The teaching of mathematical problem-solving in Swedish classrooms: a case study of one grade five teacher's practice

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In this paper we examine the teaching of mathematical problem-solving to grade five students of one well-regarded and experienced Swedish teacher, whom we call Mary. Working within a decentralised curriculum in which problem-solving is centrally placed, Mary is offered little systemic support in her professional decision making with respect to problem-solving instruction. Drawing on Lester's and Schroeder's descriptions of teaching *for, about* and *through* problem-solving, we draw on multiple sources of data, derived from interviews and videotaped lessons, to examine how Mary's problem-solving-related teaching is constituted in relation to the weakly-framed curriculum and the unregulated textbooks that on which she draws. The analyses indicate that Mary's emphases are on teaching *for* and *about* problem-solving rather than *through*, although the ambiguities that can be identified throughout her practice with respect to goals, curricular aims and the means of their achievement can also be identified in the curricular documents from which she draws.

In the following we present a case study of Mary (pseudonym), an experienced and professionally respected Swedish primary school teacher, focused on how she engages her students in mathematical problem solving. The broader study from which this paper derives has examined Mary's professional decision-making within a decentralised system's high expectations of personal autonomy with respect to what and how a teacher chooses to teach (Skott, 2001). Consequently, we have followed Mary, a mathematics and science specialist who currently teaches year five, for the past four years, examining the intersection of her professional decision-making, relationships with colleagues, teaching and the curricular and institutional structures within which she works.

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It is internationally accepted that doing mathematics means solving problems, a perspective reflected in curricula throughout the world and international tests of achievement like the OECD's Programme of International Student Assessment (PISA). In the particular context of the mandated Swedish curriculum, problem solving is simultaneously an overarching aim and one of six central content domains (Skolverket, 2011a). In other words, problem solving, as both broad aim and particular goal, is intended to form the core of the Swedish students' learning of mathematics. It also widely accepted that to become a competent problem solver, a student needs to experience mathematical problems in a variety of different ways (Andrews & Xenofontos, 2015). However, the very loose specification of the Swedish curriculum (Bergqvist et al., 2010) and unregulated textbook production have colluded in creating a situation in which teachers have little explicit or authoritative guidance with respect to what this variety might entail or how it might be managed in a typical classroom. In this paper, drawing on Lester & Schroeder's (1989) classification of teaching *for*, *about* and *through* problem solving, we examine how Mary tacitly negotiates this problematic aspect of her professional responsibilities and, in so doing, highlight a number of tensions in the manifestation of problem solving in her classroom.

Problem-solving in mathematics teaching and learning

A read of Bell's (1937), albeit romantic and at times inaccurate, biographies of many of Europe's early mathematicians confirms that problem-solving is the core of the mathematician's work, although, until recently the inclusion of problem-solving as a curricular goal for all students has been variable. For example, for more than 200 years the French curriculum has been based on an Enlightenment principle that education for all should emphasise rationality and an intellectually-based problem-solving (Holmes & McLean 1989; Lauwerys, 1959). Such perspectives contrast with the English tradition whereby experience rather than rationality underpinned the education of ruling elites (Holmes & McLean, 1989; Lauwerys, 1959). Later, the industrial revolution with its rapid urbanisation transformed public education, although mathematics was restricted to what were thought at the time to be necessary arithmetical skills (Katz, 1976). However, the launch by the Soviet Union of Sputnik in 1957 precipitated a shift of educational attention in the West (Klein, 2003) and, driven by the growth of the Bourbaki group's influence on university mathematics, the second half of the twentieth century saw problem-solving shift to the centre of school mathematics (Andrews, 2015).

Today, despite Bourbaki's curricular demise, problem-solving remains at the core of school mathematics internationally, as evidenced by recent summaries of problem-solving in, inter alia, Australia (Clarke, Goos & Morony, 2007), China (Cai & Nie, 2007), France (Artigue & Houdement, 2007), Germany (Reiss & Törner, 2007), Hungary (Szendrei, 2007), Israel (Arcadi & Friedlander, 2007), Italy (Boero & Dapueto, 2007), Japan (Hino, 2007), Mexico (Santos-Trigo, 2007), Netherlands (Doorman et al., 2007), Portugal (da Ponte, 2007), Singapore (Fan & Zhou, 2007), the UK (Burkhardt & Bell, 2007) and the USA (Schoenfeld, 2007). As part of this international movement, problem-solving has been discussed in terms of desirable competencies (Schroeder & Lester, 1989; Kilpatrick, Swafford & Findell, 2001; Niss, 2002; Kilpatrick, Martin & Schifter, 2003; OECD, 2003).

In general terms, "a mathematical problem presents an objective with no immediately obvious means of achievement" and is not "a function of the task but the individual solver's knowledge, experience and dispositions" (Andrews & Xenofontos, 2015, p. 301). Moreover, mathematical problems may be located entirely in a world of mathematics or presented in some form of context, which may be real-world (Xenofontos & Andrews, 2014). That being said,

> we consider problem-solving as the "art" of dealing with non-trivial problems which do not yet have a known, routine solution strategy to the student, but which provide opportunities for the student to develop new solution strategies. (Doorman et al., 2007, p. 405)

Such matters lead us to ask, how has the teaching of problem-solving been conceptualised? In this respect, Schroeder and Lester (1989) offer three perspectives that have been extensively and continuously used as a research framework (see, for example, Andrews & Xenofontos, 2015); Stacey, 2005; Wyndhamn, Riesbeck & Schoultz, 2000). They write of teaching *for*, *about* and *through*[1] problem-solving. In short, teaching *for* problem-solving focuses on the acquisition of skills necessary to solve problems (Andrews & Xenofontos, 2015; Hino, 2007). The objective is the solving of problems (Schroeder & Lester, 1989). Teaching *about* problem-solving focuses on teaching about the different strategies, such as Pólya's (1945), understand the problem, devise a plan, implement it, and reflect, used when solving a problem (Andrews & Xenofontos, 2015). Here, problem-solving is the content of instruction (Hino, 2007; Schroeder & Lester, 1989). Teaching *through* problem-solving occurs when new mathematical content is introduced through the posing and solving of a problem (Xenofontos & Andrews, 2014). Teaching through problem-solving does not mean that heuristics are unimportant but instead of a focus on presenting different strategies a discussion would follow where advantages and disadvantages of different strategies are discussed (Hiebert, 2003.) The three approaches exist side by side and together they hold the potential to give students a rich and deep understanding of mathematics (Schroeder & Lester, 1989).

> When teachers teach via problem-solving, as well as about it and for it, they provide their students with a powerful and important means of developing their own understanding. As students' understanding of mathematics becomes deeper and richer, their ability to use mathematics to solve problems increases. (ibid, p. 41)

Drawing on their review of the literature, Andrews and Xenofontos (2015) conclude that to become successful problem-solvers students need to acquire an appropriate knowledge of mathematics, a set of problem-solving strategies or heuristics, a regulatory meta-knowledge and a belief that mathematical problems are worth solving (Schoenfeld, 2004). To facilitate the development of such competencies, they argue that teachers should encourage their students to problematise mathematics through the asking of questions like, ”can this example be extended or generalised?”. They should grant their students the authority to define and pursue their own problems, shift the burden of accountability from the teacher to both their peers and the disciplinary norms of mathematics, and finally, provide the appropriate resources for the above to happen (Andrews & Xenofontos, 2015; Engle & Conant, 2002).

The Swedish (Mary's) context

Over the last thirty five years, the role of problem-solving in the Swedish national curriculum has changed considerably. In the 1980 curriculum an emphasis on problem-solving strategies was introduced, in ways that reflected an emphasis on teaching *about* problem-solving. The 1994 curriculum encouraged the use of problem-solving to promote and develop mathematical understanding, as in teaching *through* problem-solving (Wyndhamn et al., 2000). However, while these and subsequent curricula emphasised broad content and other goals students were expected to achieve by certain ages, a significant change emerged in 1994 with the introduction of competencies [2], making explicit that which previously had been implicit (Bergqvist et al. 2010). These broad competencies included, for example, the expectation that

> The school in its teaching of mathematics should aim to ensure that pupils [...] develop their ability to formulate, represent and

> solve problems with the help of mathematics, as well as interpret, compare and evaluate solutions in relation to the original problem situation. (Skolverket, 1999, p. 23)

The current curriculum, which presents mathematics as a problem-solving activity (Skolverket, 2011a), not only prescribes problem-solving as both content and knowledge but also as a competency. Consequently, problem-solving is now uniquely presented as both competency and required knowledge (ibid). However, the specification of the competencies is less explicit than that of the mathematical content goals (Boesen et al., 2014). This lack of clarity, or weak framing (Bernstein, 2000), means that teachers are free to choose how and when they teach problem-solving. Moreover, within the current specification can be found teaching *for*, *about* and *through* problem-solving. For example, with respect to teaching *for* problem-solving it is specified that students should "formulate and solve problems using mathematics" (Skolverket, 2011a, p. 59). From the perspective of teaching *about* problem-solving students should be taught "strategies for mathematical problem-solving" (Skolverket, 2011a, p. 62) and finally, with respect to teaching *through* problem-solving, can be found the advice that reflecting "over the limitations and possibilities within different methods and strategies gives new knowledge and creates a confidence in one's own thinking (Skolverket, 2011b, p. 7). Thus, Mary has spent her entire career within a curricular tradition in which problem-solving, albeit in vague and weakly specified ways, has played a major part. Moreover, in addition to curricular and national test expectations, Swedish teachers like Mary are influenced by the textbooks they use (Boesen et al., 2014). Indeed, within the Swedish context has been found a strong positive correlation between text book use and procedural goals and a strong negative correlation between text book use and problem-solving (ibid).

Aims and research question

In this paper we focus on one Swedish grade five classroom to explore in depth one teacher's approaches to the teaching of problem-solving. In a context like Sweden, where problem-solving has been placed at the heart of a weakly framed curriculum, it is important to understand teachers' interpretations of the expectations placed on them. In this paper, therefore, we examine not only how Mary teaches *for*, *about* and *through* problem-solving (Schroeder & Lester, 1989) but how her practice is informed by the resources available to her. To do this, we focus on the following question.

– How is Mary's teaching of problem-solving constituted in relation to teaching for, about and through problem-solving and how is it informed by the curricular materials available to her?

Methodology

The study is a single instrumental case study (Stake, 1995) with an ethnographic approach (Heath & Street, 2008) through which we aim to understand more about the teaching of problem-solving in Swedish grade five classroom. In this respect, while making no claims to her typicality, Mary can be construed as a ”telling case" (Andrews, 2016; Mitchell, 1984) that serves to make visible insights previously unavailable or hidden. Moreover, studying a single case not only facilitates a thorough analysis of the teacher within her social setting (Hammersley & Gomm, 2009) but permits a depth unlikely with multiple cases (Donmoyer, 2009; Gerring, 2006). Thus, we aim not to generalise but offer a thick description (Geertz, 1994) of one teacher's problem-solving reality that will extend our understanding of the complexity of teaching.

Acknowledging the ethnographic emphasis, data were collected over several months in two periods two years apart. The data on which this paper is based derive from transcriptions of audio recordings of two semi-structured, hour-long, interviews (Bryman, 2008) that were structured by Mary's own lesson notes and focused on how she construes mathematical problem-solving and its teaching. Data also included video recordings of four topic-based and four problem-solving lessons. A detailed synopsis was produced for each of the eight lessons that included descriptions of the lesson's events, summaries of what participants said and images of, for example, the whiteboard. These synopses were then read, annotated and confirmed by Mary. Finally, the students' normal textbook and two supplementary problem-solving textbooks were collected, with the former illustrating the teaching tradition within which Mary works.

The production of both transcripts and synopses, while themselves interpretive acts, provided opportunities for an initial thematic scrutiny of data, a process that is typically independent of theory (Roulston, 2001) that facilitated not only the capture of Mary's problem-solving-related actions but alluded to unresolved or unnoticed tensions. In such circumstances a ”theme captures something important about the data in relation to the research question, and represents some level of patterned response or meaning within the data set" (Braun & Clarke, 2006, p. 82). For example, one such tension concerned differences in the ways that Mary organised her problem-solving lessons compared to her topic-based lessons; another was found in the different ways she conceptualised important prerequisite knowledge. Such matters then informed a top-down analysis in which data were read from the perspective of helping us to understand how Mary's teaching of problem-solving was constituted in relation to teaching for, about and through problem-solving and the manner in which this was informed by the curricular materials available to her.

Findings

Overview of Mary's teaching

Before presenting our findings we offer a brief overview, inferred from four topic-based and four problem-solving lessons, of Mary's typical teaching practice. With respect to her teaching of mathematical topics, in this case angles, her lessons typically began (See figure 1) with a review, or repetition, of all the relevant concepts and procedures students were supposed to know, prior to her introduction of the new material. After about 20 minutes of introduction the students worked individually from their textbook.

When teaching problem-solving, Mary abandoned her class' regular textbook for others focused on problem-solving. The former, while including two problem-solving-related pages at the end of each chapter, does not integrate problem-solving into the mainstream content. The others, Mary's main source of problem-solving activity, offer problems alongside exemplar solutions. All books offer advice concerning the managements of students' work on problem-solving, including approaches to the grouping of students as well as problem-solving strategies. In addition, Mary encourages her students to learn a set of strategies presented under the acronym, LURBRAK. This, in English, invites children to read the text, repeat the question, underline or circle important information, decide how to calculate and explain why, sketch a solution, use mathematical language and check if the answer is reasonable. This acronym, unknown to Mary, has its origins in support for students of special educational needs working on text problems (Sterner, 2007) and has become Mary's default problem-solving process.

Mary's problem-solving lessons take place once a week and are structured differently from the norm. Typically, figure 2, they begin with an introduction to the lesson's single problem, which is followed by a long period of student work before one or two student groups present their solutions on the board for feedback, which is usually focused on the quality of the written solution. Mary expresses that teaching problem-solving separately from her ordinary lessons ensure her students' access

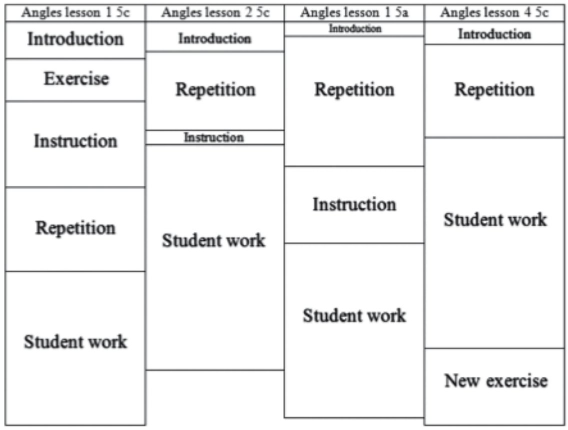

Figure 1. *Mary's angle lessons*

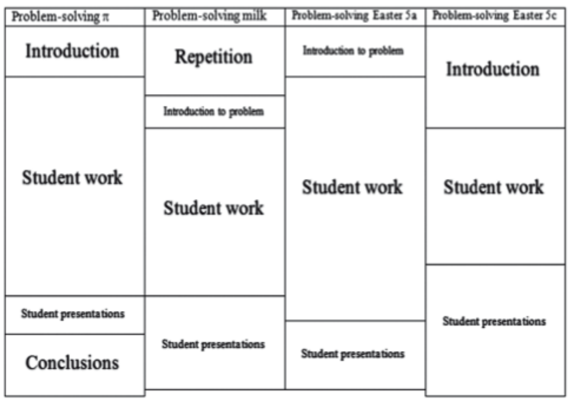

Figure 2. *Mary's problem-solving lessons*

to an important element of mathematical learning, this is also supported by the structure of the materials she uses.

The observed problems

In this paper we offer analyses of four lessons focused on three different problems; Mary teaches two classes and one of the problems was undertaken by both classes. The first problem, the *π-problem*, asked students to measure different circular objects and calculate the ratio between the circumference and the diameter. Students were given a table to complete and instructions on how to calculate the ratio. The second problem, the

Figure 3. *The milk-problem (Grevholm, 1989)*

Milk-problem, was presented as in figure 3 with, translated into English, the following text "A farmer sells milk from his farm; he measures the milk with a 5 dl measure and a 3 dl measure. One day he had to pour exactly 1 dl. How did he do that?" The final, *Easter-problem*, was seen on the Friday before Easter. Children had to decide what Easter egg (see figure 4) they would buy for a fictional family based on their decisions concerning the size of an Easter egg, how much candy it could hold and its cost.

Figure 4. *Easter egg (photo: www.karamello.com)*

From the above and what was said in interviews, it became clear that Mary has a broad perspective on what constitutes a mathematical problem. For example, with respect to the Easter-problem she commented that,

> That was what was so wonderful with the Easter-problem, that it is so wide, from big to small. It is a problem for me, maybe not for you, but it is still a problem of some kind and depends on the background fact you have. Eh, open and closed, I so want to do more open ended problems, with open answers, just to get the creativeness and not only ... they (her students) are so locked on right or wrong and that makes me go crazy.

While, with respect to the π-problem, she said, "I think this is more like an investigation; I lead them to a discovery". In such comments Mary talks of the need to offer her students something beyond closed questions, with their right or wrong answers. In so doing, she expresses that a mathematical problem is an atypical task and relative to the students involved, a view commensurable with both the broader problem-solving literature and the Swedish curriculum framework.

Teaching for problem-solving

In the following we show how teaching *for* problem-solving can be seen in Mary's observed lessons, interviews and curricular materials. As indicated above, the goal of teaching *for* problem-solving is the act of problem-solving itself; the point at which students' mathematical knowledge is focused on the solution of non-standard tasks. In this respect we regard any teaching act explicitly focused on preparing students to solve such tasks is teaching for problem-solving. For example, as shown below, Mary spoke of the need for students to acquire a set of basic competencies prior to becoming problem solvers:

Int.: What do they need to know to be problem solvers in mathematics?

Mary: The basics, the four principles of calculations, they have to ...

Int.: The basics?

Mary: The basics, yes ... If you know them the field is open and you can transform them to volume, weight and such, the whole thing. And number sense, the position system that whole thing then you have it. Then it is the methods, step by step and so. But you must have the basics.

The importance of this basic knowledge permeated both Mary's interviews and lessons. In this regard, she spoke of students struggling to solve a problem when their basic knowledge is insufficiently robust for it to be integrated in any attempt to solve it. For example, she commented, with respect to the problem in figure 5, that

> I like this because there is a house that gets in the way and they really have to see the mathematics in the whole picture, they need to know what the angle sum is. There were a couple of girls. They remembered the angle sum at last and because they know their subtraction they could solve it.

Of course, whether the task in figure 5 constitutes a genuine problem for Mary's students is uncertain. The key point is that for her it demands an integration of basic knowledge – angle sum of a triangle and subtraction – as prerequisite for teaching *for* problem-solving. Interestingly, it can

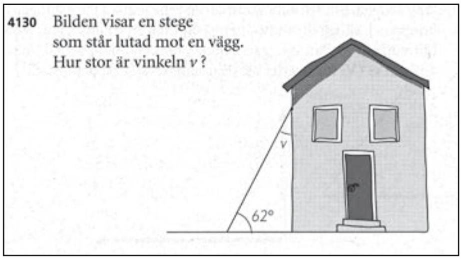

Figure 5. *"Angle sum"-problem (Undvall, Olofsson & Forsberg, 2001, p. 187)*

be argued that the structure of the regular textbook supports teaching *for* problem-solving in its locating genuine problems at the end of each chapter. However, these problems do not always draw on the material found in the pages before them.

Mary's teaching *for* problem-solving can be seen in various ways, particularly in her interactions with her students, as in her introduction to the Milk-problem, where the following occurred;

Mary: Today we will work with volume, what do you know about volume? Mikael

Mikael: You can calculate the volume of something.

Mary: And what is volume?

Emma: What something contains

Mary: What something contains. Today's problem is about volume, what something contains.

Interestingly, solving the Milk-problem does not require knowledge about volume, it is essentially a number patterns problem posed in a volume setting. However, Mary opts to reassure students about the nature of volume, highlighting, again, a focus on prerequisite mathematical knowledge in line with teaching *for* problem-solving.

In the above we see at least two tensions. Firstly, Mary's teaching *for* problem-solving seems more focused on guaranteeing students' basic knowledge than on the integration of that knowledge. There is an espoused integration, but her actions do not always align with that espousal. Secondly, as with the Milk-problem, Mary's emphasised basic knowledge does not always help the students with the problem at hand.

Teaching about problem-solving

Teaching *about* problem-solving, as described above, is when knowledge of problem-solving and problem-solving strategies provides the content of mathematics teaching. As such, students may be taught about such strategies with no explicit intention of their being used or, if they are to be used, it is their use that is privileged rather than the solution on which they are focused. In this latter respect, much of Mary's data alludes to an emphasis on learning about strategies. Indeed, it could be argued that her frequent invocation LURBRAK, as described earlier, is more focused on the teaching of strategies than their application to the solving of problems. Mary has placed on her classroom wall a poster on which the seven LURBRAK strategies have been written as a guide for students. During the Easter-problem students were told to present their solutions according to LURBRAK. In so doing, Mary's attention alludes to an emphasis on teaching about problem-solving due to its subordinating not only the solution itself to a discussion of these privileged strategies, as in this episode when Mary comments on a solution presented by two students.

Mary: That is also a way to solve this. Most of you solved according to the first example but this one emerged, it was a bit tortuous at first but this is also a way to solve it, to think outside the box, you pour back and forth you pour out

This is interpreted as teaching *about* problem-solving when the solution process and the solution itself overshadows if the solution is efficient or general.

The fact that Mary teaches one problem-solving lesson each week is also indicative of her teaching *about* problem-solving in the sense that she distinguishes problem-solving as something different from mainstream mathematics. Also, as indicated above, when students present their solutions, much attention is paid to strategies and presentation, at the expense of a discussion of the function of those strategies in the solution's construction. For example, during the Milk-problem the following ensued:

Mary: Are there any ways we can make this solution better? What do you suggest Cecilia?

Cecilia: Well you could have used pictures (inaudible)

Mary: To make it even clearer, you could have drawn a picture of these measures, 5dl and 3dl, and maybe included the volume, I agree since that is what we practice.

Here the feedback was focused not on the solution to the problem but strategies independent of their role in the solution. This invocation to draw also permeated Mary's interviews, as when she described the problem-solving process:

Mary: Start from the beginning and do it step by step. Or draw a picture. It is possible that this is my favourite.

Int.: I think solving the problem with the picture is different from illustrating the solution with a picture. They are not really the same.

Mary: No, they are different; I had some students who were solving a problem that involved two characters, Lisa and Pelle, buying something. They just drew Lisa and Pelle; it did not mean anything for the solution.

Mary clearly saw the drawing of Lisa and Pelle as unsatisfactory, due to its not being construed as part of the students' solution. In some ways this seems to conflict not only with her earlier acceptance of the drawing of the Milk-problem measures but also her frequently expressed expectation that students should use the LURBRAK-strategies, of which drawing is one. Even when students presented correct solutions without pictures they were encouraged to add one. This we interpret as teaching *about* problem-solving, since the drawing is privileged above its function in the solution.

In the above can be seen an important tension. Mary focuses on discussing available strategies rather than the quality of their contribution to problem solutions. Thus, for example her emphasis on drawing may lead students to see this as the goal rather than as a means to support, where appropriate, the solution process. In other words, in Mary's practice can be seen a focus on a variety of solutions rather than a focus on efficient or elegant solutions, which can be argued to be in line with teaching about problem-solving.

Teaching through problem-solving

Teaching *through* problem-solving is when problems are used to prompt students' learning of mathematical content. Attention is not on the solution itself but the mathematical knowledge that emerges from it, for example by searching for a general solution. In some respects it would not be surprising if lessons focused on teaching *for* or *about* problem-solving should yield little with respect to teaching *through* problem-solving. And, in general terms, this was the case. However, there was some evidence of its existence, not least because the *π-lesson* was focused on students' discovering the value of π, even if the π-task does not satisfy generally accepted criteria of what it is to be a mathematical problem (Andrews & Xenofontos, 2015). For Mary, this was a lesson based on an atypical task, which she construed as a problem.

Also, when working on a problem, Mary intervened in ways that supported students' learning of new material. For example, during the Easter-problem lesson, even though it had not been an explicit teaching goal, Mary's intervention helped Max understand that volume and weight offer different measures of the same object:

Mary: The problem is about volume but then you might buy candy by weight, you don't buy that by volume.

Max: But, how many kilos is a litre?

Mary: If you have water for example, one litre milk or water weighs one kilogram.

Max: Yes

Mary: But if you have one litre foamy candy it does not weigh one kilogram.

Max: No

Mary: But a kilo of harder candy will be heavier than the foamy candy.

Max: Yes

During interviews Mary rarely spoke of the possibility of students learning new material *through* problem-solving. However, when asked a direct question, in the interview after the observations, she not only conceded the possibility she also taught a whole lesson focused on the discovery of π. In other words, despite little evidence with respect to teaching *through* problem-solving, it was not invisible in Mary's practice. Finally, possibly explaining why so little evidence was forthcoming, only one of the books she used discussed learning *through* problem-solving. Moreover, this same book advised teachers to "value the chosen strategies", which may indicate why, as discussed above in relation to teaching *for* and *about* problem-solving, Mary tended to privilege students reporting on their strategies rather than the relationship of those strategies to the solution itself.

Discussion

We began by asking, how Mary's teaching of problem-solving was constituted in relation to teaching *for*, *about* and *through* problem-solving and how this was informed by available curricular materials. This seemed an important question in light of her working within a weakly framed curriculum (Bernstein, 2000) that forces professional autonomy (Skott, 2004). The data indicate that Mary did, indeed, teach for, about and through problem-solving. However, the extent to which these are privileged varied considerably and we address each of these, in relation to institutional structures.

Firstly, teaching *for* problem-solving permeated Mary's interviews and, as shown in figure 1, the ways in which she taught her mainstream lessons. That is, her emphases on students learning basic mathematical skills underpinned much of what she said and did. This was reflected in her problem-solving lessons, particularly with respect to the Milk-problem. However, as indicated, this may hinder rather than support students' solving the problem before them.

Secondly, an emphasis on teaching *about* problem-solving can be seen in Mary's separation of problem-solving from her ordinary topic teaching. However, this distinction provides a tension, not least because problem-solving is internationally increasingly integrated into mainstream teaching (Doorman et al., 2007; Fan & Zhu, 2007; Hino, 2007). However, the distinction is unchallenged by the Swedish curriculum and effectively encouraged by the separation of problems and exercises in Mary's everyday textbook and the problem-solving textbooks she used. Many of Mary's actions allude to teaching about problem-solving. This was seen in many interactions with students, both publicly and privately, whereby her attention was on strategies independently of their relationship to the problem under scrutiny, particularly in her repeated invocation of the LURBRAK rubric. We can also find support for this focus in the national curriculum, especially in the commentary material (Skolverket, 2011b).

Thirdly, Mary's separation of problem-solving from mainstream teaching also indicated that teaching *through* problem-solving was not a personal objective, even though the π-lesson was focused explicitly on a new mathematical relationship. Such ambiguity of intention may be explained by recourse to the national curriculum, which, while discussing the possibility of students learning *through* problem-solving, advises teachers to ”value their chosen strategies”. In such an invocation can be seen a privileging of teaching *about* over either teaching *for* or *through* problem-solving. Finally, Mary expressed an idiosyncratic view on the nature of mathematical problems – she construed the π-task as a problem because it satisfies her criterion of non-routineness, which is why this problem is discussed in relation to teaching through problem-solving. Against other criteria the π-task would have been construed as an investigation (Cockcroft, 1982), although no such distinctions exist within the Swedish framework (Skolverket, 2011b). Such ambiguity is not unknown elsewhere. For example, in an Australian context the expression ”problem-solving” typically becomes a description of any non-routine task used in support of students' learning of mathematical ideas (Stacey, 2005).

The success of the decentralised Swedish school system depends on teachers' decision-making. How Mary taught problem-solving was the result of decisions taken on many levels, and the decision to teach problem-solving once a week is one of them. Indeed, her problem-solving teaching was the result of many, typically ongoing, decisions concerning, for example, her relationship with and understanding of mathematics, the curriculum and the various resources she had available to her (Skott, 2001). In the intersection of Mary's professional decision-making, teaching and institutional structures we can see how a weakly framed curriculum and other resources contribute to tensions in her problem-solving-related decision making. This leads us back to Schroeder and Lester (1989), who argue that all three approaches – *for*, *about* and *through* problem-solving – are necessary for students to gain a deep understanding of mathematics. Mary enacted all three, although she attended more to teaching *about* problem-solving than the other two. While the source of this uneven enactment is unclear, it may be due to the lack of specificity in the Swedish national curriculum and the unregulated textbooks currently available. Indeed, although teaching through problem-solving may be a realistic curricular objective, precise specification as to its enactment, due to the very nature of problem-solving, is unrealistic (Lester, 2003; Russel et al., 2003; Schoen & Charles, 2003). These curricular resources provide full support for teaching *about* and *for* problem-solving, but little or no support for teaching *through* problem-solving. This insight is important if we want to develop how we discuss and describe problem-solving teaching. It is not enough to claim that problem-solving is important, we need to broaden the discussion as well as our descriptions of what problem-solving teaching could be.

Mary, in her decisions about teaching problem-solving communicates the importance of problem-solving and different problem-solving strategies. The communication that would be privileged by the academic mathematics, generalisation for example, was rarely visible in either Mary's teaching, or in the curriculum. In order for the students to get access to mathematics, the curriculum, as well as the teacher, need to privilege specialised forms of communication. That being said, we acknowledge that a case study of one teacher has many limitations, particularly with respect to generalisation. However, Mary is a very "telling" case (Andrews, 2016; Mitchell, 1984), in that she has highlighted the likely consequences, for a teacher who is very highly regarded by her colleagues and her students' parents, of a weakly framed curricular specification.

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Notes

1 Although Lester and Schroeder (1989) use the expression "via problem solving" subsequent research has typically used the phrase "teaching through problem solving", which is what we have done here.

2 In the Swedish national curriculum the word for competency is more related to ability (*förmåga*). We use competency rather than ability because of its connection to the international problem-solving literature.

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