# First and second language students' achievement in mathematical content areas

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This study compares Swedish first ($n$ = 2 253) and second language ($n$ = 248) students' achievement in mathematical content areas specified by the TIMSS-framework. Data on mathematics achievement from three national tests 2007–2009 in school year 9 are used. The present study found that the achievement difference between the mathematical content areas algebra and number was smaller for second language students than for first language students and this result holds with statistical significance ($p = 0.016$). The same holds for algebra versus data and chance ($p = 0.00053$). A hypothesis for further research is suggested; that students immigrating in late school years have contributed to the observed result by bringing experiences from other curricula into their new schooling.

There are about 10 % second language students in Swedish compulsory school (Statistics Sweden, 2016). Some of them have had a shorter or longer part of their school experience from a curriculum other than the Swedish. Moreover, large scale studies like TIMSS have shown that students in different countries may have a different achievement profile for the mathematical content areas. For example in TIMSS 2011 and 2007 8th grade students in for example Sweden, Norway and Finland achieved above their overall mathematics score in number and in data and chance but below their overall mathematics score in algebra while students in several countries in the Middle East and Eastern Europe showed an opposite pattern (Mullis, Martin, Foy & Arora, 2012). The purpose of the present study is to explore if there are any differences between Swedish first and second language students in this respect and what these differences may look like.

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## Complexity of multilingualism and mathematics achievement

Both migration status and language status are used in mathematics education research, though language status seems more common when it comes to investigating spoken and written classroom communication. These two ways of categorising students sometimes makes it complicated to compare different studies. For example TIMSS categorises according to whether the student speaks the language of the test at home always, almost always, sometimes or never. PISA instead categorises students according to immigrant status, such as native students, first or second-generation immigrants, where ”second-generation immigrants” are defined as native students with two immigrant guardians. These two definitions are related but not equivalent as illustrated in figure 1. For example a Swedish-speaking person migrating from the Swedish-speaking part of Finland to Sweden will remain a first language speaker. The situation of one language spoken in different countries is common. For example German, French and English are all spoken in several countries within Europe.

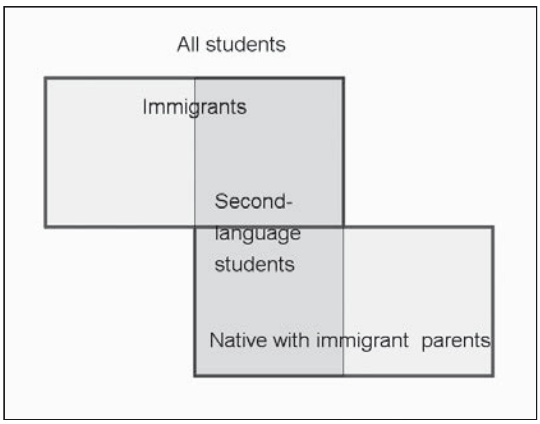


Figure 1. *Illustration of the relation between language status and immigrant status*

In Swedish student achievement statistics, it is common to categorise students by migration status though categorisation by first or second language is sometimes used. In the Swedish school system the students are assigned by language experts and governed by the school act to follow either ”Swedish” or ”Swedish as a second language” courses (SFS 1994 : 1194; SFS 2011 : 185). The two criteria for being assigned to the course ”Swedish as a second language” are that the student is considered to need support in Swedish language development *and* has a mother tongue other than Swedish. The latter condition may include immigrants as well as native students with two immigrant guardians (sometimes denoted "second-generation immigrants").

### *Mathematics achievement and multilingualism*

There are at least three components that research indicates as potential factors impacting on the achievement differences between first and second language students. The first component involves having the language of instruction as a second language. Good knowledge in the language of teaching and testing is crucial for participating in the social and mathematical activities during lessons (Gorgorió & Planas, 2001). It is also crucial for understanding test items given in for example written assessment situations (Norén & Andersson, 2016). For Swedish conditions, Parszyk (1999) reported that second language students may not yet master the skills in the language of instruction necessary for successful mathematics learning and test-taking. In TIMSS, the students report how often they speak the test language at home using the categories "never", "sometimes", "almost always" or "always". Achievement differences, between students that reported themselves as "sometimes" or "never" speaking the language of the test at home and students who reported themselves as "always or almost always" speaking the language of the test at home, have been reported from TIMSS 1997 data (Heesch, Storaker & Lie, 2000) and TIMSS 2011 8th grade data (Mullis et al., 2012). This occurred for Sweden and several other countries, but not for all of the participating countries in TIMSS 2011. The impact of being a second language student is also visible in the national Swedish assessments. Students in school year 9 (the last year of compulsory school) take compulsory national tests in mathematics and in other subjects. Results from these tests show that second language students achieve lower than first language students (Petersson, 2012).

A second component of being a second language student is the time needed for developing proficiency in the second language when this is the language of instruction and testing. For example Cummins (2008) gave an empirically based time span of five years or more to gain language proficiency good enough for successfully learning advanced academic content in school. This component specifically applies to second language students that have immigrated during the last years of compulsory school. Böhlmark (2008) conducted an analysis of siblings to control for socio-economic background. Böhlmark found that the closer to grade 9 (last school year of Swedish compulsory school) the student had immigrated, the lower the average leaving grade in mathematics.

A third component of being a second language student is the socio-economic background. Two aspects of the socio-economic background are the student's school situation and the student's home situation (Hansson, 2012; Pásztor, 2008; Skolverket, 2004; Svensson, Meaney & Norén, 2014). Hansson (2012) used data from TIMSS 2003 8th grade to study these two aspects. The student's home situation is essentially measured using the educational level of the parents. In Hansson's study parents of second language students on average had a shorter education than parents of first language students and the educational level of the parents is positively correlated to the school success of their children (see e.g. Pásztor, 2008; Skolverket, 2004). Moreover Skolverket (2004) found that schools in residential areas with a lower socio-economic status (in terms of parents' education and income) have a larger proportion of unqualified teachers than other residential areas. Hansson (2012) measured the students' school situation as responsibility in classroom for learning. The teachers' part in this responsibility is to scaffold students by actively supporting the students' mathematical work and learning. However, Hansson found a pedagogical segregation in terms of group composition and learning responsibility. This pedagogical segregation was expressed as a correlation between teachers taking less responsibility and classes with large proportions of students having lower language proficiency and parents with shorter education. A similar relation between group composition and how the classroom activities are organised has also been found by Zevenbergen (2001). Students' socio-economic background may also result in some students perceiving themselves predetermined to underachieve in mathematics, partly due to the discussion in public media, since they may think that they do not have parents that are good enough at helping them with for example home-work (Svensson et al., 2014).

A possible fourth component is that the student ”brings influences from outside [...] into the formal education process” (Bishop, 1991, p. 15). Some second language students may have had shorter or longer experience from previous schooling following a curriculum that may be different from that of the country of immigration. This fourth component is crucial with respect to the purpose of the present study of exploring possible differences between Swedish first and second language students in achievement profile for the mathematical content areas. It is known that teaching styles may vary between different countries (Andrews, 2009; Ma, 1999; Stigler & Hiebert, 1999). Moreover different curricula may emphasis different mathematical content areas. For example, in TIMSS 8th grade 2007 and 2011, students in for example Finland, Norway and Sweden achieved above their own overall mathematics average scale score in the mathematical content area of data and chance and numbers and achieved below their own overall mathematics average scale score in the mathematical content area of algebra (Mullis, Martin & Foy, 2008; Mullis et al., 2012). For example Swedish students in the mathematical content area of number achieved 19 units above and in data and chance 20 units above their own overall mathematics average scale score and 26 and 28 units below in algebra and geometry respectively (Mullis et al., 2012). All these differences were statistically significant (and the TIMSS-studies use a per mille scale with fixed centre point 500 and students typically perform within the range from 300 to 700). Of the second language grade 9 students in Sweden 2006, 73 % had mother tongues spoken in the Middle East and Eastern Europe including OSS (Statistics Sweden, 2016). For countries in this region, data from TIMSS 2011 and 2007 show that most countries achieved higher in the mathematical content area of algebra than in number and data and chance (Mullis et al., 2008; Mullis, et al., 2012), that is an achievement profile in algebra and number opposite to that of Sweden.

Now, one question is how many second language students are there that have had a considerable part of their schooling before immigration to Sweden. This number of second language students can be estimated by the number of recently arrived immigrants as follows. Year 2009 the number of sixteen year old Iraqi born immigrants were 2 177 and year 2006 the number of thirteen year old Iraqi born immigrants were 1 441 (Statistics Sweden, 2016). The increase of 736 immigrants with the same age must be due to immigration of Iraqi born students aged 13–16. Since some earlier immigrated Iraqi born students may have emigrated back to Iraq, the actual increase of students aged 13–16 may be even larger though it is likely that the number of returning immigrants is small due to the security situation in Iraq still being unstable at that time period. In the same way table 1 gives the estimated number of students that immigrated at age 13–16 during the years 2007–2009.

Of the immigrants in table 1 only a small part were from the Nordic countries and of these the majority were born in Denmark and Norway, whose languages are similar to Swedish. As a comparison with table 1, the

Table 1. *Estimated number of students that immigrated at ages 13–16 during 2007–2009 (Statistics Sweden, 2016)*

| Region | Count | Of all immigrants |
|---|---|---|
| Middle East & Eastern Europe including OSS | 4 482 | 14.6 % |
| Nordic countries | 95 | 0.3 % |
| Other countries | 2 980 | 9.7 % |
| Sum | 7 557 | 24.6 % |

number of participants in the school subject "Swedish as a second language" 2006 in grade 9 was 6 634 and of these 4 869 had a mother tongue spoken in the Middle East and Eastern Europe including OSS (Statistics Sweden, 2016). So, students that have emigrated from countries in the Middle East & Eastern Europe including OSS or have mother-tongues spoken in this area are in majority among the students with Swedish as a second language; among sixteen years old immigrants; and among sixteen years old immigrants that immigrated during the age of 13–16.

As shown above, there are several studies on overall achievement in mathematics of second language students and immigrants such as TIMSS and PISA and several studies comparing curricula and teaching styles in order to explain achievement between different countries (Andrews, 2009; Ma, 1999; Stigler & Hiebert, 1999). Despite this, few studies consider immigrants and second language students' achievement in specific mathematical content areas, for example those specified by TIMSS. One example is Petersson (2012), who noted that second language students achieved lower than first language students on test items in statistics and on mathematic concepts that are infrequent in mathematics lessons. Another example is Ufer, Reiss and Mehringer (2013), who compared learning gain from the first to the second school year in first and second language students' achievement on two kinds of test items; namely those testing mathematical procedures and those testing understanding of mathematical concepts, (though not divided into mathematical content areas). Their conclusion from their study was that the influence of language proficiency in learning gain is more pronounced for test items demanding conceptual understanding than for test items demanding procedural skills. Moreover, the language-related differences in learning gain between first and second language students starts to accumulate already at school start. Ufer et al. also pointed out that good result on procedural test items for second language students may sometimes conceal difficulties with test items on conceptual understanding.

### *Research question*

The literature review showed that second language students in Sweden achieved less well than first language students in both mathematics tests and in leaving grade (Böhlmark, 2008; Hansson, 2012; Mullis et al., 2012; Petersson, 2012). Additionally, while Swedish students achieved worse in algebra than their overall mathematics achievement in TIMSS and better in number and data and chance than their overall mathematics achievement, the achievement pattern is reversed in most countries in the Middle-East and Eastern Europe. However, only little research was found, both for Swedish conditions and internationally, on comparing first and second language students' achievements in different content areas of mathematics. As a result, this study will investigate the following research question.

– What do the achievement profiles for different mathematical content areas look like for first and second language students respectively?

Instead of focusing a possible achievement gap between first and second language students, the research question will generate new knowledge and new research questions about the achievement profiles of these two student categories in relation to mathematics content.

## Method

For the present study, a secondary analysis of the achievements on the national test part B1 in school year 9 was made. School year 9 is the last year of Swedish compulsory school, thus this test is the last test for which a random sample out of all students is possible. The Swedish National Agency for Education (which is the Swedish governmental organisation for education), has given the PRIM-group at Stockholm University the commission for constructing, devising, implementing and evaluating national tests in mathematics for compulsory school. The test instrument, categorisation of the test items into mathematical content areas and rationale for the method will be discussed in the following subsections.

### *Test instrument*

The national tests are designed to follow the Swedish mathematics syllabus and are used for evaluating the students' knowledge on an individual level as well as on school level and on national level. The test in school year 9 covers all mathematical content areas, which is not the case for tests given in earlier school years. Among all the parts of the national test, part B1 covers the widest range of mathematical content areas. The teachers' assessment of the students' responses is highly reliable for part B1 since only the students answer is assessed and no full solution is required from the students. In part B1 calculators are not allowed and the formulation of each test item is mathematically narrow in the sense that each test item covers only one mathematical content area. The mark given by the teachers was one or zero points on each test item. Part B1 has the lowest correlation between achievement and reading ability for the written parts (Skolverket, 2009, 2010), which serves to minimise the risk of testing language proficiency rather than knowledge in mathematics, since problem formulations may constitute language obstacles for second language students (Barwell, 2009; Gerofsky, 2006; Lager, 2006). Finally, the test item formulations in part B1 typically have direct questions and few real life application contexts compared with other parts of the national test, an issue of importance for second language students (Campbell, Davis & Adams, 2007). The national tests given in the years 2007–2009 were chosen since from 2007 data on first or second language as background variable were included and for these years the national tests used in the present study are publically available (PRIM-gruppen, 2007b, 2008b, 2009b), but tests from 2010 and onwards are presently still under a publication embargo.

### *Categorising the test items in mathematical content areas*

The research question calls for a definition of mathematical content areas. Several frameworks have been modelled for characterising mathematical knowledge (Devlin, 2000; Grønmo, Lindquist, Arora & Mullis, 2013; Kilpatrick, Swafford & Findell, 2001; Niss, 2002; OECD, 2013; Turner, Blum & Niss, 2015). For the present study, the TIMSS-categorisation of mathematical content areas was chosen. This choice can be motivated from the discussion earlier in the paper. There it was shown that a majority of Swedish second language students during the time period of the collected test data (2007–2009) have mother tongues spoken in the Middle East and Eastern Europe. Moreover, students in these countries showed achievement profiles different from that of Swedish students for the mathematical content areas of algebra, number, and data and chance as defined by TIMSS (Mullis et al., 2008; Mullis et al., 2012). To be able to interpret the outcome of the present study in the light of the achievement profiles found by TIMSS, it is needed to re-categorise the test items in the present study into the TIMSS framework.

The mathematical content areas in TIMSS are the four categories algebra; data and chance; geometry; and number. This categorisation is similar though not identical to that of the Swedish syllabus, which is algebra; data and chance; measurement; and number. The test items were categorised anew into the TIMSS-categories by the following process – the author and a research colleague made independent categorisations of the test items according to the TIMSS description given in Grønmo et al. (2013) and then reached agreement on a common categorisation. The final categorisation matched the one given by PRIM-gruppen (2007a, 2008a, 2009a) except for the following six cases. Test item 2007B10a (signifying test item 10a, part B1 given 2007) was categorised as *data and chance* (instead of original Swedish syllabus categorisation *algebra*). This test item asked ”Which package costs least?” given a diagram with three marked points. The other five test items in Swedish syllabus are categorised as *measurement* and were classified into the following TIMSS categories. Test item 2009B13 had the formulation ”The perimeter of a square is 8a. Write an expression for the area of the square” and was categorised as *algebra*. The following four test items were categorised as *number*. Test items 2008B10 and 2009B16, which both were on speed, time and distance. Test item 2009B4 was ”How many minutes is 0.75 h?” Test item 2009B12 was on converting the distance in a map into the distance in reality. In the TIMSS 2015 framework, scale is not included in geometry but fits well into working with ratio and proportion within the mathematical content area of number. Altogether there were 14 test items in algebra, five in geometry, 31 in number and eight in data and chance as stated in table 2.

Table 2. *Categorisation of the test items in national tests 2007–2009 part B1*

| Mathematics content | 2007 | 2008 | 2009 | Sum |
|---|---|---|---|---|
| Algebra | 5 | 5 | 4 | 14 |
| Data & chance | 3 | 2 | 3 | 8 |
| Geometry | 2 | 2 | 1 | 5 |
| Number | 10 | 11 | 10 | 31 |

The test items in TIMSS 8th grade are similar to the Swedish national test part B1 in the sense that only the answer is assessed. A difference is that the TIMSS test items are multiple choice questions with one of four alternatives being correct; while in the Swedish national test only nine of the 58 test items in table 2 (two–four each year) were multiple choice questions with one of five alternatives being correct. The remaining 49 test items were open for any response written by the student.

### *Rationale for the method*

The data were collected in a random sample by the PRIM-group for evaluation purposes. The data have the format of a table with one row for each randomly sampled student. Each row contains one column for categorising the student as a first or a second language student and one column for each test item assessed as correct (1 point) or incorrect (0 points). To answer the research question, the following measure was constructed. For each student was calculated the proportion (in percent) of correct responses on test items in number minus the proportion of correct responses on test items in algebra. Now the research question can be answered with a statistical test as follows; use a two-tailed *t*-test to find if the differences (number achievement minus algebra achievement) are equal or not for first and second language students. In the same way was tested the difference achievement in data and chance minus achievement in algebra. The achievement in geometry was not tested, since the test items in geometry were few. Geometry had only one or two test items per year as shown in table 2.

This way of comparing achievement profiles instead of absolute achievement avoids a possible trap when comparing test achievements of two student categories with different properties. First and second language students, seen as samples, have different properties due to for example socio-economic background, as mentioned earlier. One example is Skolverket (2004), who compared school achievement of native students and students with different migration status (of which some were second language students, see figure 1) and found that when socio-economic background was controlled for, the achievement differences disappeared except for those who immigrated after school start age. Since the achievement profile defined in this section is a difference within a student category, it is likely to cancel effects from the social background. Simply comparing achievements of different student categories as a measure of equity have been challenged by some researchers (e.g. Gutiérrez & Dixon-Román, 2010; Leder, 2015). Further questions must be asked. How does exploring a possible achievement gap benefit the students in the classrooms instead of just add to stereotyping different student categories? The research question in the present study examine the achievement profiles for different mathematical content areas, and they might help identifying areas of relative strength and weakness of the two student categories and thus identify possible areas for development of the mathematics classroom activities.

## Results

For the two student categories, table 3 provides the number of students and their achievement profile per mathematical content area measured as the percentage of test items answered correctly on part B1 of the national tests 2007–2009. On average the second language students achieved lower than the first language students both in total and for the different mathematics content.

Table 3. *Number of students and achievement profile per mathematics content*

| Mathematics content | First language students | Second language students |
|---|---|---|
| Algebra | 53.3 % | 46.5 % |
| Number | 60.6 % | 49.6 % |
| Data & Chance | 62.4 % | 47.8 % |
| Geometry | 44.7 % | 35.7 % |
| Total | 57.7 % | 47.0 % |
| Counts | 2 253 | 248 |

Table 3 shows that the achievement profile difference between number and algebra was smaller for second language students (*Mean* = 3.2 %, *Standard error* = 1.6 %, $n$ = 248) than for first language students (*Mean* = 7.3 %, *Standard error* = 0.53 %, $n$ = 2 253). For the first language students and the second language students the $t$-test described in the method found that the difference (= 4.1 %) between the two achievement profiles was statistically significant with $p = 0.016$. For the achievement profile difference between on the one hand data and chance and on the other hand algebra, the following holds. For the first language students the difference (*Mean* = 9.2 %, *Standard error* = 0.70 %, $n$ = 2 253) was larger than for the second language students (*Mean* = 1.3 %, *Standard error* = 2.1 %, $n$ = 248). The $t$-test found this difference (= 7.8 %) between the two achievement profiles to be statistically significant with $p = 0.00053$.

The distributions behind the averages in table 3 are plotted in figure 2, i.e. each point in figure 2 corresponds to the first and second language students' achievement on each test item. The equality-line corresponds to equal achievement of first and second language students; a point above this line corresponds to a test item for which the second language students on average achieved better than the first language students. The smaller achievement differences between number and algebra for second language students in table 3 correspond to a smaller shift downwards when the plots of number and algebra in figure 2 are compared. For first language students, the larger achievement differences between number and algebra correspond to a larger shift leftwards. In other words this means that the distribution for algebra achievement lies closer to the Equality-line than does the distribution for number achievement. The same holds when comparing achievement in algebra versus data and chance for the two student categories. In detail, for the mathematical content area of data and chance, the second language students achieved between eight and 23 percentage points lower than the first language students for all eight test items, as displayed in figure 2. For algebra, the

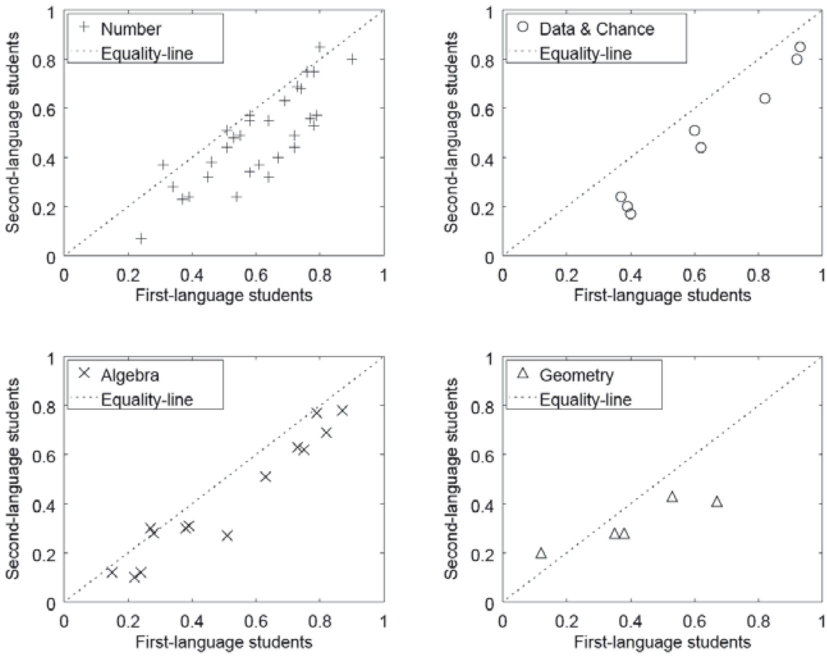


Figure 2. *Achievement profiles of first and second language students per test item*

students achieved nearer the Equality-line in figure 2 and between -3 and 13 percentage points lower than the first language students except for test item 2007B13, for which the difference was 24 percentage points. The two extremes were the following test items.

- The algebra test item 2007B13 with coordinates (51 %; 27 %); a multiple choice question on equations.
- The algebra test item 2009B17 with coordinates (27 %; 30 %); determine a quotient of two sums of identical numbers.

The distribution for number achievement is further away from the Equality-line in figure 2 than the distribution for algebra achievement. Moreover, the number achievement distribution shows a large spread. For example, the second language students achieved over the equality-line for the following two test items.

- The number test item 2008B12 with coordinates (31 %; 37 %); a multiple choice question on estimating the product of two decimal numbers.

– The number test item 2007B4 with coordinates (80 %; 85 %); calculate $7+3\cdot6$.

For ten test items in number the achievement difference between first and second language students was larger than twenty percentage points (ranging 21–32 percentage points). For example the second language students achieved well under the equality-line for the following two test items.

– The number test item 2007B14 with coordinates (54 %; 24 %); calculate $\sqrt{9+16}$.

– The number test item 2007B3 with coordinates (64 %; 32 %); fill in the missing number. 1.795 – __ = 1.705.

One observation in figure 2 is that though the second language students on average achieved lower than the first language students on the test items, the second language students still achieved higher than the first language students on two test items that were among the more difficult ones; the algebra test item 2009B17 mentioned above and the geometry test item 2007B18, which was the most difficult of all test items for the first language students.

– The geometry test item 2007B18 with coordinates (12 %; 20 %); multiple choice on the area of an equilateral triangle. 2009B17.

## Discussion

The present study set out to explore the achievement profile between Swedish first and second language students for different mathematical content areas as specified by TIMSS (Grønmo et al., 2013). The overall mathematics achievement of second language students was lower than that of first language students in the present study as seen in table 3. That is in line with several other studies (Böhlmark, 2008; Heesch et al., 2000; Mullis et al., 2012; Pásztor, 2008; Petersson, 2012; Skolverket, 2004). The contribution of the present study is that the difference in achievement in algebra in comparison to number and also for algebra compared to data and chance was smaller among second language students than among first language students. Along with this followed that the mathematics achievement profile in algebra, number and data and chance was different for first and second language students.

The second language students' smaller difference between achievements in the partitions of the mathematical content areas algebra versus data and chance and number versus algebra in table 3 raises the suggestion that immigrant students may have brought experiences from curricula with a larger emphasis on algebra and a smaller emphasis on number, and on data and chance compared to the Swedish curriculum. A large proportion of the second language students in Sweden have a background in the Middle East and Eastern Europe, and TIMSS 2011 results for 8th grade found that in most countries in these regions, the students on average achieved relatively higher in algebra than in number and in data and chance (Mullis et al, 2012). Since using algebraic symbols for numbers in several curricula is introduced around school years 6–7, this suggests that students that had immigrated in later school years are those that essentially contribute to the second language students' smaller achievement differences between algebra and the two content areas of number and data and chance. This suggestion is also supported by the pattern in figure 2; of the four test items, for which the second language students achieved better than the first language students, three were among the more difficult ones, and of these two were on algebra and geometry. A suggestion for further research is thus to distinguish between students who immigrate in earlier and in later school years and for these two subcategories of immigrants, explore their mathematics achievements with respect to specific mathematical content areas, for example, as in TIMSS.

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