

Measuring temperature within the didactic space of preschool

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The informal character of preschool mathematics, engaged in during children's play, places complex requirements on preschool teachers. It also leads to challenges in developing appropriate analytical tools for researching teacher work. In this paper a framework, the "didactic space", is described and used to analyse interactions between preschool teachers and children in relationship to mathematical learning situations. An interaction between a preschool teacher and a group of children about how to compare their temperatures is analysed, using this framework. The analysis focuses on how the teacher's contributions as well as those of the children changed as the role of the mathematics changed. The paper discusses how the didactic space offers a nuanced understanding of preschool mathematical situations, both to researchers and to teachers.

In this paper, we, as a group of researchers, explore how to make sense of the teaching and learning of mathematics in preschool, when it occurs through play. To do this, we contrast potential learning situations in preschools and schools to identify which aspects of these situations are likely to have an impact and incorporate these into an analytical framework. We then use this framework to describe changes in the teaching and learning in an interaction between a preschool teacher and a group of young children.

In recent decades, there has been much attention on young children's mathematics. Not only is this visible in national goals and curricula (see for example Kunnskapsdepartementet, 2006; Skolverket, 2010), but mathematics education research has also turned to focus on the mathematics young children can and should learn (Cross, Woods &

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Schweingruber, 2009; Ginsburg, Lee & Boyd, 2008). Yet, the contexts of school and preschools frame learning situations in quite different ways, and this affects how they should be researched.

For example, in Swedish preschools with a strong tradition of perceiving children as learning through play (Karlsson Lohmander & Pramling Samuelsson, 2015), clearly demarcated situations include "fruit time", "circle time", indoors and outdoors "play" (see for example Emilson, 2007). However, there are not situations labelled "lessons" as is typical in a school setting, even though in some preschools, situations labelled "mathematics" may occur. To reflect the lack of demarcations between subjects in preschools, in our own work, both research (Helenius et al., 2015c; Johansson et al., 2012) and development (Helenius et al., 2015b), we have considered the mathematics that preschool children engage in to be one version of Bishop's (1988) six mathematical activities. The mathematical activities are Counting, Measuring, Locating, Designing, Playing and Explaining, which respectively, answer questions involving quantification (how many? how much?); space and shape (where? what?); abstraction, hypothetical thinking and reasoning (how to? why?). From a research perspective, one advantage of using Bishop's six mathematical activities is that it is sufficient for the mathematical activity to be recognised by the researchers.

In contrast, in school it is a taken-for-granted idea that the children are there to learn, and as it has been phrased by Ference Marton, "learning is always the learning of something" (Marton & Ling, 2007, p. 39). In school, learning is characterised as achieving objectives set up in advance through a standard or curricula (Sundberg & Wahlström, 2012). The temporal and academic structure formed by the existence of lessons and subjects emphasises both how and where learning should happen. This structure strengthens the boundaries of what mathematics education in school should be, setting it apart from, for example, other school subjects or out of school activities. Children and teachers are rarely uncertain about whether or not they are engaging in mathematical learning at school.

Using Bernstein's (1971) terminology, the structures found in schools and preschools determine the classification and framing of mathematics. Classification describes how the knowledge valued as disciplinary is demarcated from other knowledge whereas framing describes the demarcation of social activities as educational activities. In Bernstein's own words framing is about who has control:

over the selection, organization and pacing of the knowledge transmitted and received in the pedagogical relationship. [...] Strong

frames reduce the power of the pupil over what, when and how he receives knowledge and increases the teacher's power in the pedagogical relationship. However, strong *classification* reduces the power of the *teacher* over what he transmits as he may not over-step the boundary between contents, *and* strong classification reduces the power of the teacher *vis-à-vis* the boundary maintainers.

(Bernstein, 1971, pp. 51–52; italics in the original)

Many preschool settings, including the Swedish preschool tradition with its long institutional and pedagogical tradition as an institution for the care and upbringing of young children, reinforced in the national curriculum (Skolverket, 2011), have different relations to learning and teaching (Vallberg Roth, 2011). The absence of formal lessons and clearly delineated subjects as well as emphasis on an integrated approach and a focus on play, in Bernstein's (1971) terms contributes to weaker classification and framing (Emilson & Folkesson, 2006). Nevertheless, the weak classification and framing of preschool mathematics may present dilemmas in regard to teaching because in a similar way to perceptions of work-based learning, "while there is much that is learnable, there may be no particular experiences that are teachable in a disciplinary or institutionalized way" (Solomon & McIntyre, 2000, p. 115). It also presents dilemmas in researching the learning interactions between teachers and children.

In this article, we describe an analytical framework, which we consider provides a nuanced understanding of how teacher and children interactions in play could contribute to children's learning about mathematical ideas. We exemplify the framework by analysing a situation relating to the Bishop's (1988) mathematical activity of Measuring. The situation is deliberately chosen because it fits the attributes of mathematical play (Helenius et al., 2016), but would not be recognised as school mathematics learning as the situation deals with aspects of measuring that are not (yet) quantified.

Learning about measuring at school and at preschool

For Bishop (1988, p. 34),

Measuring is [...] concerned with comparing, with ordering, and with quantifying qualities which are of value and importance. All cultures recognise the importance of certain things but once again, all cultures do not value the same things to the same extent.

Bishop went on to discuss measuring devices and their origin and function. Bishop's culturally situated description values measuring as the

identification of a quality, which can be compared in various ways including through quantifying the amount of the quality which is present. The development of children's comparing and measuring skills has been the focus of research, particularly since the early work of Piaget and colleagues (Piaget, Inhelder & Alina, 1960). From this work, the important measurement concepts are generally agreed to be: transitivity, iteration; and the use of identical units (Clements & Stephan, 2004; Meaney, 2011; Zöllner & Benz, 2016). Buys and de Moor (2008) described the "basic pattern of the learning-teaching trajectory" (p.23) for measurement as having three stages (p. 25):

- measuring through comparing and ordering
- measuring through pacing off using a measurement unit
- measuring through reading off with the help of a measuring instrument

Although the measurement concepts and the learning-teaching trajectories are considered to be the same for preschool children and for school children, their realisation in the different institutions can be quite different.

In Swedish preschools, both Doverborg and Pramling Samuelsson (2011) and Lembrér (2013) documented how measurement concepts were learnt and utilised by children when they were solving problems in which they were interested. In these situations, the teacher was crucial in challenging children, while they engaged in the play, but did not insist that the challenges be accepted. Similarly, Lange, Meaney, Riesbeck and Wernberg (2014) documented how a Swedish preschool teacher challenged a group of children playing with jars to put them in order. The children initially responded by grouping them according to whether they were round or had sides with corners. When the children then started testing if their feet would fit into the jars, the teacher did not force them to stay with the ordering problem. However, later she asked again if it was possible to order the jars according to size. At this point, the children explored how to do this producing a line of jars with the tallest one in the middle and jars decreasing in heights on both sides. The teacher did not change their ordering but asked questions about it. As Zöllner and Benz (2016) suggested, preschool teachers need appropriate pedagogical content knowledge to provide quality learning opportunities about measurement to children in a play setting.

In preschools where children are expected to learn through play, generally both classification and framing are weak from the children's perspective. The children, in solving problems in which they are

interested, such as ordering the jars, are unlikely to be aware that they are engaged in learning measurement concepts. This knowledge is not separated from the other learning that is occurring as they play, such as what it feels like to put their feet into the jars, thus the experiences are weakly classified. For the children, these experiences are also weakly framed, because the children control what they do when. However, it may be different for the teacher, who by being aware of measurement concepts from their teacher education or schooling, may consider there to be a strong classification between these concepts and other knowledge. Responding to this strong classification, their suggestions for engaging children in discussion and reflection would be likely to highlight aspects of the measurement concepts. Nonetheless, if teachers do not control what should be done at any particular time, they, like the children, must accept the weak framing of the situation.

In contrast, teachers in schools are expected to determine the learning situations for children, who must participate as directed. In these situations, teachers also need the necessary pedagogical content knowledge for teaching measurement concepts, but do this with a particular trajectory for this learning in mind (Clements & Stephan, 2004). For example, Clements (1999) suggested that young children needed to engage in experiences comparing lengths of objects, such as finding out all the objects in a room which are as long as their fore-arm. Then they could start to use a ruler, even if they did not have a full understanding of how it worked – “teachers may consider allowing students to use rulers *along with* manipulable units such as centimetre cubes and arbitrary units” (p. 9). Children’s own interests and problem solving is secondary to ensuring that they are provided with learning situations, identified as appropriate for their development. For both teachers and children, the classification and framing is strong. The knowledge and the order in which it should be taught to children in school is determined, often by curricula, so the role of the teacher becomes one of implementer, while the children are learners. The children have limited if any possibilities for affecting the experiences offered to them.

The didactic space

From previous research (Helenius et al., 2015a), we were aware that Bishop’s six activities provided a researcher-focused interpretation of how children were learning through interactions. To try to gain more understanding of how children’s learning was affected, we used Walkerdine’s (1988) distinction between instrumental and pedagogic tasks. Walkerdine analysed interactions in the home.

This classification used the designations *instrumental* and *pedagogic* to describe certain kinds of tasks at home and was a distinction originally devised in relation to practices involving *number* in the home. Instrumental referred to tasks in which the main focus and goal of the task was a practical accomplishment and in which numbers were an incidental feature of the task, for example in cake-making, in which the number *two* might feature in relation to the number of eggs needed and so on. In the pedagogic tasks numbers featured in a quite different way: that is, numbers were the explicit focus of the task. On such occasions the focus was predominantly the teaching and practice of counting. So, for example, a child might be asked to count her coat buttons for no other purpose than to practise the count. (Walkerdine, 1988, p. 81; italics in the original)

Nevertheless, Walkerdine (1988) noted that identifying mathematical situations that were not about number as either instrumental or pedagogic was not straightforward. Some non-number situations were clearly not one kind or the other and there were occasions in which "the mother appeared to be *commenting* on an activity or on something which had been done or seen" (p. 86; italics in the original). In these cases, there seemed to be neither an instrumental nor a pedagogic purpose to the commenting.

In our previous research, we identified both instrumental and pedagogic situations connected to each of the six mathematical activities (Johansson et al., 2012), thus suggesting contrary to Walkerdine's own doubts, it was possible to use the categorisation, in relationship to other mathematical activities. Nevertheless, we were aware that the distinctions between instrumental and pedagogic, like the categorising of the situations into Bishop's six activities were being made by ourselves as the researchers and might not be seen in the same way by the teachers and children.

In later research (Helenius et al., 2015a), we introduced the didactic space where we incorporated Walkerdine's distinction between instrumental and pedagogic situations to allow for situations where the adults and children could have different perspectives on the role of the mathematics (see figure 1). The "didactic space" utilises a "k" in order to highlight the Nordic-German rather than the Anglo-Saxon notion of "didactic". The situations within each of the four quadrants of the didactic space were described as having a particular didactic makeup (Helenius et al., 2015a). In this paper, we extend this framework in regard to how the situations can be considered as having strong or weak classification and framing. We see this as providing a more nuanced understanding of how a teacher and multiple children simultaneously engage in learning in the messy, complex situations of play in preschools.

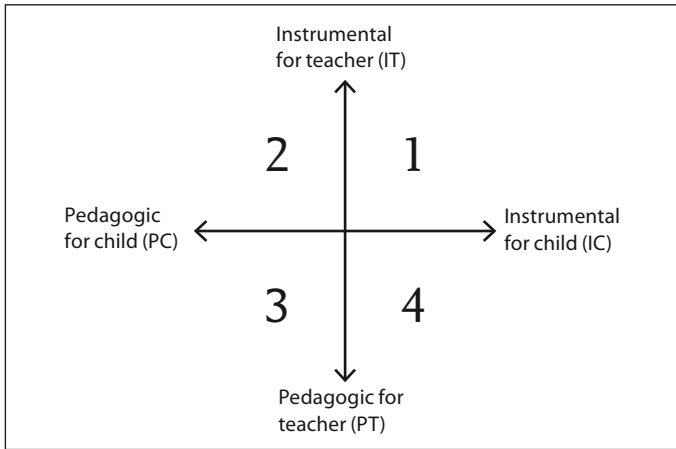


Figure 1. *Didactic space, with the quadrants numbered*

Situations would be classified as being in quadrant one, when the teacher and the child(ren) interact to solve a problem involving one or more mathematical activities. Although one participant may be more knowledgeable about how to solve the problem, the focus for all participants is on its resolution, not on the process of resolution, leaving little possibility for formal teaching. In this quadrant, the situations are both weakly classified, in regards to the mathematics, and weakly framed in regards to the pedagogy. The children, with or without the teacher, would make use of whatever knowledge and skills that they had to resolve whatever they found interesting in the problem, in any way that made sense to them.

In quadrant two, the teacher would be focused on solving a problem whereas the child(ren) are focused on teaching each other, themselves or the teacher, about some aspect of a mathematical activity. There were only a few examples of these situations in our data set (Johansson et al., 2012). A situation could be one in which a teacher is packing up materials, while the child is focused on learning about different attributes while they were engaged in the process. In this case, for the child the mathematics is more strongly classified than is the case for the teacher. However, the framing remains weak for both the child and teacher, as there are no specific constraints on how this packing up/learning about attributes should be done.

In the third quadrant, the focus for both the teacher and the child(ren) is on the mathematical activity. Usually, the teacher is the one who teaches and the child(ren) learn. Then the activities will be strongly classified, likely making the teaching also strongly framed, as discussed previously. The Pedagogic child (PC) – Pedagogic teacher (PT) combination is typical in school mathematics, which is both strongly classified

and framed. However, there may be situations where the teacher/learner roles are reversed. Examples could be the teacher making sure the children are aware of a specific feature or more formally requiring a child to pay attention to and learn the material in their own way. In these situations, the mathematical activities may be weakly classified and framed.

In the final quadrant, the teacher's focus is on teaching the child(ren) about the mathematical activities, while the child's focus is on resolving a problem. In our data set, we had many situations that we could classify as belonging to this quadrant, including the one described above where the children put the glass jars in order. This would be instrumental for the children (IC), who were trying to solve the problem of how to order the jars, but pedagogic for the teacher (PT) who was focusing on exposing the children to measuring ideas about comparing through suggestions and questions. In this case, from the children's perspective the mathematics was weakly classified and because they had the possibility for not taking up the teacher's challenge, the framing would also be weak.

We found the earlier version of the didactic space useful for tracking the continual changes in the focus of the teacher and a child within specific situations (Helenius et al., 2015a). For example, we found situations that were not planned but which we considered to be pedagogic, in that the teacher seized the moment and highlighted an aspect of a mathematical activity, thus refocusing it from its original (instrumental) character into being pedagogic.

Although acknowledging the complexity of the research we are undertaking, combining Bishop's six mathematical activities with the instrumental/pedagogic construct contributed to us identifying when and how children came in contact with the mathematical activities in preschools. Further, it described the preschool teachers' "field of choices" for achieving the goals of the curriculum of providing situations for children to engage with the mathematical activities. In identifying how the teachers in our data set engaged with the children, we were able to determine when the teacher planned the play situation and supported the children to engage in it or when the teacher made use of a spontaneous situation to highlight aspects of one or more of the mathematical activities, for pedagogic purposes. Adding insights from Bernstein's classification and framing seems to support our understanding about how the interaction between teachers and children affects the kind of learning opportunities that is offered.

In our previous research we considered interactions between one teacher and a child (Helenius et al., 2015a). In this paper, we use the didactic space to explore a measuring situation where several children are with a teacher. In particular, we are interested in what happened

when the teacher re-focused a situation towards being pedagogic and how different children reacted to this shift in attention.

Methodology

The video analysed in this paper comes from a set collected at a private preschool in a large city in the southern part of Sweden, over the course of two years, 2011–2012. We filmed situations both where the teachers had planned to involve mathematics, as well as ones in which the teachers initially did not consider there would be mathematics. Consent for collecting and publishing video recordings was gained from parents and teachers and discussed with the children.¹

In the 16 minutes video, a group of mixed-aged children were eating their lunch with the teacher. We analyse two minutes where there was animated discussion, initiated by a child, between children and between the children and the teacher about how to compare different children's temperatures. This situation was about Bishop's Measuring activity. Temperature is often considered difficult for young children to grasp because of its abstract nature (Strauss, 1987) and thus it is unlikely that a preschool teacher would initiate such a discussion. However, in this case a proposition by one of the children that one's own forehead always feels hotter than it actually is promoted both discussion and comparing the hotness of each others' foreheads.

The video was transcribed and divided into episodes. For each episode, we classified particular utterances or gestures as having either a pedagogic or an instrumental focus, for each participant in that episode. For a situation to be described as instrumental, an aspect of measuring was used to solve a problem but was not the focus of the situation. In pedagogic situations, someone, the teacher or a child, actively directed attention towards aspects of measuring that they wanted, or they wanted others, to learn.

The situation

In the video, seven children ate lunch with a preschool teacher. Of the children, five were active participants in that they made comments or felt their own or others' foreheads in our video. Although all comments are recorded, only a few of the children were in view of the camera at any moment and it is not possible to say what the children were doing who were not being recorded. We have named the five active children Lena, Rita, Annika, Emma and Hans.

The interaction about measurement began when one child, Rita, stated that you can assess if you have fever by feeling your forehead. The children and the teacher then started to feel their own and others' foreheads with their hands. Examples from the video can be seen in figure 2.



Figure 2. *Feeling foreheads*

Throughout the interaction, the problem being solved is whether someone had fever or not, or more generally, if they were warm or not. Being warm was considered by the teacher and the children as being equivalent to having fever. They begin by discussing the accuracy of using one's hand to determine whether or not someone was hotter than normal before going on to discuss who is hottest. In so doing, these children were engaged in learning about transitivity, where they were using one thing, their hand, to compare whether someone was more or less hot than themselves. The children are only at the beginning stage of understanding the concepts of transitivity (Clements & Stephan, 2004) and comparison (Buys & de Moor, 2008). No measuring instruments were used and so no specific quantifications were mentioned. However, as Grootenboer and Sullivan (2013) found, the cultural context of the situation may mean that the use of specific numerical quantities is irrelevant to the needs of the discussants. Therefore, even if the children had skills to read a thermometer, they might not have wanted or found it appropriate to solve the problem in that way.

Instead, the comparisons were described using terms about heat. Many of the children seemed to be trying out the terms when they described an instance of hotness. The teacher challenged the use of some of the terms and tried to clarify with them that a comparison is between at least two things. Consequently, we categorised interactions in which the focus was on *how* temperature could be assessed (by comparison) as pedagogic (P). On the other hand, we categorised interactions about having fever or being warm as instrumental (I).

Analysis of the situation

The interaction started with Rita and Lena discussing with the teacher how hot their foreheads were (see table 1). In this discussion, Rita brought in information learnt from her mother about how when you feel your own forehead it always feels hotter than it actually is. Lena disagreed but Rita continued with her argument, although she did acknowledge that

Table 1. *Extract of the analysis*

	Swedish utterance	English translation	Gesture	T	R	L	H	A	E
1	R	Känns det alltid varmare?	Does it always feel warmer?	Lena feels her forehead.	I	I			
2	T	Gör det det nu här varför?	Does it now, why?		P				
3	R	Det säger mamma.	Mum says so.			P			
4	T	Om man känner själv?	If one feel oneself?	Teacher feels her forehead.	P				
5	L	Nä om man känner någon annans panna.	Nah, if one feels someone else's forehead.	Teacher feels Rita's forehead.	P	P			
6	T	Du menar att om jag känner på min panna så ska den kännas varmare än din?	You mean if I felt my forehead it will feel warmer than yours?	Teacher feels her forehead, then feels Rita's forehead, Rita turns to Teacher.	P				
7	R	Nä liksom om jag känner på min då känner. Mmm då känner jag jag kan inte känna på min egen panna.	Nah, but like if I feel mine. Mmm then I feel because I can't feel my own forehead.	Rita feels her forehead, turns to Lena and then back to Teacher. Emma feels her forehead.		P			I
8	T	För att den kännas varmare än vad då?	Because it feels hotter than what?		P				
9	R	Än vad det är.	Than what it is.			P			
10	T	Än vad den egentligen är, så den är kallare än vad den kännas, va, va konstigt!	Than what it really is, so it is cooler than it feels, huh, that's weird!		P				
11	R	Eller eller jag vet faktiskt inte men jag tror det	Or I do not really know but I believe it.			P			

Note. In the extracts of the analysis, T stands for Teacher, R for Rita, L for Lena, H for Hans, A for Annika and E for Emma. Each utterance is numbered.

she did not really understand how it worked. The teacher did not situate herself as knowing anything about this but seemed truly interested in Rita's point of view.

The actions and words around utterance 1 are categorised as instrumental as they contributed to the general discussion of the problem of how to tell if you are warmer than normal. The teacher's question in utterance 2 is classified as pedagogic because it asked about why this was the case, thereby, shifting the attention to the comparison which has to be made. The situation continued to be pedagogic with the teacher, Rita and Lena reasoning about relations between feeling one's own and others' foreheads. By situating herself as someone willing to learn, the teacher provided opportunities for both Rita and Lena to take on teaching roles in which they could provide their perspectives on what kind of comparisons were needed. They therefore were situated within quadrant 3 of the didaktik space framework.

However, Emma and perhaps the other children who did not participate in the same way cannot be categorised as being in the same quadrant. By feeling her own forehead, Emma appeared to be focused on solving a problem about if she felt warm, suggesting that she could be situated in quadrant 4. The action did not in itself highlight the comparison or any other aspect of measuring. It could be that she and the other children were passively following the discussion between the teacher, Rita and Lena, but there is no explicit evidence of this. Certainly none of the subsequent actions indicate that the other children had understood the issue of the distortion that occurs when one feels one's own forehead. In Meaney's (2011) research of how a six-year old discussed measurement ideas in the home, it was noted that many of the child's examples involved comparison with an undiscussed norm. Emma's action could be a similar example.

At utterance 12, several children asked the teacher to feel their foreheads and the children felt their own and others' foreheads. The teacher tried to bring the situation towards a pedagogic focus by asking "Can you compare with someone else?" Nevertheless, in what follows, Lena focused on her own forehead being "very hot", with Hans and Annika also having an instrumental focus. Focussing on who was warm meant that the situation was an instrumental one for the participants. By utterance 15, it seemed that the teacher also had an instrumental perspective, so that the situation could be considered as being in quadrant 1. In previous research (Johansson et al., 2014), we had found that children at this pre-school often adapted and incorporated each other's actions into their explanations. Therefore, it may be that the earlier examples of feeling foreheads may have supported the other children to copy these actions as they explored the issue of who was warmest.

Table 2. *Extract of the analysis*

	Swedish utterance	English translation	Gesture	T	R	L	H	A	E
12	(Flera barn säger känn på min)	More children say, "Feel my forehead."						I	I
13	T Kan man jämföra med någon annans panna?	Can you compare with someone else's forehead?	Annika feels Lena's forehead, Lena feels her own forehead.	P			I		I
14	L Jag känner att den är jättevarm	I feel that it is very warm.	Lena feels her forehead, Annika stretches her arm out to feel Lena's forehead				I		
15	T Är din jättevarm?	Is yours very warm?	Annika feels Lena's forehead.	I					I
16	R Om man känner själv det är olika på händerna	If you feel yourself, it is different on the hands.	Lena feels her forehead takes her hand to her mouth, blows on it and then takes it back to her forehead.			P	P		
17	L Den är jättevarm.	It is really warm.							I

Note. In the extracts of the analysis, T stands for Teacher, R for Rita, L for Lena, H for Hans, A for Annika and E for Emma. Each utterance is numbered.

Rita in utterance 16 could be seen as having a pedagogic focus as she returned to her point that one's own forehead feels hotter than it actually is. It may be that Lena was following this reasoning by blowing on her hand, thus making her hand a different temperature than it was originally, before feeling her forehead again. If Lena was following Rita's reasoning, then her action could be classified as pedagogic.

A similar interaction occurred several utterances later, when the teacher again tried to focus on the pedagogic task of comparisons (utterance 21, 24 and 26. See table 3). However, the children stayed with the instrumental task of judging their own temperatures. Despite the teacher's efforts to turn the discussion into one about why comparisons were needed to determine who was hottest, the children remained with the instrumental issue of who was hot or boiling hot. Thus, this episode can be considered as being within quadrant 4, where the teacher has a pedagogic approach while the children have an instrumental one. In what followed, the teacher continued her pedagogic focus of comparing temperatures.

In the final episode, it is only Rita who took up the teacher's invitation to discuss comparisons. The other children did not follow her lead,

Table 3. *Extract of the analysis*

	Swedish utterance	English translation	Gesture	T	R	L	H	A	E
18	H Känn på mig!	Feel mine!							I
19	A Oj vad du var varm!	Oh, how hot you were!	Lena's hand remains on her forehead. Annika at the bottom of the screen seems to be turned to Hans.			I			I
20	A Får jag känna?	Can I feel?	Emma feels her own forehead.						I
21	T Var Hans varmast?	Was Hans the warmest?	Hans feels another child's forehead.	P					
22	A Får jag känna?	Can I feel?	Turns towards the other child.						I
23	H Lenas är varmast.	Lena's is the warmest.							I
24	T Är Lenas varmast? Hur kan du veta det?	Is Lena's the warmest? How can you know that?		P					
25	H Jo, Lena är kokhet.	Yes Lena is boiling hot.							I
26	T Är Lena kokhet?	Is Lena boiling hot?		P					
27	H Ja	Yes							I

Note. In the extracts of the analysis, T stands for Teacher, R for Rita, L for Lena, H for Hans, A for Annika and E for Emma. Each utterance is numbered.

as Lena had done in the first and possibly also the second episode. This time Lena appeared to be only interested in having herself declared the hottest. Consequently, this episode can be placed in quadrant 4, except for Rita who with the teacher could be said to be in quadrant 3, at least briefly. It is interesting to note that the teacher appeared to use the action of feeling Hans' and her own forehead as part of her invitation to Hans to participate in a pedagogic discussion. This was a repeat of what she had done in utterance 6. It does not seem that the children were able to notice the difference between her actions of going from forehead to forehead with their own actions of feeling just one forehead at a time. This suggests that they were at the very beginning of learning about comparisons but also that the pleasure of feeling each other's temperatures and describing them in absolute terms such as "boiling hot" had a strong

Table 4. *Extract of the analysis*

	Swedish utterance	English translation	Gesture	T	R	L	H	A	E
28	T	Hur ska vi kunna ta reda på detta?	How do we find this out?						P
29	H	Känn på min!	Feel mine!	Teacher feels Hans's forehead.					I
30	T	Det känns skönt.	It feels nice.						I?
31	L	Min då!	Feel mine!	Teacher feels her own forehead. Emma feels her own forehead.					P I I
32	T	Var din superkokhet, jämfört med vem, jämfört med Lena eller?	Was yours super boiling hot, compared to whom, compared to Lena's or?	Hans feels Lena's forehead.					P I
33	R	Nej jämfört med ingen.	Nah, compared with none.						P
34	T	Jämfört med ingen, men man kan väl inte jämföra med ingen	Compared to nobody, but you can't compare to nobody.						P
35	H	Kan du ta på min panna, nej min panna?	Can you feel my forehead, no, my forehead?	Annika feels Hans' forehead					I I
36	A	Oj du är kokhet!	Oh, you are boiling hot!						I

Note. In the extracts of the analysis, T stands for Teacher, R for Rita, L for Lena, H for Hans, A for Annika and E for Emma. Each utterance is numbered.

influence on their playing. This situation engaged the children in learning about how to describe different temperatures and that temperature was something that could be compared. This is a vital component of early measurement understanding. However in comparison with school measuring lessons, the classification between what is measurement and what is something else is blurred.

Discussion

As an analytical tool, the didactic space provides information about the role of the mathematical activity during interactions between the teacher and children while playing. The typical action of the teacher was to push the episodes towards the pedagogic by asking who is the hottest

(utterances 21 and 24), through gestures indicating that a comparison between foreheads can be made (utterances 6 and 29–31), or explicitly asking how you can know who is the hottest (utterance 13, 24 and 28). From this, it can be seen that the teacher had appropriate pedagogic content knowledge about important measuring concepts and about how to engage children to consider those concepts within a play situation. She did not force the children to take up her offers for pedagogic discussion but instead followed them into the instrumental issue of determining who felt hot. The requirements of play situations results in weak framing because the children have rights to determine what they can do and how they do it. As noted in earlier research in the same preschool (Lange et al., 2014), the teacher's role is about offering suggestions, while not insisting that the children take them up. Zöllner and Benz (2016) also made the case for why the demands on the teacher of navigating between supporting children to explore a situation through play and encouraging them to engage with important measuring ideas is complex and requires significant pedagogical content knowledge.

Looking at the whole situation from the perspective of individual children, interesting phenomena can be observed. Although she began with what seemed to be an instrumental introduction, Rita, the initiator of the situation, stayed with the pedagogic perspective. Lena made two contributions that were categorised as being pedagogic. The first one was in response to the teacher (utterance 5) and the later one as a response to a contribution from Rita. However, her remaining contributions as well as the contributions of the other children were categorised as being instrumental. This is despite the pedagogic shifts of the teacher and Rita's pedagogic contributions. Therefore, the learning opportunities for the children were not limited to those aspects of the situation, which had a pedagogic focus. For most of the children, the learning about Measuring – different degrees of hotness, using a measuring instrument (their hand), description terms – came from solving the problems that were of interest to them, in collaboration with each other.

Conclusion

The situation is characterised by the teacher seizing the moment created by one of the children referring to temperatures and the method of feeling one's forehead and its associated drawbacks. The teacher systematically tried to shift the focus to the pedagogic one, but also adapted to the instrumental focus of most of the children. Perhaps this was because she was concerned about the social aspect of the situation and wanted all children engaged. The distribution of pedagogic and instrumental

contributions is highly asymmetric, with only two of the children having more than one pedagogic encounter. From our analysis using the didactic space, we conclude that Walkerdine's (1988) distinctions of pedagogic and instrumental within the dynamics of the situation, would support both teachers and researchers to better evaluate the ways that children experience and learn from mathematical activities arising in different situations in preschool.

The didactic space also provides insights into how preschool teachers make use of their pedagogic content knowledge. As is the case for many preschool systems around the world, the Swedish preschool has specific goals related to mathematics. In the Swedish case, those goals are connected to Bishop's six mathematical activities (Utbildningsdepartementet, 2010). Yet, both formal expectations and the Nordic preschool tradition emphasise a preschool environment that builds on play and thematic work so a formal teaching approach to achieving those goals would not be appropriate. From a Bernsteinian perspective, such a learning environment is weakly framed, which means that the social settings in which learning is expected to happen are more varied and the potential learning that a particular situation enables also is less predetermined than is the case in school settings.

In Ginsburg et al.'s (2008) overview, it is acknowledged that the principle of *teachable moments* building on "teacher's careful observation of children's play and other activities in order to identify the spontaneously emerging situation that can be exploited to promote learning [...] can provide a superb learning experience for the child" (p. 7). Yet, Ginsburg et al. (2008, p. 7) counterbalance this by claiming it is not an effective educational method because:

teachers seldom attempt to exploit teachable moments, and even if they did, it's hard to see how they could effectively keep track of and productively respond to the haphazard occurrences of teachable moments in 20 or so young children, especially from diverse backgrounds.

While this quote is perhaps overly negative – for example by assuming a teachable moment needs to be connected to an individual, it is probably true that any weakly framed teaching situation will put higher demands on the teacher's pedagogical content knowledge. From the analysis using the didactic space, it could be possible to suggest that because only two of the children followed the pedagogic intentions of the teacher there was limited learning. However, the measurement concepts that arose required the children to engage with and make use of some of the fundamental ideas connected to Measuring. It may well be that having

instrumental experiences of feeling each other's foreheads in connection to how warm or otherwise they were, provided these children with experiences that could be drawn upon by contrasting them with later, similar experiences so that the children could have more possibilities to reflect on Measuring.

From a researcher perspective, the didactic space provides a nuanced understanding of the factors contributing to the learning that occurs in different interactions. It allows for possibilities to track individual children and/or the teacher's contributions and to consider how they change within a dynamic play situation. By focusing on the role of the mathematical activities within the situations, it is possible to recognise who has the pedagogic perspective and to move away from just presuming that it is the teacher alone who teaches. It also looks at how the actions of the children can change their focus as the interaction continues. In our case, the focus changed from the issue of whether it was true that you felt hotter than you actually were when you used your own hand as the measuring instrument to the enjoyment of feeling each other's forehead. This change was possible within the situation because it was implicitly assumed by all participants to be a play situation. It also resulted in changing the learning possibilities made available in the situation. However, Rita's continual focus on the original point showed how not every child was learning about the same Measuring ideas. Therefore, we consider that the didactic space provides a more nuanced understanding about mathematical learning in preschool play situations.

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Notes

- 1 The video is publicly available at www.youtube.com/watch?v=wiPm0T99Kzc

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