction are that the role is of post character, mainly in displaying results from the student–content interaction. Time is central also in the teacher–content interaction. Here, the role of the physical environment has a pre character mainly when teachers prepare for their teaching. This seems natural considering the traditional teacher–student role where the teacher is responsible for the didactical situation and appointed the power of the mathematics education (cf. Gomez, 2002).

Content in the interactions mentioned in the previous section can be mathematics, but also many other subjects. Results where mathematics is explicit concerns geometrical aspects of the physical environment. In this case, the physical environment is represented by the school building or classroom furniture. These results with geometrical aspects of furniture or buildings are very specific for mathematics and a few other mathematics-related subjects, such as physics. When we look for results specific for mathematics education we have students solving mathematical problems in groups standing up as in Liljedahl (2016), at blackboards dedicated to each group and all visible to the teacher. This case could probably be found in several other subjects where group work is common. The fact that the data in our analysis originates from only six articles, might contribute to a vague picture of the specific role of the physical environment in mathematics education. However, the picture of the role in education in general, turned out relatively rich, considering the data used. A possible implication is that when wanting to separate what is unique for mathematics compared to other subjects, data regarding other subjects in relations to the physical environment needs to be analysed. In this way, by contrasting the different subjects, a richer picture of the specific and unique role of the physical environment in mathematics education can be obtained.

Also, when using this extension of the didactical triangle, there is a possibility to emphasize the different roles that the physical environment played in each of the different relations. This specification is another strength of this analytical tool. We suggest that it can be extended to a theoretical model for further understanding of the role of the physical environment in mathematics education, other subjects, and education in general.

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