Central skills in toddlers' and pre-schoolers' mathematical development, observed in play and everyday activities

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In good mathematical development, it is important to master some central skills at the kindergarten age. Being aware of such skills for each child is useful for facilitating children's learning and development. The present study examines how kindergarten children master central mathematical skills in toddler age and preschool age. The staff in kindergartens collected data by using structured observation on the basis of observational material *The Mathematics, the Individual and the Environments* (MIO). The areas examined were Mathematical language, Logical reasoning, Shape and space, Pattern and order, Counting and series of numbers and Enumeration. The children were observed in play and everyday activities in kindergarten in two three-month periods when they were 2 ½ years of age (toddlers, $n = 1003$) and two year later when they were 4 ½ (pre-schoolers, $n = 744$). The results show a large dispersion in the children's skills as toddlers, but as pre-schoolers, most of the children have a high level of mastery. The percentage of mastery at each of the observation times is reported and discussed in light of earlier findings. In our study, the children had slower development in the numerical area than was found in other research. Reflections about how the Norwegian kindergarten tradition and the method used to collect the data influence the results are included.

Mathematical skills are important for small children here and now, in play and everyday life (Sarama & Clements, 2009; Solem & Reikerås, 2008). Children make judgements about sizes to get the largest cookies, they develop their skills about space by making mental maps to find the way to grandma's house, they learn about sorting when tidy up their toys, they use bijections when handing out one cup to each doll, and so on.

Early mathematical skills are also the basis for further mathematical development and are found to be indicators for subsequent mathematical

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achievement (Aunola, Leskinen, Lerkkanen & Nurmi, 2004; Claessens, Duncan & Engel, 2009; Hannula-Sormunen, Lethinen & Räsänen, 2015). The interacting with the environment plays an important role for the children's learning and development (Sameroff, 2009; Vygotskij & Kozulin, 2001). Kindergartens, which serve as the Early Years Education and Care Institutions for children 1–5 years of age in Norway, therefore have a particular responsibility for facilitating and supporting children's mathematical learning and development. Early focusing on mathematics gives good results and is especially important for children with a weak starting point (Hannula, Räsänen & Lehtinen, 2007; Starkey, Klein & Wakeley, 2004).

Over the last several years, an increasing number of publications about the mathematical skills of young children has indicated a growing awareness of its importance (Tsamir, Tirosh & Levenson, 2011). However, few studies on early mathematics are performed within the Scandinavian kindergarten culture (Nielsen, Tiftikci & Larsen, 2013), and there is a lack of studies describing which skills children at different ages in kindergarten express in their daily lives in kindergarten. To facilitate learning for each child, it is helpful for the kindergarten teachers to have knowledge about what characterises typical mathematical development.

The aim of the present study is to investigate mathematical skills important for mathematical development, which can be observed in children's play and daily-life activities in Norwegian kindergartens when they are 2 years and 9 months old and how their skills have developed two years later, when they are 4 years and 9 months old. We wanted to look for central mathematical skills, which, in good development, should be learned before attending school. A more detailed study of the results from when the children were toddlers has been published previously (Reikerås, Løge & Knivsberg, 2012).

The Stavanger project

The present study is a part of the ongoing longitudinal and multidisciplinary *Stavanger project – The learning child*. The project is a collaboration between the Norwegian Centre for Reading Education and Research at the University of Stavanger and the municipality of Stavanger. The objective of the project is to generate research-based knowledge about children's development from the age of two-and-a-half years to ten years. The study focuses on language, mathematics, and social, emotional and motor behaviour, and the relations between these areas at the kindergarten age and reading, writing and arithmetic at school age. In a subsequent phase of the Stavanger project, we will analyse the predictive value of skills at the kindergarten age for reading, writing and arithmetic at school age.

Small children's mathematical development

There is comprehensive mathematical development in children's earliest years, and as toddlers, they have acquired skills in a broad field of mathematics (Reikerås et al., 2012).

Through the infant, toddler and preschool ages, children gain important experience and develop important concepts in the *geometrical* area (Geist, 2009; Solem & Reikerås, 2008). Through bodily and sensory experiences, the child develops spatial skills (Sarama & Clements, 2009). By the age of 16 months, children already begin to show the hierarchical combinations that characterise adult spatial coding (Newcombe & Huttenlocher, 2000). Toddlers are capable of orientating in their nearest surroundings and use their spatial knowledge when playing with blocks (Geist, 2009).

With regard to spatial language, there is a considerable development from toddler to preschool age. The first terms acquired in toddler age are "in", "on" and "under", which initially refer to one spatial relationship with another, whereas, e.g. navigation abilities, including following instructions with spatial words, are not common before late preschool age (Sarama & Clements, 2009).

Recognition of shapes is the tool children use to establish order in their surroundings by classifying and naming objects, and at kindergarten age, children develop concepts through tactile and visual senses of different shapes as they recognise, distinguish and classify (Geist, 2009; Oakes & Madole, 2003). For example, in puzzle-making children practice the recognition of parts of shapes and putting the parts together (Montford & Readdick, 2008). Drawing e.g., when copying simple figures, gives the child opportunities to recreate shapes. This can also include transference from the three-dimensional to the two-dimensional, as when drawing a tap-pole-man, which is found to be mastered in late preschool (Golomb, 2004). Throughout the toddler and preschool age, children gain experience with patterns and produce their own (Seo & Ginsburg, 2004). Recognition and analysis of patterns are also essential components in mathematical development, for geometry, arithmetic and algebraic thinking (Sarama & Clements, 2009).

Numbers and quantitative thinking is a central mathematical area in children's development. Infants can observe numerical changes (Cordes & Brannon, 2009; Wood & Spelke, 2005), and it has been extensively discussed whether small children use an inborn subitizing skill to perceive small quantities without counting whether they use to enumerate small sets (e.g. Mix, Huttenlocher & Levine, 2002). Regardless, children need a number of experiences to develop more conscious relations with quantities (Mix et al., 2002; Sophian, 2008). Through toddler age, the child develops important quantitative skills, such as making judgments of ”more” (Brannon, Abbott & Lutz, 2004), distinguishing between one and many (Spelke & Kinzler, 2007), and making bijections (Geist, 2009). Number words become a part of the child’s vocabulary (Durkin et al., 1986). At first they were only used as individual words before these words become part of saying the number sequence (Solem & Reikerås, 2008). Although seriation is reported to be only partly mastered before the age of 5 years (Sarama & Clements, 2009), Fluck (1995) found that most toddlers could recite the number sequence up to ten or beyond. In regard to answering correctly the question of ”how many” after counting objects, which also includes bijection, Sarama and Clements (2009) find that most five-year-olds can count up to 20 to 30 objects.

The understanding of the role of units in the numerical representation of quantities is complex. The different quantitative components seem to develop as separate skills before they are combined, and the development of quantitative skills occurs throughout the entire childhood period (Mix et al., 2002).

Problem-solving of realistic every day challenges and *logical reasoning* needed related to this, are found to be even more powerful facilitators of mathematical learning in early years than are specific skills of mathematical knowledge as e.g. enumeration and spatial skills (Perry & Dockett, 2002). Even very young children reason and draw conclusions if the task is motivating and the context familiar (English, 2004; Goswami, 2001). In toddler age, children solve problems and show their mathematics mainly through action because they still are not fully able to express themselves verbally (Björklund, 2008). Although the children’s bodily and sensory experiences are central for their mathematical development in early age, the interactive and linguistic context is also important for the learning of mathematics (Björklund, 2012). Adults and other children who use language to explain, e.g. sizes, shapes, numbers, relations, directions, and quantities surround the children. In preschool age, language is the most important way of expressing mathematics for most children (Solem & Reikerås, 2008). Mathematical language develops through interaction with acquisition of skills in most mathematical areas. Verbal counting is, for instance, central in quantitative development (Mix et al., 2002), and relational language makes a powerful contribution to spatial learning (Gentner, 2008). The children’s skills in the different mathematical areas are interconnected, and the need for research that covers more than one area in the child’s mathematical development to obtain knowledge on how overall mathematical performance develops was underlined previously (Aunola et al., 2004). The present study takes this need into account.

Most research on young children's mathematical competencies has been performed in cultures with a more academic approach to early childhood education than the Nordic, which has a dominating social pedagogical tradition with a holistic approach (Jensen, 2009; OECD, 2006). Differences in mathematics at early ages have been reported from cross-cultural studies (Aunio et al., 2008; Geary, Bow-Thomas, Fan & Siegler, 1993; Stevenson, 1987; Yuzawa et al., 1999), which indicate that the existing research results cannot be attributed directly to Norwegian children.

Observations of the childrens' interaction with their surroundings are important to obtain information on small children's mathematical competencies (Björklund, 2008; Piaget & Inhelder, 2002; Säljö, 2001; Vygotskij & Davydov, 1997), but to date, little research has been done on children's natural environments (English, 2004; Tudge et al., 2008). In a Norwegian kindergarten context, where play is the main pedagogical tool (Ministry of Education and Research, 2006), research in a natural environment means identifying children's competencies as they are expressed through play and daily life activities. Data obtained in this way are needed because much of our current knowledge is based on standardised assessment or surveys on reported behaviour (Downer et al., 2010).

Though there is a growing interest in research on small children's mathematics, few studies have surveyed the mathematical knowledge of children younger than five years of age (Sarama & Clements, 2009), and studies on mathematical development including children younger than three years of age are scarce (Björklund, 2008). There is a need for more research describing what mathematics is common at different age levels. Such knowledge can help staff in kindergartens facilitate learning for each child and help prevent children from weak development.

Based on the information presented, the following research questions were formulated:

1) What mastery do children aged 2 years and 9 months in Norwegian kindergartens have of central mathematical skills?

2) How have their skills developed after two years, when they are 4 years and 9 months old?

Design and method

Recruitment and participants

All public and private kindergartens of Stavanger municipality were invited to participate in the study. The 61 public institutions were obliged to participate in the project by their owner (Stavanger municipality), whereas the privately owned institutions were invited. Twenty-five of the private institutions (approximately 50%) accepted this invitation, which yielded a total of 86 participating institutions.

The parents of children born between July 1, 2005, and December 31, 2005, who attended one of the participating kindergartens received oral and written information about the project. They were asked for written consent for their child to participate in the study, in accordance with national research ethical standards. In addition to the period of birth, no other criteria excluded a child from participating in the study. The Norwegian Social Science Data Services approved the study.

When the present study was performed, data were available for 1003 of the children (490 girls, 513 boys) when they were toddlers and for 744 (368 girls, 376 boys) of them when they were pre-schoolers.

Instrument and procedure

The children's mathematical skills were assessed using the observation material of *The mathematics, the individual and the environments* (MIO) (Davidsen et al., 2008). The material was developed in Norway for the age range of 2 to 5 years and was constructed for use in kindergartens. It aims to assess skills central for good mathematical development at the kindergarten age. The skills are limited to include the skills that children are expected to master before school age. Natural situations and children's play activities are recommended as the main observational arenas. MIO consists of three main mathematical areas that are divided into six sections:

Geometry
- Shape and space
- Pattern and order

Numbers
- Counting and series of numbers
- Enumeration

Problem solving
- Mathematical language
- Logical reasoning

There is six items included in each section; a total of 36 items. The items in MIO are presented in the result section in table 1–3. Each item represent a specific skill the staff should looked for.

There are three levels of difficulty within each section, i.e., level 1, 2 and 3, with level 1 as the easiest and level 3 as the most difficult. There is a scoring manual for the observation scheme (Reikerås, 2008). It has three explanations for each observation item. The items should be scored on a three-level scale:

- Can do: The child shows competence in almost all situations.
- Can partly do: The child is beginning to show competence, situational or with help from staff.
- Cannot do yet: Competence not yet observed by the staff.

In the present study, only *Can do* is reported.

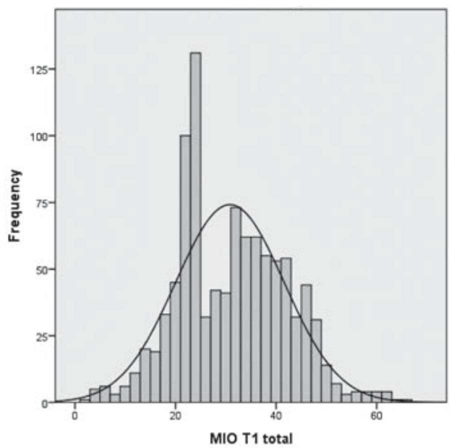
The children had to display mastery of the skill in at least two different settings before marking. In addition, two of the kindergartens' staff had to observe mastery of the item independently. This procedure strengthens our data.

A more detailed description of the Stavanger project, the material, procedure and so on can be found in Reikerås, Løge and Knivsberg (2012).

The data collection was performed by the staff. They observed the children's skills in natural settings in play and everyday activities. These data tell us what mathematical skills the children have mastered and what skills they are about to master, which make a foundation to facilitate learning. Since we have many participants, the children's results also gives us a reference frame of what level of mastery is common at different ages.

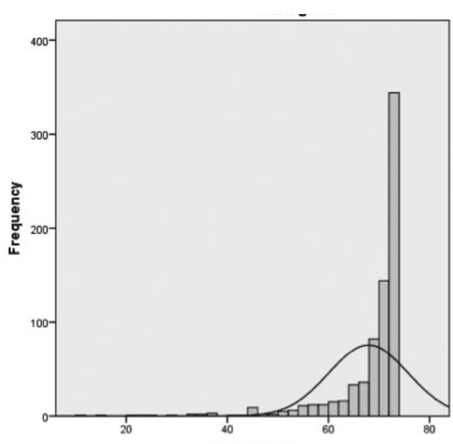
To say something about the dispersion of the results the observations were coded at three levels of achievement: 2 points for Can do, 1 for can partly do and 0 for cannot do yet. This gives a maximum total of 72 points for all items.

The method used to collect the data is gentle and non-intrusive for the children and is in accordance with the concept of authentic assessment (Bagnato, Neisworth & Pretti-Frontczak, 2010). Authentic assessment provides ecologically valid data and offers useful information about children's functioning, strengths and weaknesses (Keilty, LaRocco & Casell, 2009). Data obtained via authentic assessment are currently lacking because the majority of our current knowledge is based on standardised assessments or surveys on reported behaviour (Downer et al., 2010). For a better understanding of the development of early mathematical skills, there is a substantial need for additional research conducted in the natural environments of young children (English, 2004a; Tudge, Li & Stanley, 2008). Several studies have documented the advantages of authentic assessment compared with standardised assessment (Bagnato et al., 2010; Macy & Bagnato, 2010), and authentic assessment has become a recommended and accepted practice (Copple & Bredekamp, 2009). In the holistic Norwegian kindergarten context, play and everyday life activities are recognised as the most important educational practice (The Ministry of Education & Research, 2006). Therefore, a reliable and valid assessment of children's skills should focus on how they express and apply skills in their play and everyday life activities in kindergartens. Authentic assessment (Bagnato et al., 2010; Macy & Bagnato, 2010) represents an approach in which children's interactions with their natural physical and social environments becomes the core object of observation for providing



Mean = 30.82
S. Dev. = 10.795
n = 1,003

Figure 1. *The frequency distribution of the total score in MIO at toddler age and the mean and standard deviation*



Mean = 67.92
S. Dev. = 7.85
n = 744

Figure 2. *The frequency distribution of the total score in MIO at preschool age and the mean and standard deviation*

important insight into the development of young children's mathematical skills (Piaget & Inhelder, 2002; Säljö, 2001; Vygotskij & Kozulin, 2001).

Results

When summing up the total score on MIO for the toddlers, a large dispersion in scores appears, varying from 2 to 65, as illustrated in figure 1. The distribution of the total scores for the pre-schoolers is shown in figure 2. For this age group, we have a ceiling effect. The items observed in the present study are presented in tables 1–6. The tables also show

Table 1. *Percentage of children mastering each item in Shape and space*

Item/Level	2y 9m	4y 9m
Level 1		
Can point at different parts of the body (at least four: arm, foot, eye etc.)	97.1 %	99.6 %
Show that he or she distinguishes between different shapes. (e.g. fit the pieces into a peg board or use a sorting box)	89.4 %	98.5 %
Level 2		
Puzzles a jigsaw with 3–4 pieces into a picture	41.0 %	96.5 %
Can by request go to a fixed place in the room (e.g. The queen commands: "go to the doll corner")	54.4 %	98.3 %
Level 3		
Draws a human body (at least a head with eyes inside and legs, a "tadpole" man)	0.8 %	79.8 %
Copies simple figures (e.g. on paper, in the sand)	1.1 %	82.0 %

Table 2. *Percentage of children mastering each item in Pattern and order*

Item/Level	2y 9m	4y 9m
Level 1		
Places a picture on an identical picture (e.g. when playing a lotto game)	83.9 %	97.9 %
Is interested in rhythms and movement (e.g. "dancing", clapping hands)	89.6 %	96.1 %
Level 2		
Has knowledge about the daily routines (e.g. "When we have finished the meal, we shall go outside.")	44.2 %	96.5 %
Puts objects in a line according to size (e.g. cars, dolls)	14.8 %	89.0 %
Level 3		
Makes his or her own patterns (e.g. with beads, in drawings, while jumping)	0.7 %	73.3 %
Sorts objects according to one characteristic (e.g. shape, size or colour)	1.9 %	84.5 %

Table 3. *Percentage of children mastering each item in Counting and series of number*

Item/Level	2y 9m	4y 9m
Level 1		
Distinguishes between one and many	86.9 %	98.5 %
Uses number words (e.g. I have a thousand cars, my father is 10 years)	50.5 %	92.2 %
Level 2		
Has started pointing and at the same time using number words	32.8 %	95.3 %
Perceives number of objects up to three without having to count them	8.5 %	84.5 %
Level 3		
Counting to five while correctly pointing at objects	7.2 %	94.6 %
Can recite the number sequence up to ten	7.4 %	92.6 %

Table 4. *Percentage of children mastering each item in Enumeration*

Item/Level	2y 9m	4y 9m
Level 1		
Fetches two objects on request	82.9 %	98.1 %
Can hand out one item to each person	82.2 %	97.5 %
Level 2		
Fetches three objects on request	21.1 %	95.0 %
Shows with the fingers how old he or she is	29.3 %	96.0 %
Level 3		
Sets the table for five persons	1.1 %	83.4 %
Can answer how many there are after having counted five objects	1.8 %	87.5 %

Table 5. *Percentage of children mastering each item in Mathematical language*

Item/Level	2y 9m	4y 9m
Level 1		
Distinguishes between concepts large and small	92.5 %	98.8 %
Knows what is up and what is down (e.g. "Sit down on the floor")	92.0 %	98.4 %
Level 2		
Uses words that describe toys (e.g. the soft bear, the red car)	27.5 %	92.6 %
Follows instructions on placing (e.g. over the table, under the bench, through the tunnel)	45.9 %	94.5 %
Level 3		
Uses words describing relationship between sizes (e.g. the balloon is lighter than the stone)	1.6 %	81.8 %
Shows what is in the middle (e.g. with cars lined up in a row)	2.4 %	90.6 %

Table 6. *Percentage of children mastering each item in Logical reasoning*

Item/Level	2y 9m	4y 9m
Level 1		
Knows what outdoor clothing to put on when it is raining	79.1 %	96.4 %
Tidies toys and puts them in the right places	91.1 %	97.5 %
Level 2		
Can share equally with a friend (e.g. four or six crayons)	11.7 %	88.5 %
Fetches objects needed for an activity (e.g. hammer in the carpenter storage, doll in the doll corner)	46.9 %	97.6 %
Level 3		
Reasons out what comes first and last when dressing	3.9 %	89.1 %
Knows the difference between what has happened and what is going to happen	2.4 %	85.1 %

the percentage of children who master each item at the two assessment points.

Discussion

The aim of the study was to investigate how toddlers and pre-schoolers master central mathematical skills as observed by the kindergarten staff in children's play and everyday activities in three-month periods until the children become 2 years and 9 months and until they become 4 years and 9 months.

The distribution of the total score on MIO for the toddlers and for the pre-schoolers (figures 1 and 2) show a marked development of skills during the two-year period. The large dispersion in the total scores for the toddlers were expected due to the large number of participants and the fact that no exclusion criteria for participation had been set up. Because MIO examined central aspects of mathematical skills that are important to master before school age, it was expected that most of the pre-schoolers would have high MIO scores. This was also the case, as shown in figure 2. For the items in tables 1–6, we also find a comprehensive development from the first to the second assessment time.

In the section *Shape and space* (table 1), some of the tasks were easy for most toddlers, whereas others were difficult. The majority of the participants could already distinguish between different shapes at the first time of assessment. This was also the case for shape recognition when placing a picture on an identical picture. This corresponds with findings reporting that differentiating between and recognising shapes are important tools that infants and toddlers use to make order of their surroundings (Sarama & Clements, 2009).

Copying simple figures was much more difficult for the toddlers; 1 % had mastered this. This could be expected because it is a competence that, according to Piaget and Inhelder (2002), is mastered late in preschool age. Our findings at preschool age show that there still is a considerable amount of children who need even more time to develop this skill.

The rather complex transference of three-dimensional placing related to one's own body into two dimensions, which is mastered late in preschool age (Golomb, 2004), is also reflected in our study. Although almost every toddler could note the different body parts, the related item, drawing a human ("a tadpole man"), was difficult for most of them, even for approximately 20 % of the children two years later.

Approximately 40 % of the toddlers managed to make a picture out of a jigsaw with 3–4 pieces. The puzzle skills to the children in our study can be said to be better than in earlier research there such puzzle-making challenges involve recognition of parts of shapes are reported as more commonly mastered around the age of 4 years (Montford & Readdick, 2008).

The majority of the children were interested in rhythm and movement as toddlers, and such recognition of *pattern and order* (table 2) was also found in a study by Geist and Geist (2008). Although nearly half of the toddlers also recognised the pattern in the daily routines, making their own patterns was more difficult, having been mastered by less than 1 %. To make their own pattern was the item with the lowest percentage of pre-schooler mastery in our study.

Sorting is difficult for children under 3 years old (Sarama & Clements, 2009), and in our study, less than 2 % managed to sort objects according to one characteristic. As stated above, seriation is reported to be only partly mastered before the age of 5 years (Sarama & Clements, 2009). It is therefore not surprising that only approximately 15 % of the toddlers in our study could put objects in size order in a row. As pre-schoolers, 89 % had mastered this item.

The easiest item in *Counting and series of numbers* (table 3) was "distinguishing between one and many". The majority of the toddlers had mastered this, which is in line with other findings (e.g. Wynn, 1992). In the item using number words, the important aspect was to assess not whether the number words were used correctly but whether the children could incorporate some number words in their language. Only half of our toddlers used number words, which was a much lower result than Fuson (1988) reported. Also the percentage on the item "recite the number sequence up to ten" is lower than found in other studies. Fluck (1995) found that most toddlers could recite the number sequence up to ten or beyond. Only 7.4 % of our toddlers could, and among the pre-schoolers, there were 7 % who had not mastered this.

A third of the toddlers had started pointing at objects when using number words (not necessarily matching objects and number words), but less than 10 % carried this out correctly. This is in line with the results reported by Mix et al. (2002). As pre-schoolers, 95 % had mastered these items.

Overall, 8.5 % had mastered perceiving a number of objects up to three without having to count them (e.g. dots on a cube). It has been extensively discussed whether subitizing, perceiving small quantities without counting, is inborn and whether it is used to enumerate small sets (e.g. Mix et al., 2002). Our findings show that less than 10 % had mastered the recognition of numbers of objects up to three without counting supporting a step model.

In the section *Enumeration* (table 4), the step model is also supported by our results from the items where the children should fetch objects. The majority of the toddlers could fetch two objects on request, but fetching three was apparently much harder. One-fifth of the group mastered this, and nearly a tenth was about to gain this competence. For the pre-schoolers, these tasks seemed to be of equal difficulty. Wynn (1992) found that children learn the meaning of two approximately 9 months before the meaning of three. This learning has been characterised as a developmental step.

The majority of the toddlers in our study mastered making bijections, such as handing out one item to each person. This has also previously been reported (Geist, 2009; Sarama & Clements, 2009).

Nearly a third of the toddlers could also show with their fingers how old they were. Only 1 % had mastered setting a table for five, which is a more complicated task that combines several components. Very few, a little less than 2 %, of the toddlers were likewise capable of answering a question on how many there were after having counted five objects. The two latter items were also difficult for the children after 2 years, when they were pre-schoolers. These results reflect earlier findings on quantitative development (Mix et al., 2002; Mononen, Aunio, Koponen & Aro, 2014)

The first two items in *Mathematical language* (table 5), "distinguishing between large and small" and "knowledge about up and down", were easy for most of the toddlers. This is in line with earlier findings (Sarama & Clements, 2009).

Approximately half of the children also managed to follow instructions with spatial words as toddlers. This requires navigation abilities and in earlier research found to not be usually fully mastered before later preschool age (Sarama & Clements, 2009). Nearly a third of the toddlers naturally used words that describe toys, but very few used words for relations between sizes, which was also the case when they were asked to show what was in the middle.

This corresponds with earlier research reporting that although toddlers show problem-solving strategies, they are not yet fully able to express themselves verbally (Björklund, 2007). Somewhat unexpected was that 18 percent of the children still had not mastered describing relationships after 2 years. Because cross-cultural language differences are related to learning mathematical language (Choi & Bowerman, 1991), more research on how Norwegian children acquire mathematical language is needed.

The easiest item in *Logical reasoning* (table 6) was tidying toys and putting them in the right places; over 91 % had mastered this. Nearly half of the toddlers could also fetch objects they needed in their activity. Little research has been conducted on these types of competencies. The Norwegian tradition, which emphasises free play and activities initiated by the children versus other cultures with more academic traditions, may cause cultural differences that should be further explored.

Only a tenth of the toddlers could share equally with a friend (it was underlined in the handbook that this was not about would but could), but two years later, this percentage was near 90. Nearly 80 % of the children in the youngest age group knew what outdoor clothing they needed when it was raining, but determining what came first and last in dressing was much more difficult. Even two years later, 10 % of children had still not fully mastered this. Likewise, very few in the youngest age group knew the difference between what had happened/what was going to happen. Both these items include seriating. This is in line with several studies reporting that seriating is only partly mastered before the age of 5 years (Sarama & Clements, 2009).

Though the results in our study are mostly in line with earlier research, the results in the number sections are somewhat lower than expected. We cannot draw conclusions about causality from our data but can only reflect on this. The weak results for using number words and reciting number sequences in toddler age and the fact that there is still a group of children who cannot recite the number sequence even as pre-schoolers may reflect the idea that the kindergartens in Norway do not emphasise counting and numbers in the same way as in countries from which previous research has been reported. Using number words and reciting the number sequence are skills that are first learned as language, independent of its content (Huttenlocher, Vasilyeva, Cymerman & Levine, 2002). Children who are in a rich language environment that also includes mathematical language develop more complex language skills and acquire them at a faster pace than do children who are involved in less linguistically stimulating interactions (Huttenlocher, Vasilyeva, Cymerman & Levine, 2002). The children's results in our study indicate that quantitative language is not emphasised in particular in Norwegian kindergartens. This should be focused on more because earlier research has reported that the numerical area is critical for mathematical development (Locuniak & Jordan, 2008; Mazzocco & Thompson, 2005).

The importance of early mastery in geometry and problem solving for children's further mathematical development is still unknown (Gersten, Jordan & Flojo, 2005). It should also be noted that previous research in geometry and problem solving in general is especially limited (Dowker, 2005; Sarama & Clements, 2009). The only results in our study that were somewhat better than those previously reported were puzzle-making and following instructions on spatial words. Whether the emphasis is better on geometry and problem solving than numbers in Norwegian kindergartens is not known. Though mathematics is one of seven learning areas in the Norwegian kindergartens (Ministry of Education and Research, 2006), a study of 1000 kindergartens (Gulbrandsen & Eliassen, 2013) found that barely one half of them said that they emphasise mathematics to a large degree.

When comparing the children's results in our study with earlier research, it should be noted that most of these studies have much fewer participants and were performed with other assessment methods. Assessment methods might influence the results (Tudge et al., 2008). It has been assumed that small children in play and daily life activity show more competence than they do in clinical tasks (Baroody, Lai & Mix, 2006; Mix et al., 2002). This was not found in the current study; most of the results were at the same or lower level than the results from clinical studies. The assumptions of better results in a natural setting might have been due to the supposition that clinical tasks were more stressful than tasks in natural settings or that observations in natural settings could easily be biased, with the observer noting more or less what he or she wanted to see. In a way it also could be seen as the staff evaluating their own work with mathematics in kindergarten when collecting the data, and they may consequently want to report somewhat better results. The differences in results do highlight the need for further research to be done. Assessments that are done in clinics and natural settings are, however, different; consequently, such comparisons should be treated and interpreted with care. In our study, for example, the children had to display mastery of a skill in at least two different settings. In addition, two of the kindergartens' staff had to observe this mastery independently. We consider this procedure to strengthen our results, and we think that our study has contributed to a realistic picture of the children's mathematical competencies.

The skills examined are, as noted above, skills that are central for kindergarten age as a basis for learning in school; thus, most pre-schoolers receive high scores. A set of assessment material without this ceiling effect would give us a better overview of what children at that age level master beyond the skills that are assessed by MIO. Though there was a ceiling effect at the second time of assessment, there were still preschoolers who had not mastered even the easiest items. This and the large differences in gained competence observed at toddler age illustrate and underline the kindergartens' challenges; every child is to be given opportunities to prosper. The use of MIO together with our results can help kindergarten teachers be aware of where the child needs support in his development.

The results also naturally raise the research question concerning low performance appearing already in toddler age as an eventual predictor of future difficulties in arithmetic. Research in these areas is needed. The Stavanger project, as the present study is a part of, may contribute interesting information over a longer timeframe because the same children will be followed until they are 10 years old. In this study, the focus was directed only at the children's mathematics. However, it has been reported that development in mathematics at early ages is related to other developmental areas, which is also being studied in the Stavanger project to shed light on children's early development (Reikerås, Moser & Tønnessen, 2015; Reikerås, 2013).

The present study contributes to a foundation for a better understanding of children's early development and further research into small children's competencies. Future research may give information on whether lower or higher results are due to different data collecting methods, clinical trials versus authentic assessment, or cultural differences reflecting different pedagogical traditions.

In the present study, the focus is on how toddlers and pre-schoolers master central mathematical skills. This gives us only part of the picture of the child's mathematical development and learning. Though our data are collected in a kindergarten context, where the children interplay with other children and adults, more knowledge about the children's mathematical development should be obtained in a larger context. We have no data on how kindergartens facilitate learning, the processes by which the children learn, the kindergarten teachers' skills, and so on. More research is needed on these important aspects of mathematics in kindergarten.

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