Young children exploring probability – with focus on their documentations

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This article reports on an intervention where possibilities and limitations with problem-solving as a basis for mathematics education in pre-school class were studied. In the article we explore how 50 children use non-guided documentation when working with a problem-solving task about probability. The results show that the task was possible to work with for these young children, and in the follow-up interviews many of the children seemed familiar with the mathematical concepts used, as well as with a relevant sample space. The children's non-guided documentation showed a diversity of strategies and contributed positively to their exploration of probability, both during the lesson and in the final discussions.

This article reports on an intervention conducted in Swedish preschool classes. Preschool class is an optional year of schooling that Swedish children can attend the year before formal schooling begins. In the intervention possibilities and limitations with problem-solving as a basis for mathematics education in pre-school class were explored. We let about 150 children solve a number of – for them – advanced problem solving tasks. The children were also interviewed before and after the intervention. In this article we will present one example of these problem-solving tasks, one about probability. The focus will especially be on children's non-guided documentations when working with this task. How do the children use documentation when solving the task and how do they reflect on probability? Thus, in the article the wholeness of the intervention will not be elaborated upon or evaluated, however, the intervention will be described shortly, in order to be able to understand the design and implementation of the example presented here.

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The article is organised as follows. It starts with a presentation of the Swedish preschool class, followed by an overview of young children's learning of mathematics in general, as well as problem-solving and probability in particular. After that, the design of the intervention is presented followed by results and conclusions.

The context – Swedish preschool class

As stated, the intervention presented in this article took place in preschool classes in Sweden. Preschool classes are an optional part of the national educational system for children aged six with the aim to facilitate a smooth transition between preschool and school and prepare children for the next step of their education. Sweden has a long tradition of preschool, offered to children between the ages of one and five. From an international perspective, the Swedish preschools are situated within a social pedagogy tradition (Bennett & Tayler, 2006) where care, play and learning are to be integrated. These traditions of play in preschool and of learning in school (Pramling & Pramling Samuelsson, 2008) made the issue of transition problematic, which is why, in 1998, Sweden introduced the preschool class. To facilitate a smooth transition, preschool classes are supposed to work in their own way, preparing children for the way of working in school, but having their grounding in the traditions of play in preschool. Until now there has been no national regulation regarding which type and how mathematics should be taught in preschool class why its mathematics teaching differs, both in design and content (National Agency for Education, 2014).

Young children and mathematics

Based on UNICEF's description, early childhood is usually defined as the ages between birth and eight (Brooker, Blaise & Edwards, 2014). The pedagogical approaches as well as the content and goals of early mathematics education have changed over the years. Today, studies show that appropriately designed and implemented learning experiences make it possible for young children (up to the age of eight) to possess mathematical competences that were previously considered to be possible only for older children (from age nine and upwards) (English & Mulligan, 2013; Newton & Alexander, 2013). Further, it has been shown that the learning of mathematics for young children has great implications for mathematical achievement later on (Cross, Woods & Schweingruber, 2009; Duncan et al., 2007).

Even though research seems to be united when it comes to mathematics education being important for young children, it is less united when it comes to what *appropriately designed* and *implemented learning experiences* imply. Historically, numeracy and counting have formed the main content in early years' mathematics education and consequently this is also the area where most research has been conducted (Clements & Sarama, 2009). However, several studies over the years have shown that young children can deal with other mathematical content (Palmér & van Bommel, 2016; Saracho & Spodek, 2008; Clements & Sarama, 2009). Some researchers emphasise that young children need to be engaged in diverse, advanced and challenging mathematical activities focusing on a broad spectrum of content – for example, statistics and early algebra (Claessens & Engel, 2013; English & Mulligan, 2013; Seo & Ginsburg, 2004) while others emphasise the importance of focusing mainly on basic number facts and on applying computational procedures (Westwood, 2011). Thus, there is no consensus in terms of either the content or the design of mathematics education for young children.

As mentioned, the intervention from which the example in this article is taken from explored possibilities and limitations with problem-solving as a basis for mathematics education in pre-school class. The content in this specific example is probability. To support the example, both these concepts – probability and problem-solving – will be elaborated further below.

Problem-solving in mathematics

In the Swedish curriculum for primary schools (National Agency for Education, 2011) mathematics is described as "a creative, reflective, problem-solving activity" (p. 62). Further, problem-solving is described both as a purpose (ability to formulate and solve problems) and as core content, where mathematics teaching should help children to develop their skills to formulate and solve mathematical problems, as well as to evaluate the strategies and methods to be used (National Agency for Education, 2011). Problem-solving is not a Swedish national phenomenon but is also emphasised internationally in both policy documents and research (for example EU, 2007; NCTM, 2000). The main idea with problem solving is that children will develop important mathematical ideas and competences through working with problem-solving tasks.

According to Lesh and Zawojewski (2007), a task becomes a problem-solving task when the individual who is to solve the task has to develop strategies and/or knowledge not yet obtained in order to be able to solve it. Thus a problem-solving task is a challenge for a child since s/he does not know in advance how to proceed to solve the task. Instead, the child has to develop new (for him or her) strategies, methods and/or models to be able to solve the task. Thus, what is a problem-solving task for one child may not be a problem-solving task for another child. Further, a problem-solving task today is often not a problem-solving task tomorrow.

Problem-solving is not only present in educational settings, since children's natural curiosity leads them into problems to solve in their daily life. Solving such problems many times involves mathematics (Charlesworth & Leali, 2012). Further, problem-solving is not new in educational settings where interventions in school have been carried out successfully (Sowder et al., 1988; Cobb, Wood, Yackel & Perlwitz, 1992). Results also show that children who work with (for them) advanced mathematical problems simultaneously developed knowledge in basic mathematics (Casey, 2009; Cobb et al., 1992; Sowder et al., 1988). However, these studies have been carried out at school settings and often with children older than the children in the intervention presented in this article. Thus, there is a need to better understand how problem-solving can be conducted with young children in other settings.

Probability

Within an experiment, all events that can occur constitute the sample space and the probability of each event occurring can be calculated. For instance, if we take one marble from a bag with two marbles, one red and one yellow, two events can occur – it can be a red marble or a yellow marble. In this case each event is equally likely to happen, which means the probability of obtaining a red marble is equal to the probability of obtaining a yellow marble, which is 0.5. If we put four marbles (two red, two yellow) in the bag, and we take out two marbles at once, three different events can occur – two red marbles, two yellow ones, or one red and one yellow. These events, however, do not have the same probability. When explaining the problem-solving task in the example later on, the experiment will be described with these probabilities.

Probability is gaining importance within our daily lives and in society in general as it "attempts to quantify uncertainty as a tool for decision making" (Amir & Williams, 1999, p. 87). In the Swedish primary school curriculum for years 1–3, some basic probability is included in core content:

- Random events in experiments and games.
- Simple tables and diagrams and how they can be used to categorise data and describe results from simple investigations.

(National Agency for Education, 2011, p. 61)

Young children's probabilistic thinking has been the subject of early research (see for instance Piaget & Inhelder, 1975; Fischbein, 1975) and research also states that probability is not perceived as a simple topic either to teach or to understand (Chick & Baker, 2005; Benko & Maher, 2006). Watson adds to this by bringing in the aspect of language (Watson 2006; English & Watson, 2016) and addresses aspects like the lack of clarity of specific terms. Jones, Langrall, Thornton and Mogill (1997) tried to describe a framework for assessing probabilistic thinking. In their research they distinguished four levels of construct in different settings: the subjective level, which is intuitive based on previous experiences; the transitional level, where predictions are more structured and consider extreme cases as well as comparisons; the informal quantitative level in which students begin to list outcomes systematically and the numerical level where probabilities are described using numerical measures. They outlined four key constructs for probabilistic thinking: sample space, probability of an event, probability comparison and conditional probability. Watson (2006) noted that young children would gain from an empirical frequency-based approach to probability, which seems comparable to the subjective level from the framework of Jones et al. (1997). Some years later, Way (2003) stated some implications for planning teaching about probability. Four of these implications were: particular attention to certain concepts, take into account the limited intuitive notions children have regarding probability, clarity within the sample space, and non-limitation regarding strategies. The work of Jones et al. (1997) and Way (2003) are the starting point for the design of the probability content in the problem-solving lesson focused on in this article.

The study

Eight preschool classes (a total of around 150 children) were part of the overall intervention. These preschool classes were selected based on the mathematical interest of the teachers at the schools. According to these teachers, they had not worked with problem-solving or with probability in mathematics in their preschool classes before. Four of these preschool classes from two different schools, with a total of 50 children, took part in the problem-solving lesson reported here.

Research involving children puts extra ethical demands on the researchers (Alderson & Morrow, 2011). The ethical regulations for research provided by the Swedish Research Council (2002) were followed, where both guardians and children approved the participation. The children were given verbal information about the intervention and the interest of the researchers. The children's guardians were given written information about the study and approved their children's participation in line with the ethical guidelines.

Educational design research

The arrangement of the lessons within the intervention has been developed within the frames of educational design research. Even though this article only reports on one lesson some issues will be raised about the wholeness of the intervention. The aim with this is to make it possible for the reader to understand the origin of the lesson. Using educational design research implies designing, testing and refining interventions with the aim of developing solutions to practical and complex educational problems (McKenney & Reeves, 2012). Some of the problems investigated in educational design research have been identified mainly by practitioners, while others have been identified primarily by researchers. In the intervention presented in this article, the problem investigated was identified mainly by the researchers. The practical and complex educational problem was the existing situation with no regulations for which type of mathematics and/or how mathematics was to be taught in preschool classes. As mentioned, the aim of the intervention was to explore possibilities and limitations with problem-solving as a basis for mathematics education in pre-school class. The intervention was not about an implementation of ready-made and researched lessons, but an exploration in which we learned in and from the setting.

There are large variations in design research studies, both in direction and size, but what is common is the intent to create and study new forms of instruction and to develop theories. Theories used are both prospective (theories of teaching and learning inform the research) and reflective (theories of teaching and learning are revised, refined or improved in the research). Also common to design research studies are the iterative design and its orientation to be valid for practice (Prediger, Gravemeijer & Confrey, 2015). The research process is iterative and interventions are developed based on design requirements. These design requirements refer to criteria that frame the interventions, criteria that the intervention should meet. During the intervention design propositions are developed that refer to further specifications of what the design should look like to reach a desired situation. The intention with this iterative process is to produce practical solutions based on theoretical understanding where the results of research are relevant for education practice (McKenney & Reeves, 2012).

Design requirements and implementation

The task presented here was the fourth of five problem-solving tasks within the first design cycle in the intervention. Its design requirements were based on the research on problem solving and probability presented previously; in particular, the conditions of Jones et al. (1997) and the planning principles of Way (2003) were taken into account. Further, design requirement one is based on the age of the children in relation to their previous school-related experience regarding reading and writing (Palmér & van Bommel, 2016).

DR1 To solve the problem the children should not have to be able to read or write (Palmér & van Bommel, 2016).

DR2 The mathematical ideas in the task are understandable but not yet grasped by the children (Lesh & Zawojewski, 2007).

DR3 It should be possible to solve the task by using different strategies without limitations (Way, 2003).

DR4 The task should touch upon intuitive notions on probability, addressing the subjective level (Way, 2003; Jones et al., 1997).

DR5 The transitional as well as the numerical level within the task have to be apparent (Jones et al., 1997).

DR6 The mathematical concepts introduced in the task have to be defined (Way, 2003).

DR7 The sample space relevant to the task should be stated making the informal quantitative level apparent (Way, 2003; Jones et al., 1997).

Design requirements 1–3 (those connected to problem-solving) were the same for all tasks within the intervention while 4–7 were specific for this task. Based on these design requirements the following task was used in the intervention.

> One takes two marbles out of a bag containing four (two red, two yellow) and notes the result. Which event will occur most often if the experiment is undertaken 20 times?

Since previous research on the implementation of problem-solving had shown the importance of the teacher (Cobb et al., 1992), one of the researchers taught the lesson. This was not because the researcher would be better as a teacher, but it enabled a constant factor regarding the design requirements of the intervention in each of the four classes (during the second design cycle the teachers taught their own preschool-classes). Thus our roles were both insiders and outsiders at different times in the intervention (Alderson & Morrow, 2011). An advantage with this was that we could control the similarity of the implementation of the tasks in the four preschool classes. A disadvantage is that we are part of the data we are to analyse.

The task was introduced orally (DR1) to the children. The whole sample space (DR7) with all three events (red-red, red-yellow, yellow-yellow) was explored with the children at the board and the children then voted for the one they thought would "win". The children were asked for reasons why, to check if they understood the problem and if the mathematical idea had been grasped at that point. The outcome was assumed not to be obvious for the children, and each child made a prediction, which made it possible for us to see if our assumption was correct (DR2). During the next phase the children were asked to document the outcome of 20 draws. While marbles were drawn from the bag, children were given their own choice of documenting (DR3). Mathematical concepts were defined (DR6), although a variety of mathematical concepts can be used to describe the exercise and to solve the problem or present the solution; a selection was made to use with the children. The aim was not to limit to just a few concepts, but to be aware of possible difficulties and how to relate to them. The intuitive notions on probability were addressed by introducing the exercise with a prediction in which the children were involved (DR4). At the end of the lesson, children were to look back at their prediction and make a reflection on their own progress. A whole-class discussion was led by the researcher. The sample space was explored together with the children both at the beginning and the end of the lesson (DR7). The transitional level as well as the numerical level (DR5) was aimed at in the last stage of the exercise – coming to an explanation (DR6).

Analysis

The focus of this article is on the children's documentation and their reflections on probability as expressed during the lesson. Children's reflections were noted during the lesson, by writing, and where permission from the guardians was given, audiotaped. All written documentation from the children was collected and then analysed. The non-guided documentation was analysed using emerging categories by looking for patterns in the documentations. Differences and similarities within the diversity in documentations were looked for. The categories emerged in two phases: first, the documentation was divided into chronological and non-chronological strategies; secondly it was further divided based on the representations used. These categories will be further described below in the results.

Results and discussion

The section here is divided into two parts: one part concerns the documentation while the other part focuses on the children's reflections on probability as expressed during and after the lesson.

Documentation

Two different strategies were observed for documenting the outcome of the experiment: a chronological and a non-chronological strategy. In chronological documentation each draw is noted in chronological order (figure 1). This means that one can reconstruct the whole experiment with regard to the outcome, as it is possible to identify each draw and the order of the draws from the documentation. The documentation in figure 1 shows a draw by two dots, in corresponding colours with the marbles. Each draw is separated from the previous one using a vertical line. Thirty children used some kind of chronological way of documenting but these differed in both structure and the representations used. These children did not state a sample space in their documentation and their documentation seemed to be on the subjective level (level 1 – the subjective level, Jones et al., 1997). However one could argue that the transitional level (level 2, Jones et al., 1997) was reached as well, as the sample space had been explained prior to documentation – in the first stage of the lesson when children had to guess the winning outcome.

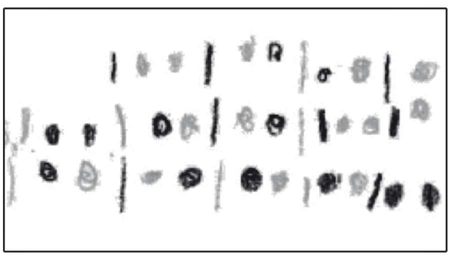

Figure 1. *Example of chronological documentation. Each draw is represented by two dots, separated from the next draw with a vertical line.*

A total of 20 children used a non-chronological way of documenting. In a non-chronological document, a reconstruction of the whole experiment with regard to the outcome is not possible. In all but two of these a table was used to note the outcome. These children listed a complete set of outcomes (level 2 – the transitional level, Jones et al., 1997). In the two other cases, the outcome was noted in such a way that it was

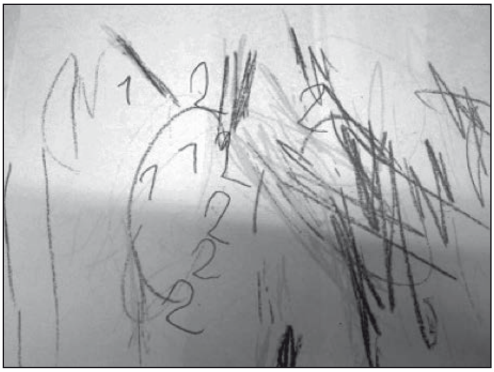

Figure 2. *Example of non-chronological documentation (not using a table) noted in such a way that neither the chronological order, nor the final result of the outcome was possible to retrieve.*

neither possible to retrieve the chronological order, nor the final result of the outcome (figure 2).

Each of these two categories (chronological and non-chronological) was then analysed using the sub-categories: *words*, *icons*, *tally marks* and *other*. We decided to distinguish between icons and tally marks in the following way. An icon is a specific representation for a specific outcome, implying that the three outcomes in our task (red-red, red-yellow, yellow-yellow) were illustrated using three different icons. This differs from the use of a tally mark where the place of the tally mark determines what it represents. Each tally mark is similar and no differences can be seen between the tally marks representing each of the three different outcomes.

Table 1. *Children's documentation in categories Chronological and Non-chronological connected to the sub-categories Words, Icons, Tally marks and Other.*

	Chronological	Non-chronological	
		Table	Other
Words	3		
Icons	26 (4)	6	2(1)
Tally marks		12 (6)	
Other	1 (1)		
Total	30	18	2

Note. Numbers in ( ) show use of number symbols

Words were used by three children (table 1), all in chronological documentation; here children wrote the outcome as in the example in figure 3. From the documentation the first draw ”gul ok gul” (yellow and yellow) can be noted, followed by the second draw written underneath ”gul ok röd” (yellow and red), and so on.

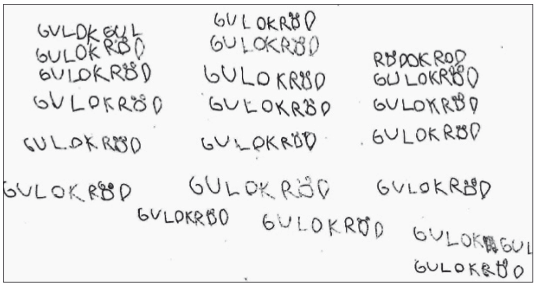

Figure 3. *Example of chronological documentation using words*
Note. röd = red, gul = yellow, ok (och) = and)

Figure 4. Example of a non-chronological documentation using icons in a table

The subcategory *icons* included all types of representations of the outcome where specific icons were used for each of the three outcomes. In figure 4 we see an example where each outcome is represented in a table by using icons. The icons used here are coloured lines representing the outcome; ordered in three piles, each representing one of the possible outcomes. The middle pile: red-yellow shows most clearly that each outcome is represented by two lines (one red one yellow). Here each outcome needs to be drawn using lines, a red line representing red-red, a yellow line representing yellow-yellow, and a yellow and red line representing yellow-red. The table in figure 4 differs from tables where *tally marks* were used to represent each draw, as is shown in figures 5 and 6 where the position of

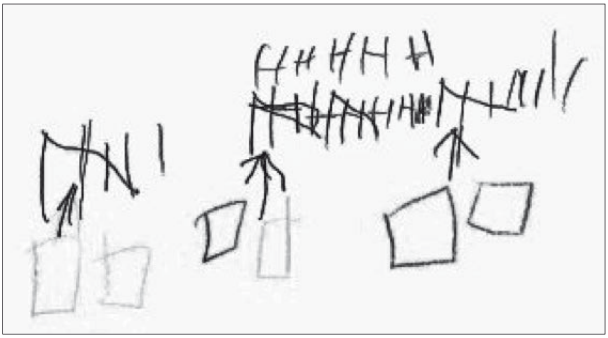

Figure 5. *Example of non-chronological documentation using tally marks in a table*

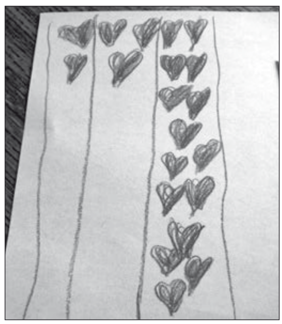

Figure 6. *Example of non-chronological documentation using hearts as a tally in a table*

the tally decides the value, not the icon per se, as in figure 4. The tally marks could differ from small (traditional) tallies (figure 5), to hearts representing a draw (figure 6). Using a table incorporates a column for each of the three outcomes. In figure 5 these three outcomes are represented by a drawing of the outcome: two squares in corresponding colour. In figure 6, the outcomes are not defined.

Twelve children used numbers in their documentation. One of these used only numbers (subcategory *other*), nothing else, which made it impossible for this child to give an answer to the problem, i.e. which combination won (figure 7).

Four children made chronological documentation with icons in combination with a numerical value for the draw. These children were the ones who could tell in the end when to stop taking marbles out of the bag (figure 8). Thus, these documentations included two dimensions, the outcome of the draw and the number of draws. Figure 8 shows an example of such a two dimension documentation. The first dimension, the outcome, was noted by using two dots in corresponding colours with

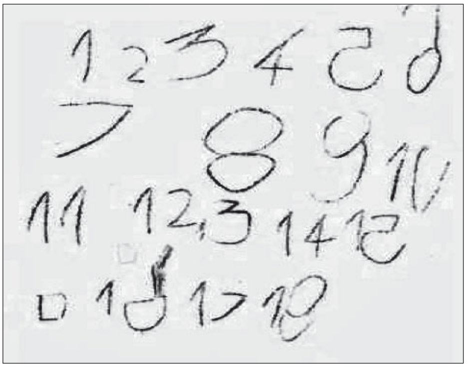

Figure 7. *Example of chronological documentation with numbers only (other)*

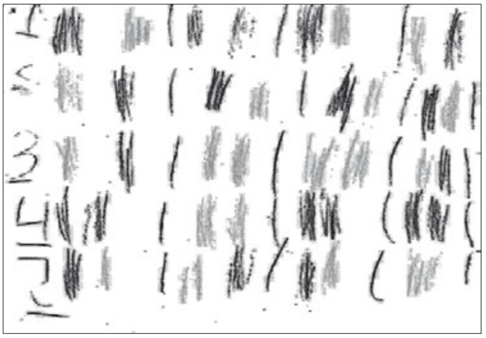

Figure 8. *Example of chronological documentation with icons and numbers making both the outcome of the draw and the number of draws visible*

the marbles, separating each draw with a vertical line (similar to the documentation shown in figure 1). The second dimension, the number of draws, was noted by the numbers 1,2,3,4 and 5. This child noted the first draw in the top left corner, the second draw below, third draw underneath again, etc. The sixth draw then was noted next to draw number one, making columns of five draws each. In all the others only one dimension was noted, the outcome of the draw. The number of draws could be counted, but was not visible in the documentation.

The children using numbers in their table did not register the numbers in a chronological order, but accounted for the number of successes for each combination (figure 9). This made it easy for them to see how often each outcome had been obtained, but they could not immediately see the total amount of draws. The total number of draws can be retrieved though by adding the top number in each column.

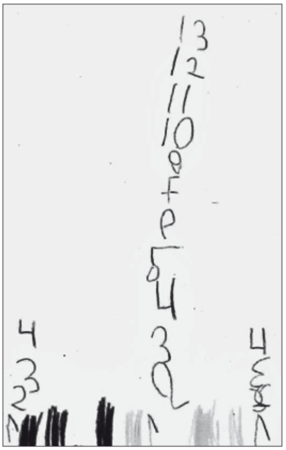

Figure 9. *Example of non-chronological documentation with table and numbers making the total of each outcome but not (without counting) the total amount of draws visible*

The diversity of documentation presented above provided input at the end of the lesson where children could look at each other's documentation. The closure of the lesson is a crucial phase: here children were asked to not only look at their own documentation, but to argue in favour of their or another form of documentation. Different questions were asked – for instance the difference between two children's documentation. The researcher then chose a chronological form and a tabular form of documentation. Children could argue for why the chronological documentation was preferred: "I can see what happened at the first draw and the second and so on", or why the table was preferable: "In a table you can easily see which combinations

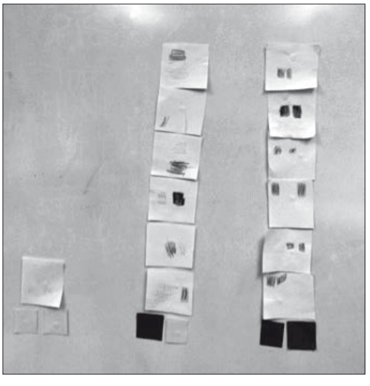

Figure 10. *Bar chart of expected outcome in one of the preschool classes*

won". Their empirical experience was of great value here as they could refer to the experiment (Watson, 2006; Jones et al., 1997). A discussion about the two dimensions (number of draws and the outcome of the draws) to keep track of was initiated and made it possible for the children to argue about the way the numbers were used in different ways. Pinpointing critical aspects of importance gave children the opportunity to learn about documentation, and not merely how to document (Hiebert & Grouws, 2002). One child expressed the following: "I always do things right and my table is good, but next time I will use her table instead, because that one was less trouble and more easy to see [who won]".

Children's reflections on probability as expressed during the lesson

The sample space was developed together with the children and they predicted the most likely event based on subjective judgements (level 1 – the subjective level, Jones et al., 1997). When the children were asked to make a guess, a bar chart (figure 10) appeared on the white board. Arguments for their choice ranged from "I like red more than yellow" to "you can only take two reds but you can take red and yellow more often". Although the events did not have an equal chance of occurring, this statement suggests thoughts in line with equiprobability bias. Children's arguments show how informal knowledge of probability influences formal learning (Amir & Williams, 1999).

During the experiment, children expressed their surprise when red-yellow appeared as often as it did. They asked the researcher if she really couldn't feel or see the colours. This is in line with what Amir and Williams (1999) found regarding random devices: the children gave arguments suggesting the outcome was being influenced, in this case by "feeling difference between the colours". As compared to Jones et al. (1997), the children expressed their concerns for fair probability, which is situated within the transitional level (level 2). When all 20 draws had been made a discussion started on why red-yellow won. The mathematical explanation was not given by the researcher, but emerged through suggestions from the children. Some children suggested looking at the way the different outcomes could be obtained. The idea of unequal chances of getting two specific marbles depending on the outcome was suggested. The children expressed an awareness of having more choices to make red-yellow compared to the one-colour options. Moreover, one of the children suggested it would have been equal if we had had four different colours in the bags.

After the documentation of the event, the children's judgements went from the subjective level to a transitional and informal quantitative level (levels 2 and 3, Jones et al., 1997). They now predicted the most likely event, based on quantitative judgement, using numbers to compare probabilities. In a follow-up interview after the intervention, the task was touched upon and a clear majority of the 50 children remembered what the task was about and what the outcome was. Some of these children could give a mathematically-related explanation while others gave more everyday explanations:

> Fun. Because you could get yellow and red in two different ways.
>
> Red and yellow won. Because you shook the bag and mixed them.
>
> There were two chances of getting yellow and red but one chance to just have red or yellow.

The last quote indicates a numerical level – (level 4, Jones et al., 1997) as the child compared probabilities and assigned equal probabilities to likely events and non-equal to other events.

Conclusions

The starting point of the intervention presented in this article was the case of no regulations for which type of mathematics and/or how mathematics should be taught in Swedish preschool classes. Based on this we wanted to explore possibilities and limitations with problem-solving as a basis for mathematics education in pre-school class. In this article one problem-solving task about probability from the intervention has been presented with special focus on children's non-guided documentation. How do the children use documentation when solving the task and how do they reflect on probability?

The seven design requirements for this task (DR1–7) were based on previous research on problem-solving and probability. However, this research had seldom included children as young as in this study. Connecting our results back to the design requirements, only 3 out of 50 children used words or letters in their documentation, so DR1 (to solve the problem the children should not have to be able to read or write) seems justifiable. During the introduction the children were asked to guess the most likely outcome as requested (DR4 – the task should touch upon intuitive notions on probability). Some of the children were able to give an explanation that touched on the mathematical idea, while others showed signs of a primary understanding (DR2 – the mathematical ideas in the task are understandable but not yet grasped by the children). When talking about possible outcomes, the sample space relevant to the task was described (DR7 – the sample space relevant to the task should be stated). The children's documentation showed a diversity of strategies in line with DR3 (it should be possible to solve the task by using different strategies without limitations). At the end of the lesson, the children responded to each other's documentation and gave arguments for strengths and weaknesses (peer modelling). The researcher chose appropriate documentation to discuss and drew the children's attention to specific solutions (indirect adult modelling). In this discussion both the transition and the numerical level within the task became apparent (DR5 – the transition as well as the numerical level within the task have to be apparent). Finally, in the follow-up interviews many of the children seemed familiar with the mathematical concepts as well as with a relevant sample space, indicating that DR6 (the mathematical concepts introduced in the task have to be defined) had been accomplished during the task. Thus, all of the design requirements seem to have been taken into account in the task.

Further, the analysis of the documentation offers some additional design properties. In future cycles, *to make visible the different categories of documentation* (table 1) will be added as a *feasible* design requirement. The feasible aspect indicates that it is not possible to control this design requirement, since the children are free to choose their own strategies for documentation. This free choice entails that different children show different levels of probabilistic thinking, as described by Jones et al. (1997).

In summary the results show that the task was possible for these young children to work with and that the non-guided documentation contributed positively to children's exploration of probability, both during the lesson and in the final discussions. In the follow-up interview a clear majority of the children remembered what the task was about as well as the outcome. This supports the design requirements used and in further implementations also the additional design properties presented above will be considered.

References

Alderson, P. & Morrow, V. (2011). *The ethics of research with children and young people. A practical handbook*. Los Angeles: SAGE.

Amir, G.S. & Williams, J.S. (1999). Cultural influences on children's probabilistic thinking. *Journal of mathematical behaviour*, 18 (1), 85–107.

Benko, P. & Maher, C. A. (2006). Students constructing representations for outcomes of experiments. In J. Novotná, H. Moraová, M. Krátká & N. Stehlíková (Eds.), *Proceedings of the 30th conference of the international group for the psychology of mathematics education* (Vol. 2, pp. 137–144). Prague: PME.

Bennett, J. & Tayler, C. P. (2006). *Starting strong II. Early childhood education and care*. Paris: OECD.

Brooker, L., Blaise, M. & Edwards, S. (2014) (Eds.). *The SAGE handbook of play and learning in early childhood*. Los Angeles: SAGE.

Cai, J. (2010). Commentary on problem solving heuristics, affect, and discrete mathematics: a representational discussion. In B. Sriraman & L. English (Eds.), *Theories of mathematics education. Seeking new frontiers* (pp. 251–258). London: Springer.

Casey, B. (2009). Applying developmental approaches to learning math. In O.A. Barbarin & B.H. Wasik, (Eds.), *Handbook of child development & early education*. New York: The Guilford Press.

Charlesworth, R. & Leali, S.A. (2012). Using problem solving to assess young children's mathematics knowledge. *Early Childhood Educational Journal*, 39, 373–382.

Chick, H. & Baker, M. (2005). Teaching elementary probability: not leaving it to chance. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough et al. (Eds.), *Building connections: research, theory and practice (proceedings of MERGA28)* (pp. 233–240). Sydney: MERGA.

Claessens, A. & Engel, M. (2013). How important is where you start? Early mathematics knowledge and later school success. *Teachers College Record*, 115, 403–434.

Clements, D. H. & Sarama, J. (2009). *Learning and teaching early mathematics. The learning trajectory approach.* New York: Routledge.

Cobb, P., Wood, T., Yackel, E. & Perlwitz, M. (1992). A follow-up assessment of a second-grade problem-centered mathematics project. *Educational Studies in Mathematics*, 23 (5), 483–504.

Cross, C.T, Woods, T.A. & Schweingruber, H. (2009). *Mathematics learning in early childhood. Paths toward excellence and equity*. Washington: The National Academies Press.

Duncan, G.J, Dowsett, C.J., Claessens, A., Magnuson, K., Huston, A.C. et al. (2007). School readiness and later achievement. *Developmental Psychology*, 43 (6), 1428–1446.

English, L.D. & Mulligan, J.T. (2013). Perspectives on reconceptualizing early mathematics learning introduction. In L.D. English & J.T. Mulligan (Eds.), *Perspectives on reconceptualizing early mathematics learning* (pp. 1–4). Dordrecht: Springer.

English, L.D. & Watson, J. (2016). Development of probabilistic understanding in fourth grade. *Journal for Research in Mathematics Education*, 47 (1), 28–62.

EU (2007). *Nyckelkompetenser för livslångt lärande. En europeisk referensram.* Luxembourg: Byrån för Europeiska gemenskapernas officiella publikationer.

Fischbein, E. (1975). *The intuitive sources of probabilistic thinking in children.* Dordrecht: Reidel.

Hiebert, J. & Grouws, D. A. (2002). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Charlotte: Information Age.
Jones, G. A., Langrall, C. W., Thornton, C. A. & Mogill, A. T. (1997). A framework for assessing and nurturing young children's thinking in probability. *Educational Studies in Mathematics*, 32, 101–125.
Lesh, R. & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*, (pp. 763–804). Charlotte: Information Age.
McKenney S. & Reeves, T. (2012). *Conducting educational design research*. London: Routledge.
National Agency for Education (2011). *Curriculum for the primary school, preschool class and leisure time centre 2011*. Stockholm: National Agency for Education.
National Agency for Education (2014). *Förskoleklassen – uppdrag, innehåll och kvalitet*. Stockholm: National Agency for Education.
NCTM (2000). *Principles and standards for school mathematics*. Charlotte: National Council of Teachers of Mathematics.
Newton, K. J. & Alexander, P. A. (2013). Early mathematics learning in perspective: eras and forces of change. In L. D. English & J. T. Mulligan (Eds.), *Perspectives on reconceptualizing early mathematics learning* (pp. 5–28). Dordrecht: Springer.
Palmér, H. & Bommel, J. van (2016). *Problemlösning som utgångspunkt – matematikundervisning i förskoleklass*. Stockholm: Liber.
Piaget, J. & Inhelder, B. (1975). *The origin of the idea of chance in children*. Routledge and Kegan Paul.
Pramling, N. & Pramling Samuelsson, I. (2008). Identifying and solving problems: making sense of basic mathematics through storytelling in the preschool class. *International Journal of Early Childhood*, 40 (1), 65–79.
Prediger, S., Gravemeijer, K. & Confrey, J. (2015). Design research with a focus on learning processes: an overview on achievements and challenges. *ZDM*, 48, 877–891.
Saracho, O. N. & Spodek, B. (2008). Trends in early childhood mathematics research. In O. N. Saracho & B. Spodek (Eds.), *Contemporary perspectives in early childhood education* (pp. vii–xx). Charlotte: Information Age.
Seo, K.-H. & Ginsburg, H. P. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D. H. Clements & J. Sarama (Eds.), *Engaging young children in mathematics: standards for early childhood mathematics education* (pp. 91–104). Mahwah: Lawrence Erlbaum.
Sowder, J., Sowder, L., Cobb, P., Yackel, E., Wood, T. et al. (1988). Creating a problem-solving athmosphere. *The Arithmetic Teacher*, 36 (1), 46–47.

Swedish Research Council (2002) *Forskningsetiska principer inom humanistisk-samhällsvetenskaplig forskning*. Stockholm: Vetenskapsrådet.

Watson, J. (2006). *Statistical literacy in schools: growth and goals*. Mahwah: Lawrence Erlbaum.

Way, J. (2003). *The development of children's notions of probability* (unpublished doctoral dissertation). University of Western Sydney.

Westwood P. (2011). The problem with problems: potential difficulties in implementing problem-based learning as the core method in primary school mathematics. *Australian Journal of Learning Difficulties*, 16 (1), 5–18.

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