Teachers' pedagogical mathematical awareness in diverse child-age-groups

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In this study we investigate preschool teachers' attention to number sense, number sequence, geometrical shapes and patterns, and their pedagogical awareness regarding these areas in their work with younger and older preschool children. The specific research questions are: Are there differences in teachers' attention to the different mathematical content areas depending on the age group they are teaching? And if mathematical content areas show differences between age groups, what pedagogical awareness levels constitute these differences? The study shows that the frequency of working with mathematical content is higher for all areas regarding older children, but teachers' choice of mathematical content area is not necessarily depending on the age of the children. Teachers' frequency of observing and engaging children in mathematics is lower among those working with younger children but the difference is in general quite small. Significant and substantial differences appear in teachers' attention to children's mathematizing initiatives and problematizing number sequences, geometrical shapes and patterns.

Sweden is one of a few countries that have a national curriculum for preschool which includes children from one year to five years of age. In Sweden, 95 % of all 3–5-year-olds are enrolled in preschool or similar pedagogical practice and close to 90 % of all 2-year-olds attend preschool or an equivalent practice (National Agency for Education, 2014). The same curriculum is in use for both older and younger children and the guidelines consider the same content to be implemented as a national strategy to facilitate equality in early education.

Mathematics is one of the topics described in the preschool curriculum. It states that children in preschool should be offered opportunities to explore and use mathematics in a practice that takes its point of departure

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in children's interests and flow of ideas and initiatives (National Agency for Education, 2011). Such a practice should integrate play and learning, strive for learning goals, but in accordance with the national guidelines, not assess specific knowledge outcomes. Teachers are free to decide which pedagogical practice and working methods to use. Altogether, this is a demanding didactical challenge for the teachers working with preschool children and in particular for those working with the youngest children, 1–3 year olds. One of the pedagogical challenges is presumably to find and follow children's perspectives and ideas while at the same time having a mission to offer children opportunities to develop their knowledge and skills within the mathematical knowledge area. An additional challenge when working with the youngest children in Early Childhood Education (ECE) may be that their mathematical concept development is at an emerging stage and their experiences of mathematical phenomena differ to a large extent from older children and adults. Due to children's varying mathematical experiences and the pedagogical freedom, there is a need for more studies on teachers' professional work, both what content they offer children to develop knowledge about and how they approach the pedagogical content.

This article is based on a study where a questionnaire was developed to investigate teachers' approaches to their pedagogical work with mathematics in preschool. The instrument used in the study is based on the idea of pedagogical content knowledge, PCK (Shulman, 1986; Ball, Thames & Phelps, 2008) and contemporary theory in early childhood education, *developmental pedagogy* (Pramling & Pramling Samuelsson, 2011). In this particular investigation presented here, we aim to compare teachers' attention to four mathematical content areas: number sense, number sequences, geometrical shapes and patterns, and in particular if there are differences between their work with children aged 1–3 and children aged 4–5.

In the following text we first describe the field of knowledge on children's mathematics learning, followed by a review of contemporary studies on preschool teachers' role in children's mathematical development. The method and instrument used in the study are thereafter presented, followed by the results from the analysis and a concluding discussion.

Learning mathematics in the early years

Even though 1–3-year-old children may have less experience of mathematical activities than 4–5-year-olds and their conceptual understanding of mathematical principles and operations is at an emerging stage, studies bring forth a competent child that is capable of making meaning and

exploring mathematical phenomena in daily activities. A large body of research shows for example that infants seem to have an intuitive knowledge of numerosities, meaning that they are able to determine whether a quantity changes in number or magnitude (Wynn, 1998; McCrink & Wynn, 2004). Butterworth (1999) calls these abilities the core for numerical development, and states that the child's social environment determines the direction of this development. However, these competencies are not easily recognized, because they are intuitive and non-verbal, yet they are the foundation for more advanced mathematical skills and mathematically meaningful verbal notions.

Research of children's number sense development (Hannula, 2005), gives reason to highlight children's intuitive or spontaneous attention to mathematical phenomena. Hannula, Mattinen and Lehtinen (2005) show for example that 41 % of a sample of 34 three-year-old preschoolers who participated in a numerical attention intervention program paid no or low attention to numerical features of tasks and everyday routines (see also Mattinen, 2006). These children are at risk of under-achieving in formal mathematics education in later school years. However, when they are encouraged to discern and make sense of numerical features (through social pedagogical activities), a raised awareness of these features of their surrounding world triggers their development not only in instructive activities but in spontaneous as well (Hannula, Mattinen & Lehtinen, 2005). Spontaneous attention to number has also shown to predict later mathematical achievements (Hannula-Sormunen, Lehtinen & Räsänen, 2015). Early interventions, where children's attention is directed towards mathematics and specific features within the knowledge area, have shown to be successful and particularly in preschool curricula where teachers' attentiveness to children's learning trajectories are emphasized (see Clements & Sarama, 2008 for a study on Pre-K classroom intervention programs).

Number sense is defined differently by different researchers, but there is a consensus that number sense concerns an intuitive knowledge of quantities and the part-whole-relationship of numbers (Dehaene, 1993; Jordan, Kaplan, Ramineni & Locuniak, 2009). This intuitive knowledge is considered the basis for representing numbers symbolically and for developing arithmetic skills. Early numeracy skills refer to children's developing understanding of relations within and between numbers, which has shown to be predictive for later arithmetical competence (Aunio & Niemivirta, 2010; Aunola, Leskinen, Lerkkanen & Nurmi, 2004; Jordan et al., 2009; LeFevre et al., 2010).

In addition to numerical relationships, there is evidence that young children, under the age of two, make use of spatial and geometrical cues in

problem solving (Huttenlocher, 2008). Spatial attention is closely related to mathematical knowledge in general. Geometry, number sequencing, comparing quantities and making patterns (LeFevre et al., 2010) where parts and whole, structure and order are central, all make use of spatial attention. Even though spatial reasoning that involves geometry and pre-algebraic thinking are considered important, they are nevertheless often neglected areas in mathematics education for young children (Clements & Sarama, 2011).

Furthermore, number competencies also involve a linear representation of numbers where for example three precedes four in a stable order (Gelman & Gallistel, 1978) and also indicates an increase in number. Numbers then correspond to a specific position and order, which supports counting and comparison of quantities (Dehaene, 2001; Resnick, 1983).

Pedagogical preconditions for mathematical development

The social environment plays an important role in children's mathematical development, and there seems to be a specific relationship between the linguistic milieu and verbal stimulation and cognitive abilities, not least spatial reasoning. Levine et al. (2010) have in a large-scale longitudinal study found evidence for early linguistic stimulation as a link to more complex spatial problem solving abilities. However, the key seems to be that children are offered a rich linguistic environment, meaning that adults' vocabulary is used in flexible and nuanced manners together with children during their first three years. Adults' interaction with children in play that is accompanied with instructions as well as verbal challenges supports children to direct attention and discern critical features of a problem, which consequently develops their reasoning skills, both verbally and non-verbally. Based on such evidence there is sufficient support for engaging also the youngest children in preschool in communicative and problem solving activities about mathematical phenomena.

An earlier study of Swedish early childhood teachers' pedagogical mathematical awareness (Björklund & Barendregt, 2015) has shown that teachers in general plan for mathematics as a learning goal and content in preschool practice. They communicate mathematical meaning to children and encourage mathematical exploration on children's initiatives. However, the study also showed that teachers rarely problematized mathematical concepts and principles with children or visualized the complexity of number concepts, shapes and patterns. This is in line with reported misconceptions among early childhood teachers, that mathematics should be limited to easy number operations and labelling of geometrical shapes rather than exploring and comparing features in an attempt to understand the characteristics of shapes and number

(Lee & Ginsburg, 2009). Björklund and Barendregt's study further confirmed earlier research and claims that numbers and numerosity form an over-represented knowledge area, whereas geometry and pre-algebra more often are neglected areas for learning and development in early childhood education (Clements & Sarama, 2007, 2011; Lee, 2010).

Shulman (1986) and Ball, Thames and Phelps (2008) argue that teaching mathematics presumes specific pedagogical content knowledge. Teachers possess skills to handle a group of children and organize for learning (pedagogical knowledge). Teachers, like most other educated persons, also have mathematical knowledge. But it is not enough to have general good mathematical skills (content knowledge), the teacher also needs sufficient *pedagogical content knowledge* about how to organize for learning situations that meet the learners' experiences and needs of a specific mathematical content, which includes knowledge of effective use of available resources. Teaching mathematics thereby looks different depending on the specific child group and the teacher's knowledge of mathematics and mathematics learning in particular. Contemporary research in early childhood education further emphasizes teachers' communicative skills and competencies to take the child's perspective in any learning situation (Pramling Samuelsson & Pramling, 2013; Sheridan, Pramling Samuelsson & Johansson, 2009). This is reflected in the teachers' pedagogical work both in what they offer children to explore and learn about and how content for learning is introduced and taught.

Based on contemporary research in mathematics learning, the early years matter for developing a conceptual base and necessary experiences that will facilitate more advanced concepts and mathematical skills in later years. Research (for example Clements & Sarama, 2008; Lee & Ginsburg, 2009) further shows that teachers play a central role in what knowledge children are enabled to develop in institutional educational settings like preschool. Teachers' pedagogical content knowledge is therefore critical for mathematics learning taking place in early childhood education practices.

Aim

In the study described in more detail below, we will investigate teachers' attention to four mathematical content areas that the contemporary field of knowledge presents as central for children's mathematical development: number sense, number sequence, geometrical shapes and patterns. Furthermore, we will focus on teachers' pedagogical awareness regarding these areas in their work with younger and older preschool children. We consequently want to explore the following:

– Are there differences in teachers' attention to the different mathematical content areas depending on the age group they are teaching?

– If mathematical content areas show differences between age groups, what pedagogical awareness levels constitute these differences?

Method

Participants

Participants were gathered from three municipalities in southern and middle parts of Sweden. The questionnaire was distributed at the beginning of the school year to 147 preschool teachers with a return response of 79 % (116 complete answers). The teachers answering the questionnaire constituted three groups, those working mainly with 1–3-year-olds, 4–5-year-olds and 6-year-olds. This particular study focuses on the teachers working with the youngest and the middle age group in preschool settings ($n = 105$).

Group A: teachers working with 1–3-year-olds , $n = 30$

Group B: teachers working with 4–5-year-olds, $n = 75$

The difference in number of participating teachers working with younger and older age groups approximately mirrors the number of age specific groups in Sweden, where one third of all preschool groups enroll primarily 1–3-year olds (National Agency for Education, 2014).

Instrument

To answer the research questions, we applied an instrument for assessing teachers' pedagogical awareness of mathematics in early childhood education. This instrument has been developed for the project *Learning about space* (Björklund, 2014). It is shaped as a questionnaire combining the idea of PCK (Ball et al., 2008) and contemporary theories of powerful child-centered early childhood education known as developmental pedagogy (Pramling Samuelsson & Pramling, 2011; 2013). The design of the questionnaire is based on a similar structure used in ECERS (Early childhood environment rating scale) for assessing quality in early childhood settings (Sylva et al., 2006). The instrument in our study does, however, contain statements to which the teachers respond often (2), occasionally (1) or never (0). The instrument includes knowledge of mathematical content related to students' learning, but also didactics (curriculum knowledge) and context (teaching, using available resources).

In this particular study we direct specific attention to the mathematical content that involves number sense (M1), number sequences (M2), geometrical shapes (M3) and patterns (M4). Each mathematical content area contains six levels of pedagogical awareness, where the first two basic levels concern the teacher's attention to his/her own use of mathematics in preschool practice, followed by two levels concerning the teacher's attention to children's use of mathematics, and finally two levels of habits of involving children in mathematical exploration. We discern the following six awareness levels deriving from the theoretical standpoint and central features of developmental pedagogy (Pramling Samuelsson & Asplund Carlsson, 2008):

A1: Teachers' attention to possible mathematical content in preschool

A2: Teachers' attention to their own use of mathematics in daily (preschool) life

A3: Teachers' attention to mathematics in children's daily activities

A4: Teachers' attention to children's mathematizing initiatives

A5: Teachers' communicative encouragement for children to explore mathematical meaning

A6: Teachers' problematizing mathematical phenomena

Each mathematical content area includes the above described awareness levels, giving a total of 24 items for our analysis.

Discussion of results

The results of our analysis will be presented in two sections: the first section answers the question whether there are any differences in mathematical content areas offered as learning objects to diverse age-groups in preschool, the second section explores the awareness levels that constitute those differences.

Differences in mathematical learning content

Our first question is whether there are differences in how often different mathematical learning content is offered to children, depending on age group. Table 1 presents the average results per mathematical content area over all six levels of awareness.

Table 1. *Accumulated results per mathematical content area for different age groups*

Content area	Group	Mean	Std. Deviation
Number sense	1–3 year olds	1.7	.32
	4–5 year olds	1.9*	.22
Number sequence	1–3 year olds	1.2	.47
	4–5 year olds	1.6*	.30
Geometrical shapes	1–3 year olds	1.2	.46
	4–5 year olds	1.5*	.39
Patterns	1–3 year olds	1.1	.56
	4–5 year olds	1.4*	.47

Note. * Significant difference ($p < 0.05$) based on a Mann-Whitney U test

The table shows that for all content areas – *number sense*, *number sequence*, *geometrical shapes* and *patterns* – there are significant differences in the frequency with which teachers say they work with the different mathematical contents; older children receive more mathematical attention and support than younger children for all mathematical areas.

For both groups, the occurrence of mathematical engagement decreases from a high engagement in number sense, to a slightly lower engagement in number sequence and geometrical shapes, and the least engagement for patterns.

The analysis shows that the difference between the age groups is smallest for number sense. For both age groups, the attention to this mathematical learning content is relatively high. For the other three types of mathematical learning content the differences between the younger and the older age group are larger.

Pedagogical awareness

As shown above, older children are involved in activities in all mathematical content areas more often than younger children. We now direct our attention to investigate the difference in the teachers' awareness levels that constitute this result. Table 2 presents the results for each separate awareness level within the mathematical content area for each of the groups (A and B).

Our expectations on teachers' pedagogical awareness were that the level of encouragement for mathematical exploration and the discernment of children's own initiatives to mathematization would be higher among the older child-age-group. The earlier study of the whole group (Björklund & Barendregt, 2015) showed a decreasing level of awareness regarding

Table 2 *Averages and standard deviations for each of the awareness levels per content area for group A and B*

Content Area / Awareness level	Teachers working with	
	1–3 year olds Group A (SD)	4–5 year olds Group B (SD)
Number Sense		
A1: Numerals are visible in the preschool room	1.4 (0.77)	1.8 (0.47)*
A2: The teacher uses numbers in dialogues with children	2.0 (0.18)	2.0 (0.16)
A3: The teacher directs children's attention to numerals and numbers appearing in daily activities	1.7 (0.52)	1.9 (0.39)
A4: The children discern numbers spontaneously	1.2 (0.76)	1.8 (0.43)*
A5: The teacher encourages the children to describe quantities with numbers	1.8 (0.37)	1.9 (0.29)
A6: The teacher varies his/her way of presenting and representing number and order	1.7 (0.46)	1.8 (0.43)
Number Sequence		
A1: A number sequence is visible in the room	1.1 (0.88)	1.4 (0.75)*
A2: The teacher counts out loud	1.8 (0.48)	1.9 (0.28)
A3: The teacher counts out loud in different ways	0.7 (0.63)	1.2 (0.52)*
A4: The children count in their play	1.2 (0.77)	1.7 (0.52)*
A5: The teacher encourages the children to count out loud	1.6 (0.61)	1.8 (0.39)*
A6: The teacher follows up on children's initiatives and problematizes the children's counting act	0.7 (0.72)	1.2 (0.52)*
Geometrical Shapes		
A1: Shapes are observable in the preschool environment	1.6 (0.61)	1.7 (0.57)
A2: The teacher uses shapes actively in the practice	1.0 (0.64)	1.4 (0.60)*
A3: The teacher directs children's attention to shapes in their surroundings	1.4 (0.48)	1.6 (0.52)*
A4: The children discern shapes in their surroundings on their own initiatives	1.0 (0.71)	1.5 (0.52)*
A5: The teacher encourages the children to compare shapes	1.3 (0.65)	1.4 (0.57)
A6: The teacher follows up and problematizes shapes that are discovered by the children	0.8 (0.61)	1.3 (0.53)*
Patterns		
A1: Patterns are visual in the preschool environment	1.4 (0.85)	1.5 (0.62)
A2: The teacher constructs patterns in the daily practice	1.1 (0.47)	1.3 (0.63)
A3: The teacher directs the children's attention to patterns in their surroundings	1.0 (0.72)	1.4 (0.59)*
A4: The children are discerning patterns themselves	1.0 (0.74)	1.6 (0.59)*
A5: The teacher encourages the children to create patterns	1.2 (0.75)	1.5 (0.55)
A6: The teacher follows up and problematizes patterns that children discover	0.8 (0.71)	1.2 (0.67)*

Note. * indicates a significant ($p < 0.05$) difference between groups based on a Mann-Whitney U test

communicative encouragement and problematizing activities, which gives us reasons to assume that the younger children are deprived of these stimulating activities to a larger extent than older children. When we look at the two age groups separately, some interesting patterns appear which will be discussed taking the awareness levels one at a time.

Teachers' attention to possible mathematical content in preschool (A1)

The first pedagogical awareness level focuses the teachers' attention to their mathematical surroundings as potential resource for mathematics education in preschool.

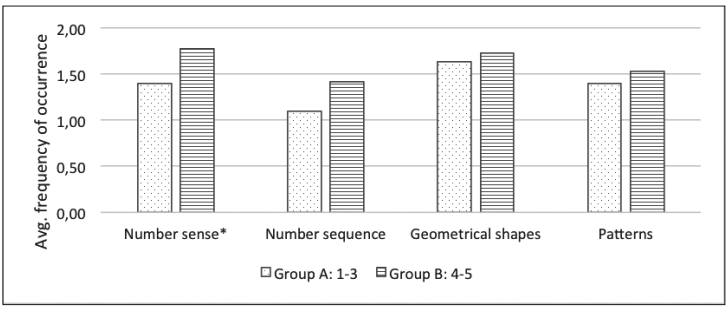

Figure 1. O*verview of teachers' attention to possible mathematical content for younger (group A) and older (group B) preschool groups, ranging from 0 = never to 2 = often. * indicates a significant (p < 0.05) difference between groups based on a Mann-Whitney U test*

There is only a significant difference between the age groups regarding number sense, a content area that gives fairly high responses in both groups. Geometrical shapes are recognized often in the surrounding preschool environment both for younger and older children, but number sequences only occasionally for the younger age group.

Teachers' own use of mathematics (A2)

By paying attention to teachers' own use of mathematical concepts, symbols and principles in daily preschool practice, we see the teachers act as role models to the children and introduce cultural norms and values of mathematics, for example how to execute counting procedures or how to make use of geometrical shapes and patterns.

Figure 2 confirms earlier research that teachers' own use of mathematics in preschool does not often discern geometry and patterns as mathematical content. They do, both with older and younger children,

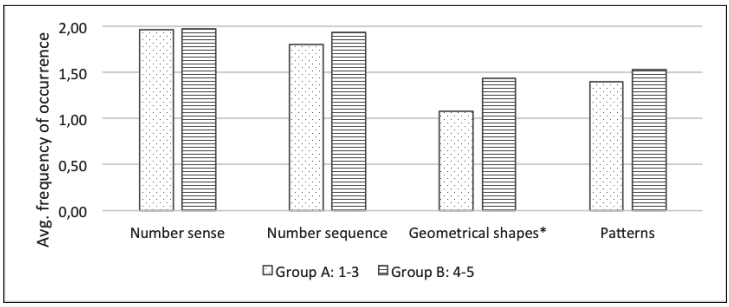

Figure 2. *Teachers' attention to their own use of mathematics in daily preschool practice with younger (group A) and older (group B) preschool children, ranging from 0 = never to 2 = often. * indicates a significant ($p < 0.05$) difference between groups based on a Mann-Whitney U test*

use numbers and counting on a daily basis, but the attention is less often focused on geometrical shapes and patterns. However, a Mann-Whitney U test shows that teachers' attention to their own use of geometrical shapes is significantly higher for those working with older children.

Teachers' attention to mathematics in children's daily activities (A3)

Attention to mathematics in children's daily activities concerns habits of directing children's attention to mathematical phenomena initiated by the teacher. This reflects the teachers' attention and knowledge of mathematical phenomena as they appear in the preschool environment and during different activities. However, the answers may also give a view of the teachers' choice of mathematical content that is presumed to be relevant to the specific group of children.

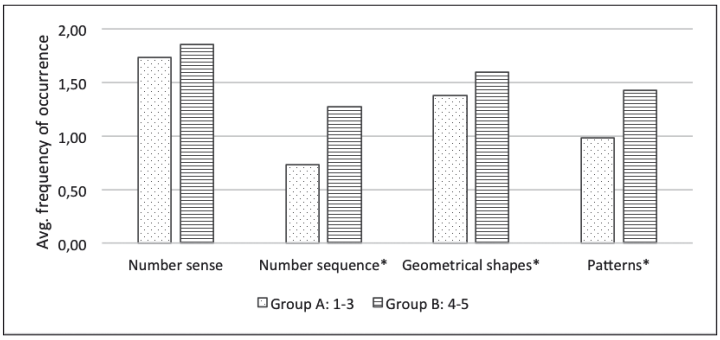

Figure 3. *Teachers' tendencies to direct attention to mathematics in children's daily activities in their work with younger (group A) and older (group B) children, ranging from 0 = never to 2 = often. * indicates a significant ($p < 0.05$) difference between groups based on a Mann-Whitney U test*

Looking at this kind of awareness shows that teachers' responses in both groups follow the same pattern, however, there is a remarkable drop for the younger group when it comes to counting out loud in different ways. This indicates that younger children, when exposed to a number sequence or rhyme, primarily encounter the traditional number line, starting from one (1, 2, 3, ...). Children are rarely exposed to counting that starts on an arbitrary number or backwards (for example 10, 9, 8, ...) or counting on every second number (2, 4, 6, ...) – variations that could reflect the relationship between number and in particular the ordinality of numbers.

Teacher's attention to children's mathematizing initiatives (A4)

The teachers' attention to children's initiatives concerns teachers' awareness of children's spontaneous reasoning about mathematical phenomena and their attention to how children discern mathematics in their surroundings.

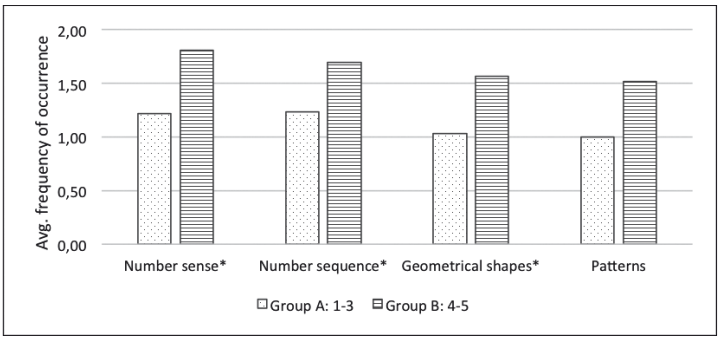

Figure 4. *Teachers' attention to children's mathematizing initiatives among older (group B) and younger (group A) preschoolers, ranging from 0 = never to 2 = often. * indicates a significant ($p < 0.05$) difference between groups based on a Mann-Whitney U test*

Figure 4 shows that for all mathematical content areas the younger children's initiatives receive significantly less attention from the teachers. One interesting finding is that while for the earlier discussed awareness levels, number sequences in general received less attention compared to the other mathematical areas, this difference was not so clear when it came to paying attention to children's own habits of taking initiatives to use and explore the counting rhyme. Children, still with an over-representation of the older ones, are in general observed to count and use numbers frequently, even though teachers themselves seldom direct attention to different number sequences (presented as A3).

Teachers' communicative encouragement (A5)

Figure 5 shows the results concerning teachers' communicative encouragement for children to explore mathematics. Although it might be expected that teachers would pay a lot more attention to communicative encouragement to explore mathematical phenomena with older children, this is not the case. Only for number sequence there is a significant difference between the groups.

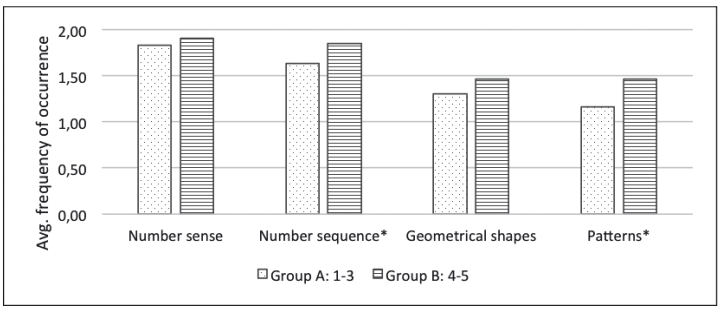

Figure 5. *Teachers' communicative encouragement to younger (group A) and older (group B) children exploring mathematical meaning in different activities, ranging from 0 = never to 2 = often. * indicates a significant ($p < 0.05$) difference between groups based on a Mann-Whitney U test*

Teachers problematizing mathematical phenomena (A6)

Problematizing mathematical phenomena concerns the habit of introducing and exploring mathematical concepts and principles in a goal-oriented way, using different means to visualize mathematical ideas and abstraction together with the children. This is expressed as highlighting

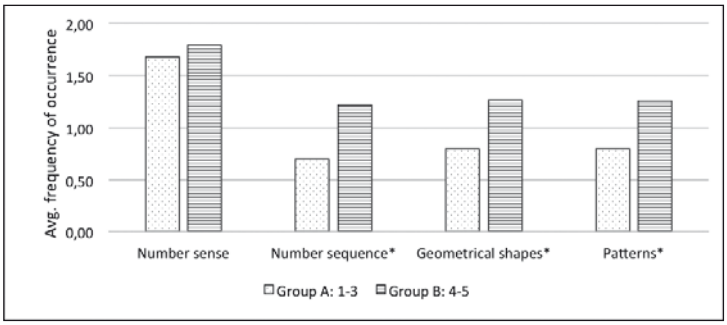

Figure 6. *Teachers' acts of problematizing mathematical phenomena in pedagogical activities to explore mathematical meaning, relevant to younger (group A) and older (group B) children, ranging from 0 = never to 2 = often. * indicates a significant ($p < 0.05$) difference between groups based on a Mann-Whitney U test*

contrasts that direct attention to some specific features of the mathematical content in ways that reflect the children's learning trajectories. The results are given in figure 6.

Problematizing ideas is considered a quite advanced pedagogical approach to stimulate children's cognitive development (Klein, 1991). This presupposes knowledge of the subject, children's learning trajectories and how to organize a learning situation that facilitates discoveries of principles and concept generalization (Ball et al., 2008). Figure 6 shows clearly that this way of working didactically is rare and in particular with the youngest preschool children. A Mann-Whitney U test shows that the difference in problematizing behavior between the age groups is significant ($p<0.05$) for all mathematical content areas, except number sense ($p>0.05$). This is probably related to the fact that number sense is the mathematical area that teachers consistently pay attention to both for the youngest and older preschool children.

Summary and discussion

Our study is based on teachers' subjective responses to the frequency of mathematical interaction and activities in their preschool work. There is a potential veridical risk using such data, both concerning the reliability in the answers and the fact that the instrument encourages reflection on teachers' own practice and thereby may work as a self-tutoring material. However, Sheridan (2009) reports on both subjective and external evaluations of a similar kind, where preschool quality generally is valued higher by the teachers themselves than by external evaluators. On the other hand, teachers from the externally evaluated highest quality preschools tend to value their practice lower than the external evaluators. This is considered to be an effect of reflective awareness of their own pedagogical professional work and areas for development. Conclusions drawn from our study have to be considered in relation to this risk, but should also be valued as showing tendencies in pedagogical practice and as a basis for further study.

In the pedagogical theoretical framework that this study is based on, teachers' communication and their challenging the children's conceptions of content and meaning making are the motor for development and learning (Pramling Samuelsson & Asplund Carlsson, 2008). This pedagogical competence builds not only on teaching methods or subject knowledge, but rather on a complexity of knowledge about mathematics and mathematics learning in particular, as well as knowledge about the individuals and the group of children that teachers are actually working with. Together, this knowledge and skills form the basis for providing

children with opportunities to explore and learn mathematics. It was thus our intention to investigate these preconditions for Swedish children's mathematics learning in preschools. This was done through the questionnaire focusing on teachers' habits of using, observing and encouraging mathematics as a content and goal for learning in preschool. Our aim was in particular to investigate any occurring differences between teachers working with the younger and the older preschool children.

Results from our analyses show that there is an increase in the frequency of mathematical activities when comparing the group working with younger and older preschool children. Older children are provided with more challenges in all mathematical areas. The difference, to the older children's advantage, is significant for teachers choosing numbers as content for learning in preschool, when teachers act to support children in discerning and communicating number sequences, concerning teachers' observations of children taking the initiative to explore and use mathematics (all content areas), and concerning problematization and challenging current conceptions (significant for all mathematical content areas except number sense). The results are in accordance with earlier research that claims numbers to be the main learning content offered to preschool children and geometry and spatial aspects of mathematics as a neglected area (Clements & Sarama, 2011; Lee, 2010). Our study contributes to the body of research with a specific comparison of younger and older preschool children, showing that the pattern is similar and teachers' choice of mathematical content area is not necessarily depending on the age of the children.

Attention to children's initiatives to use mathematics received similar attention among all mathematical areas, but for the younger children this was less frequent. This may be an effect of teachers' expectations of children's competencies but also children's actual skills and knowledge. Younger children do not have the same possibility to express their discoveries and exploration verbally, which makes it more difficult for teachers to recognize children's initiatives expressed in action. However, intuitive knowledge of numbers, shapes and structure emerge and develop long before children express their understanding in verbal expressions (Butterworth, 1999; Wynn, 1998). According to the large body of research on toddlers' and infants' emerging mathematical competence, there should be sufficient support for teachers to direct attention to the youngest preschool children's expressions of mathematical focus and reasoning skills. However, there are also children who do not pay attention to numbers spontaneously (Hannula, 2005). These children should in particular be offered opportunities to interact with peers and teachers, exploring and communicating about mathematical content.

These results give reasons to highlight the question of equality in mathematics education that young children receive in Swedish preschool. Earlier research and studies of intervention programmes with older preschool children (Aunola et al., 2010; Hannula, 2005; Clements & Sarama, 2008) have convincingly shown that experiences and attention to mathematics in daily activities in preschool have effects on later mathematics development and school achievements. Empirical studies of mathematics intervention programmes with younger preschool children are on the other hand sparse. However, a small-scale study with 2–3 year olds by Doverborg and Pramling Samuelsson (2000) shows that children participating in daily activities communicating about quantities develop a more flexible and conceptual understanding of small numbers, compared with a similar age-group without intervention. The study emphasizes that learning basic ideas of mathematical content (numerical as well as spatial) are not merely a question of the frequency teachers expose children to mathematical content, but the quality of challenging activities children are engaged in. The question we aim to raise is therefore concerning the quality and awareness of the learning content offered to children of different ages. Our concern is that younger children should, not least according to national guidelines, be offered opportunities to learn on their own terms to the same extent as older children. Exploring and communicating children's emerging mathematical concepts should therefore be of central interest in an equality education in the early years. Beginning this kind of activity only when children in later preschool years are able to show the competence to verbalize mathematical content on their own initiative, may thus hamper their mathematical development.

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