Different perspectives on possible – desirable – plausible mathematics learning in preschool

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This article addresses the question of what is considered possible – desirable – plausible in preschool mathematics. On the one hand, there is a growing consensus that preschool mathematics matters, on the other hand, there are different opinions about how it should be designed and what constitutes an appropriate content. In the article we provide an overview of similarities and differences found in eight articles published in a thematic issue of NOMAD on preschool mathematics. The overview is based on Bernstein's notions vertical and horizontal discourses, and how content for learning is described as basic or advanced mathematics. The aim is not to evaluate or rate the articles but to illustrate diversity regarding possible – desirable – plausible in current research of preschool mathematics.

There is a growing consensus in the pedagogical research community that early mathematics matters. A large number of studies show that competencies acquired in the early preschool years have effects on later school achievements (Aunio & Niemivirta, 2010; Doverborg & Pramling Samuelsson, 2011; Hannula & Lehtinen, 2005; Levine et al., 2011; Mazzocco et al., 2011; Pruden et al., 2011; Siegler & Ramani, 2009). Naturally this has had impact also on the political agenda in Nordic as well as global discussions about preschool education's purpose, goals and educational practice. Even though preschool mathematics in recent years has received increased attention, both in politics and in research, there are large differences when it comes to perceptions of what preschool mathematics is, how it should be designed and what constitutes an appropriate content. Research approaches to mathematics education are often argued for in bright contrast to another. These argumentations have often shown to be nonproductive since they lead to either-or controversies without shades and creates sets of false choices (Casey, 2009; Clarke, 2006). However, it is

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not only within mathematics education different approaches are argued. For example, in the introduction of a handbook of play and learning in early childhood the editors conclude that a consensus about the definition of play and learning as well as the definition of early childhood will never be achieved (Brooker, Blaise & Edwards, 2014).

In this article we will provide an overview of the included articles in this thematic issue of NOMAD regarding how appropriate design and content for preschool mathematics are – explicitly or implicitly – expressed: Which similarities and differences can be found in the articles concerning what preschool mathematics is, how it should be designed and what constitutes an appropriate mathematical content? We will use Bernstein's (1999) notions *vertical and horizontal discourses* to distinguish between different designs of preschool mathematics. Furthermore we will use Claesson, Engel and Curran's notions (2014) *basic and advanced content* to distinguish between the mathematical content being emphasized as appropriate. Together these four notions can be used to describe different contexts of mathematics in preschool.

Children come to understand their world – including mathematics – through active participation and interaction with other people in different contexts. What is made possible for children to learn depends on what we invite them to participate in where context is not something to be acted upon but something to interact with. Children will learn regardless of which people and which context they encounter, but they will learn different things (Rogoff, 2003; Wertsch, 1998). How we design (mathematics) activities in preschool (for example in terms of vertical or horizontal discourses – basic or advanced content) influence what the children are invited to participate in and by that which mathematics they can learn. Thus, in this article we will provide an overview of the articles in this thematic issue of NOMAD regarding how appropriate design and content for preschool mathematics are expressed. Further we will discuss possible implications of the similarities and differences found.

Design of mathematics teaching

As mentioned we will use Bernstein's (1999) notions *vertical and horizontal discourses* to distinguish between different designs of preschool mathematics. Bernstein used these notions to distinguish between school knowledge (vertical discourse) and everyday common sense knowledge (horizontal discourse). A vertical discourse is characterized by coherence of content, hierarchically interconnected procedures, specialized language, systematically organized activities and by a focus on general knowledge. A horizontal discourse is characterized by its location within

communities, high relevance in the situation, every-day language, segmentally organized and maximized encounters with persons and habits.

Horizontal discourse and vertical discourse can be understood as two extremes. If connecting these extremes to the design of preschool mathematics you will find preschools which present mathematics in separate, hierarchically systematically organized "lessons" on one side. On the other side you will find preschools where mathematics is considered to be part of everyday activities and by that there is no need to make it further explicit to the children. And, of course, between these extremes there are infinite designs of preschool mathematics.

The design of preschool mathematics matters, since it influences what preschool children are invited to participate in. According to Devlin (2000) differences in how individuals solve what may look as "the same" mathematical task can be connected to what the task is portrayed as being about. It is not necessarily easier for children to work with mathematics in a design involving play, pictures or manipulatives since these may make the situation more complex and by that it becomes harder for children to identify what they should pay attention to. Thus children can become absorbed by doing and by that look as being very engaged in the activity but still not learn the intended. Then you could think that the best design would be to structure tasks in a way where no distractions are possible. However, this has shown to drain all excitement and interest since the learner then risks only experiencing a series of instructions (Mason & Johnston-Wilder, 2006). In a recent study (Björklund, 2014) this complexity of designing preschool mathematics is illustrated in three ways of how teachers are approaching "the same" mathematical learning object. The first way of approaching mathematics learning objects was to give individual children traditional tasks ("I give you x number of items, can you divide them into half?"). This engaged the children in activities where their primary focus was on the "doing", to be given a task to solve, whereby the mathematical content became irrelevant to the children. Another approach for teaching mathematics could be found in hidden learning objects within every-day problem solving tasks. These tasks were carefully designed to provide children with opportunities to explore the mathematical content, but the context of solving an every-day problem (for example doubling a recipe in order to make a dough) directed attention to the finished product, not to elaborate on the mathematical principles that would solve the problem. The third way of approaching mathematical learning objects was framed by a narrative context. The narrative (such as a story, a play or game) was used to give relevance to the mathematical content (for example, a farmer had six animals; we have to divide them equally into two folds). This latter

approach seemed to help direct the children's attention to the intended learning object, while containing a structure that emphasizes the mathematical content. If connecting these examples to a continuum between the two extremes vertical and horizontal discourses, the first ("I give you *x* number of items, can you divide them into half?") can be seen as an example within the vertical discourse; the second (doubling a recipe in order to make a dough) as an example within the horizontal discourse; and the third (a farmer had six animals, we have to divide them equally into two folds) somewhere in the middle.

Of course, which design of preschool mathematics children are invited to participate in can and will differ within one single preschool where neither of the two extremes vertical and horizontal discourses seems to be "the" desirable design of preschool mathematics. Further, different children can contextualize a designed task very differently, for example as a mathematical issue, a practical one and/or an illustrative one (Pramling & Pramling Samuelsson, 2008). Thus, the design is part of – but does not constitute – the context of preschool mathematics.

Content of mathematics teaching

From a historical and general preschool perspective, content has been less focused on than the activities containing the content (Pramling Samuelsson & Asplund Carlsson, 2014). Content is, however, part of the context of preschool mathematics in which children are invited to participate and lead to the question of what an appropriate mathematical content is in preschool. We will use the notions *basic and advanced mathematics* (Claesson et al., 2014) to distinguish between the desirable mathematics content in preschool as it is presented in the articles in this thematic issue. However, we will not define the notions exactly in the same sense as Claesson et al. (2014). They define mathematics content as basic or advanced depending on whether the majority of children have mastered the content or not. If more than half of a group of children have mastered a specific content they consider it to be basic. Thus, basic mathematics imply mathematics content that the majority of the children already know but that still is new for others while advanced mathematics is new content for the majority of the children. Then, to be able to distinguish between basic and advanced mathematics Claesson et al. (2014) needed to screen the pre-knowledge of the children which of course is not possible in our situation. We will not define basic and advanced mathematics based on pretests but instead on the mathematics content promoted in the articles. What kind of content, if any, do the articles imply? Do the

authors refer to and/or promote basic mathematics or do they stress the use of more advanced mathematics? Thus, we here base our analysis on the authors' description of the content.

Number and small quantities are the most frequent content in preschool mathematics education (Björklund & Barendregt, 2016) and also the area in which most research has been conducted. However, also geometrical experiences and concepts about space, shape, pattern, and order are common in preschool mathematics (Sarama & Clements, 2009). Probability, combinatorics, statistics, measurement and problem solving are less frequent but have in research shown to be possible content in preschool (see for example English, 1991; Åberg-Bengtsson, 2006). Of course, all content can be elaborated on a basic or on an advanced level and a relevant question is if it matters which approach is taken? In their studies Claesson et al. (2014) found that all children benefit from exposure to advanced content in mathematics while they do not benefit from basic content coverage. "We find that all children, regardless of their early childhood care experiences, benefit from more exposure to advanced mathematics content" (Claesson et al., 2014, p. 426).

Connecting design and content

If we connect design and content we get the context of mathematics teaching that children are invited to participate in their preschool. We connect *horizontal* and *vertical* discourse with *basic* and *advanced* content and find that this context can offer children very different learning opportunities, as illustrated in figure 1.

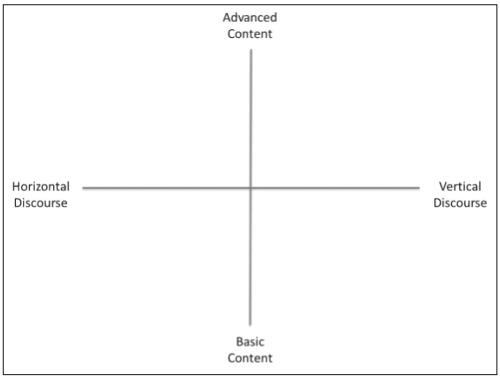

Figure 1. *Connecting horizontal and vertical discourse with basic and advanced mathematics*

The two extremes horizontal and vertical discourses are to be understood as two very different design approaches of preschool mathematics. On the one side (horizontal discourse) it is sufficient that mathematics is part of every-day activities and routines with no need to make the mathematical content explicit for the children. On the other side (vertical discourse) mathematics itself is the starting point with no need for applications. Thus, every-day is the starting point in the horizontal discourse and mathematics is the starting point in the vertical discourse. Along the line there is a gradual shift and somewhere in the middle there is a shift concerning everyday life or mathematics being the starting point for the design of preschool mathematics education. These differences (not the extremes) can be understood as the difference between the following two approaches:

- Today we are about to take a walk to the park. What kind of mathematics can we make visible for the children as they participate in this activity?
- Our children have few experiences of length. Which activities can we arrange and invite the children to participate in where they can experience length?

The two extremes on the other axis, basic and advanced content, are to be understood as differences when it comes to which mathematics content that is stressed. As mentioned, we will not pre-define what basic or advanced mathematics is, but instead focus on how the mathematics content is described in the articles. Do the authors refer to and/or stress what can be considered as basic mathematics or do they stress the use of more advanced mathematics? Both questions above, those illustrating horizontal and vertical discourse, can be connected to either basic or advanced mathematics. The more advanced mathematics the higher we will end up in the figure.

Results

There are eight articles in addition to this article in this thematic issue and in this section we will do a brief presentation of them, when discussing which context of mathematics teaching that is being promoted. Due to the limited sample of articles we cannot draw any generalized conclusions of their respective themes' relationship to the design and content of their described mathematics education; they are to be seen as examples of how content and design are handled by the authors. Not all authors discuss content and design to an equal extent; therefore we have

chiseled out their main point made in the articles and what discourse they thereby may represent according to our framework.

All four areas in the figure are represented by our small sample, which tells us that the diversity within the field of early mathematics education is large. Our aim is, as earlier stated, not to make an evaluation of best practice, rather to give an overview of the current situation in mathematics education in preschool.

Promoting a horizontal discourse

Four articles express a horizontal discourse promoting the child's initiatives to mathematize meaningful experiences from their lived world. Mathematics is described as a dynamic act of exploration in which the teacher follows the child's intentions.

The article *Locating learning of toddlers in the individual/society and mind/body divides* by Tamsin Meaney (in this issue of Nomad) highlights the question of what young children can and should learn about mathematics. It is argued that learning for young children should be considered as problem solving with both the mind and the body. Further, a need to change the conception of mathematics from a static set of knowledge and skills to a view of mathematics as dynamic, fluctuating activities involving both mind and body is emphasized. The empirical examples in the article focus on children locating themselves in space, on the ground and on vertical bars. These examples are characterized as problem solving since the children experience difficulties in placing themselves in specific locations. The author writes that the children solve location problems relevant to themselves. The teachers are not involved in direct teaching but let the children explore location with their bodies by setting up the boundaries of the play. Based on these examples the author argues that the learning of preschool mathematics should not be considered as a process of acquiring a set of stable facts and skills but as dynamic problem solving where the doing leads to the knowing and not the other way around. Thus, this article seems to give prominence to a context of preschool mathematics located within a horizontal discourse as characterized by a high relevance in the situation and a segmental organization with maximized encounters with persons and habits.

The article *Measuring temperature within the didaktic space of preschool* by Ola Helenius, Maria L. Johansson, Troels Lange, Tamsin Meaney and Anna Wernberg (in this issue of Nomad) presents a framework for analyzing interactions between preschool teachers and children in situations involving mathematics. The authors want to explore how to make sense of the teaching and learning of mathematics in preschool when it occurs

through play. The framework combines instrumental and pedagogical dimensions to analyse teachers' and children's contributions in activities. The empirical example analysed is about measurement and was chosen since it fits Bishop's attributes of mathematical play. The example chosen in this article clearly promotes a horizontal discourse. The teacher trying to turn the situation from instrumental to pedagogical can be understood as a movement towards a vertical discourse. However it is written that the teacher do not "force the children to take up her offers" which still emphasises a horizontal discourse.

The article *Initiating a conceptualization of the professional work of teaching mathematics in kindergarten in terms of discourse* by Per-Einar Sæbbe and Reidar Mosvold (in this issue of NOMAD) is based on an everyday Norwegian preschool situation. The authors motivate this by explaining that Norwegian preschool tradition is more focused on everyday activities and free play than on classroom lessons. In the article "teaching" implies preschool teachers' communication of mathematics and is illustrated by how a preschool teacher turns a Lego play activity into a mathematical discourse through questions and affirmations. In this article part of the aim can be understood as trying to distinguish and describe the context of mathematics in preschool where the used definition of discourse includes both design and content. The empirical example is based on an activity within a horizontal discourse (high rele-vance in the situation, segmentally organized) but if imagining figure 1 the activity is quite close to the middle as the teacher uses the situation to teach (communicate) mathematics.

In the article *What characterises mathematical conversations in a Norwegian kindergarten?* by Trude Fosse (in this issue of NOMAD), Bishop's approach to define mathematics is used and it is made explicit that this approach does not rely on school curricula of mathematics. Bishop describes six universal mathematical activities that can be found in all cultures and in the article these activities are used to analyse a conversation. In the article it is argued that if a conversation is to be seen as mathematical it must be based around mathematics and structured such as it invites the participants to inquire further mathematics. Furthermore, the participants are to be interacting and reflect on the mathematics and the conversation should promote further learning. The author argues that such conversations either can be carefully planned or arise more or less spontaneously. Based on the reference to Bishop and on the examples this article seems to imply a horizontal discourse of preschool mathematics. However, the promoted characteristics of mathematical conversations put it near the middle in figure 1.

Towards a vertical discourse

Some of the articles take a mathematics curriculum as starting point or emphasize the need for teachers with content knowledge who enable planned learning in mathematics. Researchers argue for the necessity of teachers being well aware of learning goals and a goal-directed approach to facilitate learning for every child that is interpreted as an approach heading towards a vertical discourse.

The article *Central skills in toddlers' and pre-schoolers' mathematical development, observed in play and everyday activities in Norwegian kindergartens* by Elin Reikerås (in this issue of NOMAD) presents a large scale study of preschool children's mathematical development. In this study children are observed in play and everyday activities located within a horizontal discourse characterized by a high relevance in the situation and a segmental organization with maximized encounters with persons and habits. However, it is not expressed whether these are also the situations in which the children are to learn mathematics. Quite the opposite the study contains no data on how the involved preschools facilitate for learning or on learning processes. It is the preschool teachers who conduct the empirical part of the study and the results are sometimes lower than expected. These expectations are based on previous research in other countries. Based on the results sometimes being lower than expected it can be understood as the author implies that a more vertical discourse may be needed.

In the article *Professional development in early mathematics: Effects of an intervention based on learning trajectories on teachers' practices and beliefs* by Julie Sarama, Douglas H. Clements, Christopher B. Wolfe and Mary Elaine Spitler (in this issue of NOMAD), the focus is on the effects of a research-based model for scaling up educational interventions on teachers' practices and beliefs in early mathematics. In the article it is argued that the most critical feature in a high-quality educational environment is a knowledgeable and responsive adult. The authors contrast this towards the low level of mathematics content and pedagogical content knowledge of most preschool teachers. Even though this article is not really about the context of mathematics in preschool the educational intervention presented is based on learning trajectories developed by the researchers. These learning trajectories consist of coherent connections of mathematical subject-matter goals, descriptions of developmental progressions towards these goals and instructional tasks and strategies that will help children to move along that path. Based on the structure of these learning trajectories this article seems to give prominence to a context of preschool mathematics located within a vertical discourse as

characterized by coherence of content, hierarchically interconnected procedures and systematically organized activities with focus on general knowledge.

In the article *Young children exploring probability – with focus on their documentations* by Jorryt van Bommel and Hanna Palmér (in this issue of NOMAD), the children are a bit older (six years old) than the child-groups described in the other articles. Through educational design research, possibilities and limitations with problem-solving as a basis for mathematics education in pre-school class is explored. The empirical example focused on is about probability and the results show different strategies used by the children in their documentation as well as their reflections on probability as expressed during the observed lessons. The empirical example in this article is within a vertical discourse as characterized by specialized language, systematically organized activities and general knowledge. However the activity is organized as play and thus located quite near the middle in figure 1.

Basic discourse

A basic discourse is not expressed explicit in any of the articles but can be recognized based on how content is described in two of the articles.

The article *Professional development in early mathematics: Effects of an intervention based on learning trajectories on teachers' practices and beliefs* by Julie Sarama, Douglas H. Clements, Christopher B. Wolfe and Mary Elaine Spitler (in this issue of NOMAD), characterized to take a strong vertical discourse above, can also be found taking an approach towards a basic discourse. They emphasize the necessity of a knowledgeable and responsive adult with subject knowledge and competence to implement a curriculum into early childhood practice. However, the authors contrast this towards the low level of mathematics content and pedagogical content knowledge of most preschool teachers. The developmental progression in the learning trajectories developed by the researchers, imply that the content is to be understood as mainly – but not exclusively – basic.

The article *Central skills in toddlers' and pre-schoolers' mathematical development, observed in play and everyday activities in Norwegian kindergartens* by Elin Reikerås (in this issue of NOMAD), above characterized as maybe implying a need for a more vertical discourse, is based on a test containing skills central for good mathematical development at the kindergarten age. Such mathematics is for example to distinguish between one and many and to place a picture on an identical picture when playing a lotto game. Regarding content the highlighted mathematics of importance for children to master in preschool age and the conclusion that

the results indicate a lack of focus on the numerical area imply that the content promoted is mainly basic.

Advanced discourse

An advanced discourse is more or less recognized in three of the articles. What stands out is the emphasis on an active communicating teacher who may either be on the horizontal or vertical line of the theoretical figure we use, but strives to challenge known concepts and introduce new ones to the children.

In the earlier described article *Young children exploring probability – with focus on their documentations* by Jorryt van Bommel and Hanna Palmér (in this issue of Nomad), the authors explicitly stress advanced mathematics as starting point. This since problem solving makes it possible to cover both advanced and basic mathematics in the same activities. Their content of choice would however be considered advanced in the broader context of Swedish preschool and preschool class.

Another article *Teachers' pedagogical mathematical awareness in diverse child-age-groups* by Camilla Björklund and Wolmet Barendregt (in this issue of Nomad) is about differences in mathematical content and pedagogical approaches that emerge when comparing how teachers express themselves working with younger and older children. Four mathematical content areas: number sense, number sequence, geometrical shapes and pattern, constitute the core in a questionnaire where teachers answer to how frequent they engage children in mathematical activities. The study does not specify the content to be made object of learning by the teachers, however, the questionnaire is theoretically designed to emphasize and promote communicative and problematizing activities that extend children's experiences. This article highlights differences between the mathematical content that younger and older preschool children are provided and discusses in particular the lack in problematization of mathematical phenomena with children. This indicates that the authors support a view on preschool mathematics that challenges children's conceptions, locating the study in the advanced discourse. The instrument used in the study further emphasizes mathematical language and specific communication of mathematical phenomena, which we would characterize as a vertical discourse. However, the closeness to children's lived world and experiences draws the study more to the center of the model.

The article *What characterises mathematical conversations in a Norwegian kindergarten?* by Trude Fosse (in this issue of Nomad) uses Bishop's approach, aiming to define mathematics in preschool as separate from school curricula of mathematics. However, in the article it is argued that

if a conversation is to be seen as mathematical it must be based around mathematics and structured such as it invites the participants to inquire further mathematics. Furthermore, the author makes references to the zone of proximal development, which can be understood as promoting a content more advanced than basic.

Discussion

In this final article in the thematic issue on preschool mathematics, we have provided a comparison between eight articles regarding how appropriate (as considered by the authors) design and content are – explicitly or implicitly – expressed. The aim has not been to evaluate or rate the articles but to highlight similarities and differences in the different contexts of preschool mathematics promoted: Which similarities and differences can be found in the articles concerning what preschool mathematics is, how it is to be designed and what constitutes an appropriate mathematical content?

As mentioned, design and content of preschool mathematics influence the context of preschool mathematics that the children are invited to participate in. Even though there is no one-to-one relation between the context of preschool mathematics and what children learn, its design and content influence which mathematics preschool children can learn. It also has implications for children's images of what mathematics is about and how you learn it.

The articles discussed in this article offer a plural view on preschool mathematics. Even though not all articles address the issues of design and/or content, explicit indicators can often be found implicit. In some articles a horizontal discourse is clearly emphasized and in others a vertical discourse is just as clear. Together the articles illustrate a spread along the whole horizontal axis in figure 1 and movements in both directions along this axis is argued for.

Similar goes for the axis basic–advanced content even though the question of content is less explicit in the articles than the question of design. Thus, the articles have more focus on *how* than on *what* which is in line with the historical and general preschool perspective where content has been less focused on than the activities containing this content (Pramling Samuelsson & Asplund Carlsson, 2014). However, content not being emphasized in the articles does not mean that these researchers do not care about the content in preschool mathematics. It is rather a result of the specific aims in several of the articles not concerning content.

Cultural issues may explain some of the differences identified but there are also differences within countries. As mentioned, research sometimes

results in either-or controversies without shades that create sets of false choices (Casey, 2009; Clarke, 2006). We do not believe that this is the case in the reviewed articles where the variation between them is not to be seen as a problem, quite the opposite. The differences indicate a research field not characterized by mainstream research but instead characterized by research where different approaches and different arguments are included. We believe that this plurality is positive in relation to reinforce stronger arguments based on strong research and as such the plurality will help to move preschool mathematics research further and deeper. The plurality also indicates approaches to mathematics education being complementary – not competing – in relation to particular forms of learning in particular settings.

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