Stimulating critical mathematical discussions in teacher education: use of indices such as the BMI as entry points

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The main purpose of our research project is to gain insight into, and develop teaching on indices and their applications in society. In this paper, the focus is to present insights into teachers' reflections when discussing the Body Mass Index (BMI). Skovsmose´s concept of mathemacy, and source criticism, are chosen as conceptual framework. The data analysed were collected in a numeracy across the curriculum class with practising teachers. The findings show that the practising teachers engaged in meaning making of the index formula, and they critically discussed how BMI is used in society and the role the BMI index can have in our lives. We gain insight into the potential of such an index for developing teachers' awareness of the application of mathematics to the real world and the issues it raises, both for the teachers and for ourselves.

Development of students' critical thinking with the help of mathematics is one of the main concerns in critical mathematics education. This is in line with the Norwegian mathematics curriculum: "Active democracy requires citizens who are able to study, understand and critically assess quantitative information, statistical analyses and economic prognoses" (Ministry of Education and Research, 2010, p. 1). Mathematical modelling is one way the critical perspective can come to reality. There is research showing that modelling can, and should, be introduced already at the primary school level. Among the arguments there are the opportunities for children: to make sense of the mathematics in the modelling contexts offered (Doerr & English, 2003), to meet and explore informal mathematical concepts (English, 2006), to develop mathematical reasoning processes (English & Watters, 2004), to connect mathematics with reality, and to see mathematics as a critical tool for analysing issues in their own lives (Greer, Verschaffel & Mukhopadhyay, 2007).

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Yet, Greer and Skovsmose (2012) pointed towards the fact that mathematical modelling, even at university level, is failing to address some crucial issues that go beyond simply finding models that can answer to a particular problem or situation. These issues concern critiquing mathematical models, and discussing "the roles played by mathematics in action in our societies, and the limitations of applying technical solutions to human problems" (p. 12).

A possibility for introducing critical discussions in the classroom is to identify the practices and situations where the mathematics is implicit, but still plays an important role in defining the phenomena (Jablonka, 2003). One such practice is indices, for instance the Body Mass Index (BMI) and the Human Development Index (HDI). Indices are widely used in our society. Mathematical concepts and operations are used in indices to decide how to measure phenomena in society by giving them some numerical values, while at the same time contributing to define a phenomenon. The latter is what Skovsmose (1998) called the formatting power of mathematics based on his argument that "social phenomena are structured and eventually constituted by mathematics" (p. 197). Likewise, D'Ambrosio (1990) argued that mathematics education has a role in building a just and democratic society. Mathematical indices are all around us, D'Ambrosio (1990) added, and mathematics education has the duty "to prepare citizens so that they will not be manipulated and cheated by indices" (p. 21).

The mathematics, in indices and other kinds of models, often remains hidden and is overshadowed by the index itself, the formula that is used. It is the results of this formula that are used by mass media and politicians and thus become the focus of further analyses about what measures need to be implemented. The mathematics behind the formula stays implicit, without being critically examined or discussed in relation to the values it brings about.

Aguilar and Zavaleta (2012) underlined in their review of studies about mathematics and democracy that teachers need mathematical and pedagogic skills to develop their students' critical democratic competences. They need to be able to identify situations with mathematical potential, to recognise the formatting power of mathematics in those situations, and be open and prepared to create and support a classroom culture that fosters critical discussions. If mathematics is implicit, as is the case with indices, it can be difficult for students and teachers to identify it. As a result, they might not see the opportunity to create their own meaning and understanding of the phenomena and the numbers assigned to it. In addition, teachers might have difficulties in thinking about possibilities of using indices in their own teaching.

As teacher educators, we will, with our research, respond to the challenges that Greer and Skovsmose (2012) raised. The main purpose of our research is to gain insight into, and develop, teaching on indices and their applications in society. For this, we have chosen a critical mathematics education perspective in line with Skovsmose (2005), both in our research and teaching.

In this paper, we analyse an arranged situation where practising teachers are invited to discuss the mathematics in the BMI index, as a known tool for measuring obesity in our society, and the influence of indices in society. The focus in this paper is to gain insights into teachers' reflections when discussing the BMI. How do they demonstrate critical reflections on how BMI is used in society and how BMI influences the interpretation of obesity? How is the mathematics involved in BMI expressed?

Indices

In this text, we focus on indices. In general, an index number can be interpreted as an average value based on measures of two or more quantities from a relatively large sample taken from a population or a set. A typical attribute of indices is that the result of the averaging process is a single number. One of the most well-known examples of an index number is the Body Mass Index. Within the metric system, BMI is computed as the ratio between weight and square of height. In other words, if m is the weight of a person and h is the height of the same person, the BMI-coefficient for this person is calculated from the formula

$$\text{BMI} = \frac{m}{h^2},$$

where m is usually measured in kilograms and h is given in meters.

An index number could be considered as a kind of prototype taking care of certain aspects of a set. Comparing a random sample from the set to this prototype, the index number will serve as a standard – a benchmark. Usually, an index is determined to lie in between some specified limits. For example, the World Health Organization states that adults 20 years and older, should have a BMI in the interval from 18.5 to 24.9 to indicate normal weight.

Though the final formulas in indices are often relatively simple, the mathematical formulation-process may involve advanced statistics and different mathematical concepts.

Only a few of several, possible parameters characterising a data set, can be used to define an index. These parameters will therefore usually not be complete enough to apply to all individuals.

The index concept can be approached from a mathematical modelling perspective. Mathematical models are translations of measurable real world phenomena into mathematically interpretable contexts. As calculations based on measures of different variables from the real world, indices may be regarded as mathematical models. There are however some differences between general mathematical models and indices as models. General models often rely on complicated equations to be solved by computers and result in many different solution variables. Indices on the other hand rely on an easy-to-use formula, resulting in a single number.

Exploring the use of mathematics in indices

When modelling a situation from reality, students have to make decisions about which variables to take into account. Skovsmose (1992; 1994a) argued that it is in this process that students can discover how mathematics influences our perception of reality and our society in general, and become able to transform it. Thinking about modelling as critique (Barbosa, 2006), indices, as one kind of model, can be discussed in terms of their mathematical construction and their impact on how society perceives social phenomena.

Hall and Barwell (2015) drew on Skovsmose's concept of formatting power to explore how mathematics has influenced the defining of obesity as a concept. The arguments in the paper are built around critical limitations with BMI as an index and consequences for its use, and recommendations are given for BMI as a good starting point for discussions about the formatting power of mathematics in the classroom.

A study that takes the Human Development Index (HDI) into consideration in working with mathematics teachers in mathematical modelling sessions, is presented by Julie (2002). The teachers preferred to use simple mathematics for extending the HDI and remained in the same categories as the original HDI formula does. Nevertheless, Julie pointed out the importance of mathematics teachers meeting modelling not just as a vehicle for learning mathematics, but also as content in itself. Further, critical reflections about the consequences of the use of mathematics in a societal context are important, because as Vithal (2012) also put it: "the thinking tools and language of mathematics do not by themselves provide the full means for criticising its applications in society" (pp. 2–3).

In our research, we use indices, BMI being one of them, based on the perspective that indices have a potential to foster critical reflections. In addition, the simplicity of the computations by using the index formula, as with the BMI, may facilitate mathematical discussions in the classroom.

Conceptual framework

In this section we go into details of the theoretical position for the study by discussing the formatting power of mathematics and the mathemacy concept. Furthermore, historical source criticism and its connections and importance for critical discussions about the BMI index are explored.

The formatting power of mathematics and mathemacy

When highlighting the links between democracy and mathematics in an educational context, Skovsmose (1992) brought forward the thesis of mathematics' formatting power: "[...] mathematics makes a real intervention in reality, not only in the sense that new insight may change interpretations, but also in the sense that mathematics colonises part of reality and rearranges it" (p. 6). In this sense, mathematics is important for society because it helps to express and evaluate phenomena. On the other hand, the ways people use mathematics result in changes to the society (Vithal, 2012). Mathematical concepts and operations are used to construct indices that serve different purposes. An example is found from Hall and Barwell (2015) who argued that the mathematics behind the BMI index has shaped the concept of obesity in our society.

A rapid development of technology and science imposes some requirements in terms of people being able to understand more mathematics and criticise its use in society (Vithal, 2012). Such requirements can be used to define what Skovsmose (1992; 1994) termed mathemacy. The concept points towards critical contents in mathematics education (Skovsmose, 2004, p. 13) and it is connected to mathematical, technological and reflective knowing. Six levels of reflections were then identified (Skovsmose 1992; 1994) to indicate different types of knowing. These levels were related to students reflecting in problem solving situations, while in this study we have related them to practising teachers discussing BMI and its applications in society.

At the mathematical knowing level there are reflections about procedures and concepts of mathematics, as well as about applying mathematics in solving problems. In discussions about the BMI index, questions indicating mathematical knowing would address issues about the mathematical formula of the index, its meaning and functioning, fluctuation in BMI values, and if an adjustment of the mathematical formula would represent a better index.

Technological knowing is related to reflections about the contexts where mathematics is used. Questions related to technological knowing applied in our examples of the BMI would include discussions about how suitable it is to measure obesity by using the index, what the meaning of a single number obtained from the mathematical formula is for the single person, and if we do need such an index to decide about obesity.

The last component is reflective knowing, reflections at a meta level, which "[...] has to be developed to provide mathematics with an element of empowerment" (Skovsmose, 1994a, p. 117). At this level, one can discuss how the use of a single index such as BMI can influence our perceptions of the phenomena obesity, or what the use of the mathematics does to us, and to society in general.

Source criticism

Source criticism is an integral part of historical methodology and describes the evaluation of historic texts to determine their reliability and factual information-value. Interpreting such source material requires certain skills within language and reading. If indices are considered as texts, then being source critical to indices and algorithms requires some mathematical skills.

Source criticism can be divided into two subcategories: external and internal source criticism. External source criticism deals with provenance, genuineness and originality of the source material. Of these three concepts, provenance is relevant for our study. In terms of indices, provenance would deal with questions about who has developed the index, and in what circumstances, or for what purposes it was developed.

Internal source criticism examines the text's content and continues with the answers to the question of provenance, and examines them further. Having identified the author, his/her knowledge about the content and motivation for writing, we can determine the text's historical and ideological context, its reliability and evidential value (Goetz, 2000; Lund, 2011).

In terms of indices, internal criticism would deal with questions such as: What does the algorithm behind the index actually compute? Is a certain index appropriate to address a specific problem? What does the result actually tell us? Furthermore, will knowledge about the author of the index provide information about (ideological) context and appropriateness?

Source criticism supplies the mathemacy concept by giving some specific questions in order to define in what ways, and to what extent, teachers are being critical in their discussions. Source criticism is therefore used together with the mathemacy concept in this study as analytical tools to investigate the kinds of reflections teachers use when discussing BMI.

Method

The data for this paper were collected during a multidisciplinary course (30 ECTS)[1] on numeracy across the curriculum. The participants in the course were twelve primary school teachers from different subjects and from different parts of Norway. The aim of the course was to provide practical and theoretical insights on numeracy across the curriculum and to contribute in developing and qualifying practising teachers to become mentors in the topic at a school and municipality level. Two of the authors of this paper were the lecturers of the course (one from mathematics education and one from social science education) and had the responsibility for the organisation and implementation of learning activities for one day on campus. The practising teachers were first lectured on the different uses of mathematics in history and economics, including an introduction to indices in general and the Human Development Index (HDI) as an example. They were then randomly divided into two groups with the task of discussing BMI for about an hour.

The groups received a question sheet with a picture of an athlete with a BMI of 35.8 kg/m^2. The purpose of the question sheet was to structure and guide the discussions towards the mathematics in the indices, the role that indices have in our society, and how teachers see possibilities and challenges in using indices to promote critical thinking with their students. After a few minutes discussion, the groups were given the BMI formula and the cut off points for six weight categories. The two lecturers observed one group each, and did only intervene to ensure that the questions were addressed.

A core of this study concerns making explicit the formatting power of indices (BMI) in society. Teachers' discussions about BMI and its use are identified and discussed in terms of the concepts mathematical, technological and reflective knowing. In addition to the concept of mathemacy, we use a source criticism perspective. Being critical about an index means to examine it from multiple points of view. First, it means examining the formula, the mathematics, and what the index tells us. These are forms of internal criticism and can be seen in discussions involving mathematical knowing. Further questions are: who uses the index, what are the purposes for those who use it, how does the intended purpose of the index fit the ways the index is used by media and authorities. Such questions can be regarded as internal criticism, and can lead to discussions involving technological and reflective knowing. Lastly, external criticism questions about who has designed the index and for what purposes, can also be identified in discussions involving technological and reflective knowing.

The discussions were audio-recorded and then transcribed. The data presented in this text comes from one of the groups. Focusing on one group proved to generate a sufficient amount of data to get insight into the teachers' different reflections about the BMI, and it provides a more coherent analytical approach. All the authors took part in transcribing and analysing the data. Our analytical perspective was to discuss and understand the teachers' utterances, and through that identify what kind of knowing and source criticism took place. Numerous examples of technological, mathematical, and reflective knowing were identified, and the excerpts presented in the following are chosen because they are rich and distinct examples of the different types of knowing.

Analysis and discussion

This section is structured according to the conceptual framework. Excerpts from the group's discussion are thus analysed and discussed applying mathematical, technological and reflective knowing respectively. Aspects from source criticism are integrated in the discussions of all three levels of knowing.

Mathematical knowing

By using the mathematical knowing perspective, we identified discussions about the mathematical formula of the BMI and its components, about mathematical operations and unit measures, as well as efforts to make sense of the formula by connecting it to some physical aspects.

Prior to the following excerpt, the practising teachers discussed the use of BMI in society. They gave arguments for BMI, they talked about exceptions from the "rule" of BMI such as people who are thin but still have an unhealthy lifestyle. They are then challenged by the teacher educator to talk about BMI and its design. The discussion takes a different direction when they start talking about what the index really measures, and they go more in depth into the mathematical expression:

T1 I think it is very difficult to think that one also measures area ... [laughter]

T2 Surface area [in norwegian: flatemål] ... yes ... [laughter]

T1 ... Talking with the students about this ... and then you take kilograms and then you divide it by the area of the body ...

T2 Yes, but ...

T1 Hmm ... there is something wrong, isn't it?

T1 says it is "difficult to think that one also measures area". It seems that including area when working with the BMI is somewhat illogical. T2 supports this by repeating "area" with different words, namely "surface area", and with a "yes", and then they laugh. T1 elaborates: you divide weight "by the area of the body". The excerpt ends with T1 explicitly asking: "there is something wrong, isn't it?"

The mathematical formula of the BMI is the main focus in this excerpt. The teachers discuss the formula and they question the measures used, in particular why area is a part of the formula. It seems that they would like to associate some physical aspect with the squared height in the BMI formula, ending up discussing if this object could be an "area" or a kind of "surface area" of a body. After some reflections, T1 ends up with questioning the validity of this presumption. These reflections can be interpreted as steps in the process of trying to obtain understanding of what physical aspects the formula is dealing with. The discussion concentrates on the mathematical concepts of area and surface area. This can be explained by the fact that the denominator in the formula has unit m^2, which is the unit of area. This is perhaps not the most expedient way to comprehend the formula, but it can have a potential from a teaching perspective. When it comes to mathematical operations, the teachers mention the mathematical operation division. They do not specifically mention mathematical concepts like proportionality and inverse proportionality, which are central in the construction and understanding of the BMI formula.

How to get a physical understanding of the BMI-coefficient is also a theme in the discussions amongst the teachers. In e.g. biological or medical studies, it is often convenient to operate with a mean value of the BMI-coefficient, calculated for some specific group of individuals. This implies that the physical aspects measured by the BMI-index could be perceived as for example the total mass of some population compared to the mean of the squared heights of the individuals.

In this excerpt, the teachers do not seem conscious about the opportunity of using BMI as a mean-value for a larger population group, as well as a measure for single individuals. Their reflections may be regarded as a first step toward achieving mathematical knowing about the BMI formula. The teachers are trying to make sense of the mathematics they see in the formula. They question the formula, or their understanding of it, when T1 asks if there is something wrong. T1 also situates the area issue of BMI in a teaching context with students. This illustrates the two-sided focus for the teachers. On the one hand they wear their student hats and focus on understanding the BMI, and on the other hand they put on their teacher hats and focus on the BMI in a school context. This is about mathematical knowing: teachers try to think of how they can make sense of the formula in a school context. The aspect of internal source criticism is also present here. The teachers are discussing the index to find a meaning behind the formula: what does the formula tell them?

To engage in questioning the mathematics involved in models as indexes is an important part of developing mathemacy. The practising teachers demonstrate mathematical knowing and use internal source criticism to some extent. However, the discussion indicates also a need to further develop mathematical competency where practising teachers get insight into how index numbers are developed and how a ratio could be understood and taught in classrooms. Based on the mathematical knowing reflected by the teachers in this study, other indices, like the HDI, seem complicated to discuss. However, to facilitate for interpretation of basic mathematical concepts in relation to indices as discussed in this section, can be a fruitful starting point.

Technological knowing

Discussions about the use of the index in different contexts are identified as technological knowing. In order to decide about the appropriateness of the index in the different contexts one needs to think about who is using the index, for what purpose, and how the results are interpreted and similar issues that we identified as internal criticism. Several excerpts from the data are regarded as being of a technological nature, and two typical excerpts involving discussions about who and for what purpose are presented and analysed in the following.

The teachers have been discussing the fact that BMI does not consider the difference between fat and muscles. Examples of people being defined as overweight in different contexts have also been discussed:

T2 How can a public health nurse define her as overweight based on some numbers ...?

T4 Surely, she has a standard to follow ...

The first teacher questions the use of numbers by nurses to define someone as overweight. T4 justifies the nurse's use of the numbers by saying it provides her with ”a standard to follow”. The focus here is on the use of the index in a school context by nurses to keep track of obesity in children. Does it make sense in this context to use a number obtained from the formula? Discussing the use of mathematics in different contexts indicates a use of technological knowing.

By using the words ”define her as overweight based on some numbers” the teacher points towards the role the index has in society – it can be used to define people. When saying "How can a public health nurse" the practising teacher points towards the role people who use the index may have. At the same time, a pattern is identified in the data: the teachers provide arguments for and against the use of the index. The two utterances above exemplify such opposing arguments, as T2 provides a pro argument while T4 provides a counterargument.

The discussion continues:

T2 But why should that standard ... I am questioning such a standard ...

T3 But can they conclude without taking into consideration lifestyle and diet ...?

T4 But they plot them on the graph according to the child's own development ... as long as you follow ...

T2 It does not make it any better ... oh, I feel that I am getting irritated ...

T2 is questioning the use of BMI as a standard. T3 questions if "they", the public health nurses, can conclude "without taking into consideration lifestyle and diet". The teacher implies that one concludes from a standard without considering lifestyle. T3 challenges the "expert", the nurses, in their use of the index by critically pointing to other variables that need to be taken into account when defining people as being overweight. T4 refers to how they (nurses) use graphs where a child's development can be tracked. In this way, a new model and representation is introduced, a graph which illustrates a development in time. T2 says that this does not help, and rejects T4's suggestion for using an individualised graph. Questioning those who are using the index and especially the way results are used, shows that internal source criticism is in play.

There is sometimes a blurry distinction between technological and reflective knowing. The first conversant is critical towards the use of BMI as a standard. Others in the group have earlier discussed an example of an adopted girl from South America who did not fit into the growth charts percentiles for European children. This was used as a counter-example to the use of BMI and is referred to later in their conversation. This discussion has a potential to go beyond a technological level, where the appropriateness of the mathematical solution/formula in the context of monitoring children's growth charts can be explored. At a reflective level, the role of mathematics in how we view children's growth and development is focused upon.

Reflective knowing

The following excerpt is identified as reflective knowing. Practising teachers are discussing BMI, its formulation and purpose. T2, who earlier was critical to the use of BMI, comes up with an argument for the use of BMI as a tool at a societal level, to set a standard that we need, that everyone can refer to:

> T2 But I also think that from a societal perspective, and being used sensibly, then this can be a good tool ... what else can a public health nurse or others or myself use ... if we don't have any standards to follow? I am one of those who trust that the experts have a competence that I don't have. But then I also could think that, no ... she is perfectly fine.

One important difference from the earlier example about technological knowing is that here the role of mathematics is discussed in terms of it being necessary in order to set some standards. It is a reflection at a meta level about the use of mathematics represented in indices. The argument for using BMI as a standard is accompanied with external source criticism: "being used sensibly, then this can be a good tool". This implies that as long as the index is used in a societal perspective to set some standards, then it is a good thing to have. At a personal level, they indicate that one should be careful to not interpret it literally. One has to take into consideration further variables to evaluate a child's development. The way the index is used is thus important in order to say something about its value.

In T2's utterance: "I am one of those who trust that the experts have a competence that I don't have", we can identify a confession of being a non-expert and a firm belief in experts, those who have the competence to decide about these things. However, in the next sentence the teacher says "But then I also could think that, no ... she is perfectly fine". This "but" and "no" indicate a resistance against trusting experts and by saying "she is perfectly fine" the teacher indicates an opposing opinion to the experts and their standards. The use of pro and con arguments is a pattern in the discussions. There is a continuous alternating between trusting the experts and being critical. The criticism can be regarded as a kind of external source criticism where the teachers use examples and question experts' judgement. The following excerpt underlines this aspect of alternating argumentation where the same teacher (T2) questions who decides what is "normal", by elaborating the previous utterance with an example:

> T2: I am a little bit worried ... about who is to decide what is right when it comes to weight. I had an example with a 14 years old girl, the daughter of a friend of mine, who was told that her weight was a little high, like you mentioned ... and in my eyes she was certainly not ...

In this excerpt, T2 explicitly focuses on "who is to decide what is right when it comes to weight". The example underlines the teacher's critique against using standards on an individual child when she says ”was told that her weight was a little high [...] and in my eyes she was certainly not”. The teacher reflects on the power that standards as BMI have when used at an individual level. The BMI influences how we see each other.

Being critical towards the use of BMI and who decides what is right, provides opportunities to think about alternative solutions. Skovsmose (1992) underlined that the object for reflective knowing is a suggested technological solution to some problems. The practising teachers problematise how BMI is used on individuals and offer alternative solutions: evaluating one’s lifestyle (see the first excerpt) and apply common sense to decide if a child is overweight or not. These alternative evaluations are not mathematical or objective criteria, but more of a general nature. One way to develop further reflective knowing would be to decompose the BMI formula and modify it, as teachers in Julie’s (2002) study were encouraged to do with the HDI formula. This requires a combination of mathematical, technological and reflective knowing.

Conclusions

In our study, teachers engage in critical reflections when asked to discuss questions about BMI and its use in society. They engage in in-depth reflections about the formula for the BMI and try to make sense of the mathematical concepts and operations used in it. This made it possible to identify mathematical knowing and different entry points to reflections. In line with Julie’s (2002) study, the teachers do not engage in deeper mathematical discussions, such as changing components or proposing mathematical concepts that could better serve the intended aims for the BMI. We identify the need for developing further such reflections about and with mathematics. One way to achieve this can be a more active role of the teacher educators to ask questions whenever they see the potential of discussing deeper mathematics.

At the technological knowing level, teachers engage in finding uses of the BMI and are critical towards such uses by arguing with personal and non-personal examples. They often question the standards set by BMI and the experts who use them. We identify a pattern in the teachers’ argumentation: they alternate between trusting the experts and being critical towards them. In this part, as well as at the reflective knowing level, teachers organise their discussions around pros and cons regarding the use of BMI – from different points of view. When they present different arguments, they help each other to argue better about the usefulness and the validity of the index. This way, the teachers jump between having confidence in, and lacking confidence in the experts who have defined the index and those who use it. They have developed some degree of scepticism that makes them question the use of mathematics in the index, which is the attitude we wish to develop with students as well. These results show that using indices in a teacher education context can lead to critical discussions about the role of indices and mathematics in our lives and in society in general. In addition, participating and being able, themselves, to discuss in critical ways, can give teachers more confidence and ideas to orchestrate critical discussions about mathematics with their own students.

When using Skovsmose's types of knowing, we find it very useful to integrate source criticism in order to have some more concrete questions to look for in teachers' discussions. This combination of theories is a novel approach and makes a valuable contribution to the operationalisation of Skovsmose's theory about different types of knowing. Questions about who is using the index, why it is being used, and what consequences it implies for the people involved, proved important for becoming aware of the formatting power of mathematics.

Having seen the potential of using indices for initialising critical discussions in the classroom, we think further research will be valuable for finding ways of developing more reflections concerning the mathematics used in indices. One way to do this is to ask practising teachers to discuss the use of indices in teaching mathematics in their own classrooms. This can stimulate them to consider what mathematics can be made available to students at a certain age, and ways for finding meaning in the mathematical formulas. Such discussions have the potential to develop reflections at a mathematical level both for the teachers and their students. Drawing attention towards possibilities of using student-formulated indices to measure phenomena that are important for them can also be fruitful in this aspect. As Skovsmose (1992) argued, it is in the process of making decisions about what variables to take into account when modelling a situation from reality that students can discover how mathematics influences our perception of reality and our society in general.

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Notes

1 30 ECTS correspond to half a year full-time studies or, as in the case of the participants in this study, one-year part-time studies.

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