Professional development in early mathematics: effects of an intervention based on learning trajectories on teachers' practices

JULIE SARAMA, DOUGLAS H. CLEMENTS,
CHRISTOPHER B. WOLFE AND MARY ELAINE SPITLER

We evaluated the effects of a research-based model for scaling up educational interventions on teachers' practices in preschool mathematics. The original participants were from 106 classrooms for 4-year-olds in two distal city districts serving low-resource communities, with 42 schools randomly assigned to one of three groups, of which the two treatment groups were the same throughout preschool (thus, there were 72 treatment classrooms). The intervention, a professional development program based on young children's mathematical learning trajectories, had a substantial positive effect on teachers' instructional practices, some of which mediated student outcomes. Teachers also demonstrated sustained levels of fidelity as long as six years after the end of the intervention. Notable is these teachers' encouragement and support for discussions of mathematics and their use of formative assessment. Finally, teachers taught the curriculum with increasing fidelity over the following six years without support from the project.

Teaching is a complex enterprise, and teaching mathematics is particularly complex (National Research Council, 2009). Further challenges confront teaching mathematics in preschool, as settings and organizational structures vary far more than do those at any other age level in the U.S. (National Research Council, 2009). The workforce in those settings, their backgrounds, and their professional education are similarly diverse. Because research suggests that the most critical feature of a high-quality educational environment is a knowledgeable and responsive adult and that high-quality professional development is essential to innovation (e.g. National Research Council, 2009; Sarama & DiBiase, 2004; Schoen, Cebulla, Finn & Fi, 2003), scaling up professional development has special

Julie Sarama, *University of Denver*
Douglas H. Clements, *University of Denver*
Christopher B. Wolfe, *Saint Leo University*
Mary Elaine Spitler, *University of Buffalo, SUNY*

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challenges in preschool contexts, including the diverse workforce, the equally diverse knowledge of teachers, and the low level of mathematics content and pedagogical content knowledge of most preschool teachers (Copley, 2004; Sarama & DiBiase, 2004).

In the present study, we evaluated the effects of a research-based model for scaling up educational innovations on teachers' practices in early mathematics in the short and long term. We also evaluated whether teachers' practices mediated the effects of the intervention on students' outcomes, including both mathematics and transfer to language and literacy competencies.

Background and need

Recently there have been calls for additional empirical studies of the effects of professional development, including the effects on teachers' practices (Ball, Thames & Phelps, 2008; Desimone, 2009). Researchers have called for more observational studies of actual teacher practices in the classroom to complement self-reports (Fishman, Marx, Best & Tal, 2003), more sophisticated tools for measuring the effects of professional development (Desimone, 2009; Ingvarson, Meiers & Beavis, 2005), more testing of specific features of professional development (Wayne et al., 2008), examinations of the long-term effects of professional programs (Antoniou & Kyriakides, 2013; Avalos, 2011) and in-depth studies of specific instructional practices that can form a common core of practices to align with a common core for teaching (Ball & Forzani, 2011). The growing international focus on the use of children's learning trajectories in professional development programs, the need for continuing research on the effects of professional development on teacher practices, especially in the preschool, and the recent intense attention to the instructional practices of learning trajectory based instruction motivate this paper.

The need for professional development is indicated by teachers' knowledge, beliefs, and practices regarding mathematics. Some researchers have noted the integration, interrelationship, and complex interplay between beliefs and practices (Clarke & Hollingsworth, 2002; Fennema et al., 1996) and inconsistencies between them (Einarsdottir, 2003). Several have described the bidirectional, cyclic nature of beliefs and practices, as beliefs influence practice and practice influences beliefs (Clarke & Hollingsworth, 2002; Fennema et al., 1996; Guskey, 2002). In this paper we distinguish between the two, and focus on practices.

The critical interaction in education is between the teacher and the student, with student learning reliant upon teacher instructional practices (Ball & Forzani, 2011). Professional development programs for preschool teachers seek to improve teacher practices and thereby, student achievement. There is growing evidence of the effects of professional development on changes in teachers' instructional practices, including increased use of problem solving, increased teacher attempts to encourage student discussions of strategies, and increased listening to students (Borko, 2004). Researchers have also found increased use of hands-on activities, increased emphasis on thinking strategies, increased efforts to challenge and extend children, more effective use of materials, more feedback for students, increased engagement of students in higher order thinking, and increased use of formative assessment (Bobis et al., 2005; Borko, 2004; Fishman et al., 2003; Ingvarson et al., 2005). In one long-term study of the effects of professional development, 77 % of participants changed and sustained at least one teaching practice (Boyle, While & Boyle, 2004).

Theoretical framework

Although core features of effective professional development have been identified and a consensus reached (Desimone, 2009), evidence for this set of features is still weak (Wayne et al., 2008). Features include active learning; focus on subject matter content and how children learn; coherence linked to school/state standards; attention to teacher beliefs and prior knowledge; emphasis on training that is ongoing, continuous, and embedded in classroom/school, with feedback; and review of student work within communities of learners (Desimone, 2009; Garet et al., 2001; Sarama & DiBiase, 2004). Currently there is increased interest in the critical role of instructional practices in the teaching/learning process (Ball & Forzani, 2011; Penuel, Confrey, Maloney & Rupp, 2014).

Use of theory, demonstrations, practice, and feedback, especially from coaches, increases the positive effects of information-only training (Foorman, Santi & Berger, 2007; Pellegrino, 2007; Showers, Joyce & Bennett, 1987). Effective professional development eschews "one-shot" interventions, begins with a specific strategy or curriculum, and weaves together content, pedagogy, and knowledge of child development and family relationships (Schoen et al., 2003; Sowder, 2007).

During the last two decades, interest in incorporating knowledge of children's mathematical thinking into professional development through learning trajectories (also termed growth points, learning progressions or cognitively guided instruction) has grown (Bobis et al., 2005; Fennema et al., 1996; Franke, Carpenter, Levi & Fennema, 2001; Sztajn, Confrey, Wilson & Edgington, 2012; Wilson, Sztajn, Edgington & Confrey, 2014). This is the reason that we placed research-based learning trajectories at the core of our theoretical framework (Clements, Sarama, Spitler et al., 2011; Sarama et al., 2008). Learning trajectories are ”descriptions of children's thinking and learning [...] and a related, conjectured route through a set of instructional tasks” (Clements & Sarama, 2004, p. 83). Thus, learning trajectories (LTs) have three components: a goal (that is, an aspect of a subject-matter domain students should learn); a developmental progression, or learning path along which students move through levels of increasingly sophisticated thinking; and instructional tasks and strategies that helps them move along that path. These three components map directly on what teachers must know and be able to do to be effective. Knowing the goals of LTs means understanding the mathematical content they are to teach, and in a different and deeper way than often presented in textbooks or standards (e.g. Hill, Rowan & Ball, 2005; Ma, 1999). Knowing the developmental progressions means understanding how their students think and learn about that mathematics, including possible misconceptions and creative pathways they may take in learning (Ball & Forzani, 2011; Clements & Sarama, 2014a; Sztajn et al., 2012) and how to assess the understandings and competencies of their students. Knowing the instructional tasks and strategies means knowing how to present, represent, and discuss that content (e.g. Ball & Forzani, 2011; Shulman, 1986). Knowing the interconnections among the three components means being able to use knowledge of students to plan and modify instruction using research-based instructional strategies (e.g. Ball & Forzani, 2011; Shulman, 1986). Possessing such integrated knowledge may be one way to prevail over many preschool teachers' dislike of mathematics (Sarama & DiBiase, 2004) and their careless attitude towards the subject (e.g. declaring that mathematics is ”covered” by providing materials and incidental exposure, Lee & Ginsburg, 2007).

The scale-up model based on our theoretical framework is called TRIAD, for *Technology-enhanced, research-based, instruction, assessment, and professional development*. The model's acronym suggests that successful scale-up must address the triad of essential components of any educational intervention and that the model is based on research and enhanced by the use of technology. The 10 research-based guidelines in the TRIAD model are described in other publications (Clements, Sarama, Spitler et al., 2011; Sarama & Clements, 2013; Sarama et al., 2008). The critical guideline, arguably for the fidelity of implementation, but also for the research questions addressed in this article, is the following.

Provide professional development that is ongoing, intentional, reflective, goal-oriented, focused on content knowledge and students' thinking, grounded in particular curriculum materials, situated in the classroom and the school. A focus on content includes accurate and adequate subject-matter knowledge both for teachers and for students. A focus on students' thinking emphasizes the LTs' developmental progressions and their pedagogical application in formative assessment (i.e., if students are not progressing, specific strategies and additional time with the teacher are suggested). Grounding in particular curriculum materials should include all three aspects of LTs, especially their connections. What the teachers know about mathematics and the learning and teaching of mathematics predicts the quality of their teaching (National Mathematics Advisory Panel, 2008). Learning trajectories also provide a shared language for teachers in working with each other and other groups (Bryk et al., 2010). Situated in the classroom does not imply that all training occurs within classrooms. However, training outside of the classroom remains focused on and connected to classroom practice and is completed by classroom-based enactment with coaching. In addition, the guideline indicates that professional development should be designed to encourage sharing with colleagues, prepare teachers to implement a well-described curriculum, and formatively evaluating teachers' fidelity of implementation (Bryk et al., 2010; Guskey, 2000; Klingner, Ahwee, Pilonieta & Menendez, 2003; Pellegrino, 2007; Schoen et al., 2003; Showers et al., 1987; Sowder, 2007).

TRIAD's theoretical framework postulates that effects on students occur through teachers' practices and beliefs (see figure. 1, cf. Romberg, 1992; Sarama & Clements, 2013). Thus, the positive outcomes of interventions such as TRIAD (e.g. Clements, Sarama, Spitler et al., 2011) are primarily realized through their effects on teachers. This study examines the effects of TRIAD on teachers' practices.

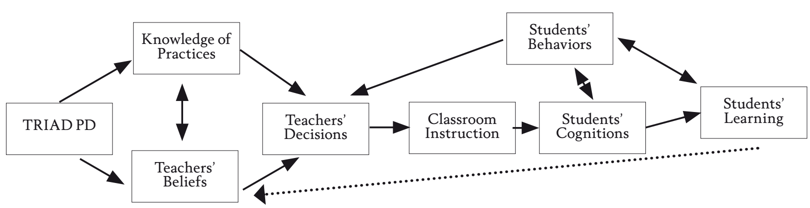


Figure 1. *Model for research and curriculum development*
(the dotted arrow indicates a hypothesized path that is not tested in this study)

We also report on sustainability, by which we mean the continued use of the resources provided in the TRIAD intervention over time, with a focus on the maintenance of core beliefs and practices (cf. Baker, 2007), the need for evaluations of sustainability past the implementation of the intervention, especially by developers or researchers (Baker, 2007;

McDonald, Keesler, Kauffman & Schneider, 2006), and the "shallow roots" of many reforms (Cuban & Usdan, 2003).

In summary, there is a critical need for professional development in preschool mathematics education, and for measurement of the impact of professional development on teacher practices, and, ultimately, student outcomes. This study is part of a larger study of an implementation of the TRIAD model in the domain of early mathematics (Clements, Sarama, Spitler et al., 2011; Clements, Sarama, Wolfe & Spitler, 2013, 2015; Sarama, Clements, Wolfe & Spitler, 2012; Sarama, Lange, Clements & Wolfe, 2012). In this study we examined the effects of this learning trajectory-based professional development program on teachers' practices in their classrooms. We hypothesized that (a) an intervention focused on knowledge of and use of research-based learning trajectories would cause teachers to score higher than control teachers on an observational measure of high-quality mathematics education practices, and (b) that the increase in specific practices related to the use of learning trajectories would account in part for student achievement. We also hypothesized that (c) this comprehensive intervention would result in lasting fidelity to these teaching practices.

Methods

Participants

The original participants were from 106 (72 treatment) preschool (4-year-old) classrooms that were embedded in elementary schools in two urban public school districts in the U.S. At the time the final data set was collected, two years after the 72 treatment preschool teachers participated in the study, 14 of the 72 had retired or were assigned to a different grade level; 64 was the maximum number of teachers with data by time-point, by subsection ($M = 2.4$ per school). Data from 64 all-but-one female teachers in 26 schools across both sites are included here. About 89 % had a Masters' degree or higher. They had an average of 17 years of teaching experience (range 1 to 33, *SD* 8), and their average number of years with preschool teaching experience was 6.8. About 94 % believed themselves to have the support of their principals. As a measure of morale, 76 % of the teachers believed that "staff members in this school generally have school spirit." The students they taught were four-year-olds (51 % female) of mixed ethnicity (53 % African American, 21 % Hispanic, 19 % White, 3.7 % Asian Pacific, 1.8 % Native American, and .6 % other). Most (82.33 %) received free or reduced lunch, 13.5 % had limited English proficiency, and 10 % had an IEP. The original randomized pool included 1,375 children. Treatment was assigned at the school level, randomly, by site. All preschool teachers in each school participated; each teacher implemented the intervention for all students in the class, however, data were collected on a maximum of 15 per class ($M = 13.6$ per classroom). District mandated maximum class sizes differed by site (18 at site 1, 24 at site 2).

Procedure

Schools were randomly assigned to one of three groups, of which the two treatment groups were the same throughout preschool (one of the two had follow through in the subsequent grades); the third was a control group. The 72 treatment classrooms received *Building blocks* training using the TRIAD model. Preschool teachers within these schools were notified of their group assignment in the prior year, and teachers in the TRIAD groups received appropriate training, and taught the *Building blocks* curriculum, substituting those activities for the district's mathematics activities, whereas teachers in the control group taught the regular district mathematics curriculum (see below) without involvement from the research team. The first year of the project involved teacher training and classroom implementation only. In the second year, students received a pre- and post-assessment on their early mathematics knowledge and skills using the TEAM (Clements, Sarama & Wolfe, 2011), and teachers continued to engage in professional development sessions and classroom observations. Students also were assessed on an assessment of expressive oral language competencies, using the *Renfrew bus story* (RBS) (Glasgow & Cowley, 1994) approximately five months following the mathematics posttest, in fall of their kindergarten year.

Assessors trained especially for the project administered all assessments. Project assessors were primarily masters-level retired preschool or elementary school teachers or graduate students in education with experience working with students. Each assessment involved specialized training including background information on the measure, administration procedures, and practice on administration. Specifically, each assessor was required to complete video recordings that were coded in-house by senior project staff. Similar training was provided to coders. Assessors and coders needed to achieve a level of item administration or item coding of 98 % accuracy or higher to become certified, and those who did not meet the criteria were not selected for these tasks. Assessors administered the measures to students individually in an open space (e.g. library, hallway) within the school. All assessments were recorded to facilitate scoring.

Intervention / Professional development and control group

Teachers participated in 8 days of professional development during the school day in the first year and 5 days in the second year, learning each of the three components of the *Building blocks* learning trajectories. To understand the goals, staff presented the core mathematics concepts and procedures for each topic. For example, they described the system of verbal counting based on cycling through 10 digits and the concept of place value (based on content similar to that presented in National Research Council, 2009). To develop understanding of the developmental progressions of levels of thinking, teachers were taught the mental ”actions on objects” that constitute the defining cognitive components of each level.

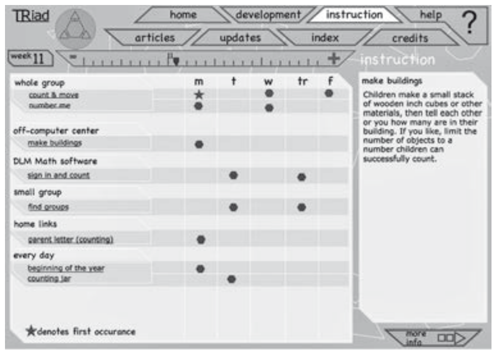
Training

Consistent with research, the TRIAD professional development included brief presentations followed by tasks and small-group work. As an example from geometry, teachers explored the *Building blocks shape set*, a collection of widely varying geometric figures. They discussed research on preschoolers' understanding of shape, then sorted the shapes by attributes, sometimes using Venn diagrams, as well as participating in other activities such as ”back-to-back,” in which one teacher chooses a shape from one shape set at random and then must describe it without using its name sufficiently that another teacher can choose it from a matched set. They constructed shapes from ”parts” (e.g. sticks and angle connectors) and by composing and decomposing regions. The sessions continued to provide hands-on experiences in rooms set up to mirror the structure of preschool classrooms, with an emphasis on interactions with colleagues around common issues.

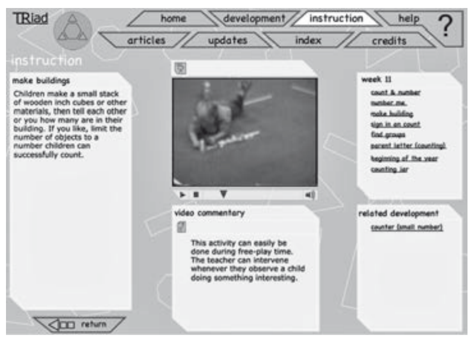
Technology support

To further develop teachers' competence teaching with LTs, a main activity used a technological tool, the *Building blocks learning trajectory* (BBLT) web application. BBLT provides scalable access to the LTs via descriptions, videos, and commentaries (see figure 2). Each level of each *developmental progression* is connected to correlated instruction. That is, teachers might choose the ”instruction” view, then click an activity and not only see an explanation and video of the activity ”in action,” but also immediately see the level of thinking that activity is designed to develop, in context of the entire LT. In this way, the BBLT connects the components of the innovation, encouraging teachers to view the LTs from both a developmental and pedagogical perspective. Finally, teachers discussed and practiced how to use LTs as formative assessment – interpreting students' thinking and selecting appropriate instructional tasks for the class (e.g. compacting the curriculum if most students can learn it at a faster pace) and for individuals (e.g. assigning students to small groups or modifying activities within groups to match instructional tasks to developmental levels of individual students).

The user reads the description that appears on the right. If she chooses "More info" the screen "slides over" to reveal the expanded view shown below.



Here she can see multiple video examples, with commentary. Clicking on the related developmental level (student's level of thinking), yields the view on the next page.



This developmental view likewise provides a description, video, and commentary on the developmental level—the video here is of a clinical interview task in which a student displays that *level of thinking*.

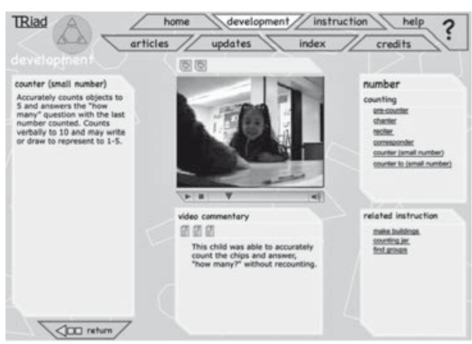


Figure 2. *Building blocks learning trajectory (BBLT) web application*

Coaching

As mentioned, training does not work well without coaching. In the classroom, coaches worked with teachers throughout the year, visiting teachers in their classrooms no less than once per month, usually twice, discussing students' learning and teaching based on the LTs. In addition, project coaches observed and provided support to teachers and completed implementation fidelity evaluations. Coaches participated in the same professional development as the teachers. Before this, they participated in an additional day of professional development on coaching and administering the Fidelity instrument, conducted by project staff. Additional meetings for coaches occurred throughout the year. They then worked with teachers throughout the remainder of the project, visiting teachers in their classrooms about twice per month.

Control group

In both districts, there was a greater focus on mathematics during the study period than there had been in prior years, due to the introduction of new comprehensive programs, including professional development on those programs, that included mathematics. The first district implemented *Where bright futures begin* (Bredekamp, Morrow & Pikulski, 2006), featuring 10 thematic segments (e.g. Animals everywhere) that included considerable mathematics. Topics were geometry and spatial sense, patterns, time concepts, measurement, classification and data collection, numbers and operations, problem solving, reasoning, and communication, and mathematics materials included 34 mathematics concept cards, as well as everyday mathematical classroom objects, such as counters and cubes. Mathematics activities were taught primarily during small group time, although sometimes during whole group instruction. The second district introduced O*pening the world of learning* (Schickedanz, Dickinson & Charlotte-Mecklenberg Schools, 2005) that included mathematics, usually in small group activities. Topics included number concepts, number words, one-to-one-correspondence, cardinality, basic computation, geometry, and measurement; domains consisted of number sense, numeration, spatial sense, measurement, geometry, and patterns.

Measures

For purposes of this paper, we analyzed data from two classroom observation instruments: a classroom administered instrument used to measure the mathematical environment and teacher/student interactions, a classroom administered fidelity instrument.

Classroom observation

Classroom observation of early mathematics – environment and teaching (COEMET), was created based on a body of research on the characteristics and teaching strategies of effective teachers of preschool mathematics (e.g. Clarke & Clarke, 2004; Clements, Sarama & DiBiase, 2004; Fraivillig, Murphy & Fuson, 1999). The COEMET measures the quality of the mathematics environment and activities with an observation of three or more hours and is not connected to any specific curriculum. Thus, it allows for intervention-control group contrasts, no matter what the source of the enacted curriculum. There are 31 items, all but four of which are 5-point Likert scales. An example of one of the three items in the section ”Personal attributes of the teacher” is, ”the teacher appeared to be knowledgeable and confident about mathematics (i.e., demonstrated accurate knowledge of mathematical ideas and procedures, demonstrated knowledge of connections between, or sequences of, mathematical ideas).”

Assessors spend no less than a half-day in the classroom, for example, from before the students arrive until the end of the half-day (e.g. until lunch). All mathematics activities are observed and evaluated, without reference to any printed curriculum. The COEMET has three main sections, classroom elements, classroom culture, and specific mathematics activities (SMA). Assessors complete the first two sections once to reflect their entire observation. They complete a SMA form for each observed mathematics activity, defined as one conducted intentionally by the teacher involving several interactions with one or more students, or set up to develop mathematics knowledge (this would not include, for instance, a single, informal comment). Inter-rater reliability for the COEMET, computed via simultaneous classroom visits by pairs of observers (10 % of all observations, with pair memberships rotated) was 88 % (i.e., 88 % of the 27 Likert items were coded the same by both assessors); 99 % of the disagreements were the same polarity (i.e., if one was agree, the other was strongly agree). Coefficient alpha (inter-item correlations) for the two instruments ranged from .95 to .97 in previous research (Clements & Sarama, 2008; Clements, Sarama, Spitler et al., 2011). Maximum possible scores for each Likert-based subtest are as follows: classroom culture total score, 45; SMA total score, 95; and verbal interaction scale, 50. The ranges for the remaining subtests were as follows: 8.2–92.5 min., time-on-task; 1.5–14, number of math activities; 0–6, number of computers students were using to engage with the intervention’s software.

Fidelity observation

The main data source for studying sustainability of the TRIAD intervention was a classroom observation measure, rather than teacher self-reports, dubious instruments which make up the main body of evidence of sustainability (Timperley, Wilson, Barrar & Fung, 2007). Based on the same research base as the COEMET, the Fidelity instrument was designed to evaluate whether or not, and to what degree, teachers were faithfully implementing the specific components of the implemented curriculum – thus this measure could only be reasonably applied to intervention classrooms (Clements, Sarama, Spitler et al., 2011). Comprised of multiple subsections, observers rate agreement with curricula based statements on a 5-point scale Likert (0 = neutral/not applicable). The General curriculum subscale includes 5 items for a possible total of 14, and a range in reliability estimates (Cronbach's alpha, a measure of internal consistency, that is, how closely related a set of items are as a group, $\alpha = .60 - .66$). The Whole group activity subscale includes 7 items for a total of 28 points and a range in reliability estimates ($\alpha = .81 - .83$). The Small group subscales includes 25 with a possible total of 75 and a range in reliability estimates from $\alpha = .90 - .95$. Finally, the Computer subscale includes 12 items for a possible total of 40 and ranges in reliability from .73 to .84.

An example of the Likert scale items within each subscale, with potential answers varying from "Strongly disagree" to "Strongly agree" is: "The teacher facilitated students' responding: e.g. elicited many solution methods for one problem, encouraged elaboration of students' responses, waited for and listened attentively to individual students, responded to errors as learning opportunities." Project coaches collected data using this instrument, after participating in several training meetings and conducting inter-rater reliability visits over two years. Inter-rater reliability, determined the same way as the COEMET, was 95 %.

Results

Classroom observations – the COEMET

As reported previously (Clements, Sarama, Spitler et al., 2011), TRIAD classes had higher scores than the control classes on the Classroom Culture subscale ($g = 1.23$), SMA subscale ($g = .78$), total number of math activities observed in SMAs ($g = 1.02$), and the number of computers on and working for students to use ($g = 0.90$). The substantial difference in the number of math activities observed in SMAs raised the issue of whether this variable was a proxy for total time allocated to math activities. However, the mean time on task was 27 minutes for control, and 32 minutes for TRIAD, which was not significantly different (Clements, Sarama, Spitler et al., 2011).

Mediation

Previously, we tested the mediational role of teacher's processes at the global level. Here we review those results and use the new analyses to extend them. The mediational hypothesis was that the COEMET components are influenced by the TRIAD intervention and they in turn cause changes in the outcome variable. Three scores derived from the COEMET mediated the effects of TRIAD (Clements, Sarama, Spitler et al., 2011). The classroom culture component, the total number of computers on and working for students, and the total number of math activities significantly, partially mediated the impact of treatment group on math and language student outcomes. These analyses utilized variance estimation across three levels (school, classroom, and child) and represent aggregated totals reflective of larger conceptual chunks. These large-grain size construct differences between treatment and control groups provide support for the efficacy of the TRIAD program in its totality. The specific practice changes enacted by treatment teachers relative to control teachers, however, have not been investigated. The current analysis seeks to expand on the change in teacher practices at the individual level, thus describing and highlighting the importance of exposure to high quality professional development in mathematics.

Analyses of individual COEMET items

More detailed analyses for this article showed a pattern of individual items that accounted for differences on the three components. Given the focus on change within individual items, a series of independent samples *t*-tests were conducted comparing the two research groups on the individual items within each measure. A Bonferoni correction to account for multiple tests of significance was utilized. Significantly higher scores on aspects of the mathematics environment within the classroom were found for the experimental group as compared to control (table 1). One striking distance is the difference in computer usage between the treatment and control groups on item 4 in table 1. Another set of significant differences can be found in how the teacher interacted with the student. Teachers in the experimental groups were found to demonstrate more to "support listener understanding" (item 23) and "observe, listen(ed), and take notes" (item 27).

Table 1. *COEMET items – mean of time 1 and time 3*

Classroom observation of early mathematics environment and teaching	Control	Experi-mental
Classroom culture		
Environment and interaction		
1. Teacher actively interacted	4.71 (.552)	4.93 (.211)*
2. Other staff interacted	4.07 (.930)	4.63 (.563)*
3. Used teachable moments	3.31 (.879)	3.80 (.668)*
4. Students used math software	2.10 (1.37)	4.01 (1.06)*
5. Environment showed signs of math	3.44 (.814)	3.93 (.492)*
6. Student math work or thinking on display	3.07 (.993)	3.36 (.698)
Personal attributes of the teacher		
7. Teacher knowledgeable about math	3.79 (.579)	4.03 (.359)
8. Teacher showed she believed math learning can be enjoyable	3.63 (.721)	3.97 (.492)*
9. Teacher showed curiosity/enthusiasm for math	3.36 (.829)	3.76 (.628)*
Specific math activity		
Mathematical Focus		
10. Teacher understanding	3.94 (.220)	4.00 (.193)
11. Content developmentally appropriate	3.94 (.291)	4.00 (.234)
Organization, teaching approaches, interactions		
12. Engage mathematical thinking	3.69 (.460)	3.96 (.729)
13. Pace appropriate for developmental level	3.87 (.417)	3.98 (.193)
14. Management strategies enhanced quality	3.82 (.472)	3.96 (.249)*
15. Percent teacher involved in activity	4.70 (.451)	4.73 (.403)
16. Teaching strategies developmentally appropriate	3.81 (.498)	3.97 (.245)*
Expectations		
17. High but realistic expectations of students	3.77 (.451)	3.93 (.263)*
18. Acknowledged or reinforced effort of students	3.87 (.325)	4.01 (.184)*
Eliciting students' solution methods		
19. Asked students to share ideas	3.37 (.730)	3.73 (.418)*
20. Facilitated students' responding	3.65 (.621)	3.89 (.317)*
21. Encouraged students to listen/evaluate thinking of others	3.25 (.648)	3.49 (.532)*
Supporting students' conceptual understanding		
22. Supported describers thinking	3.57 (.567)	3.79 (.372)*
23. Supported listeners understanding	2.99 (.678)	3.39 (.584)*
24. Just enough support provided	3.71 (.526)	3.93 (.252)*
Extending students' mathematical thinking		
25. Elaborated math ideas of students	3.21 (.589)	3.57 (.485)*
26. Encouraged mathematical reflection	3.24 (.569)	3.52 (.499)*
Assessment and instructional adjustment		
27. Observed, listened and took notes	2.48 (.686)	3.04 (.603)*
28. Adapted tasks to accommodate range of abilities	3.37 (.581)	3.51 (.432)

Note. * $p < .05$

Sustainability of fidelity

Previously, we found that teachers exposed to the TRIAD intervention continued to demonstrate high levels of fidelity to the underlying curriculum two years past the end of the external intervention, without continued project support (Clements et al., 2015, recall these are descriptive data, as fidelity measures could only be collected from intervention teachers). Teachers appeared to first learn to implement the instructional components (whole group, small group, centers, computer center) and made continual improvements in such implementation, but made substantial growth in the General curriculum subscale only when they had some skill and confidence in the separate instructional components. This appeared to allow them to synthesize the components into a coherent, positive, classroom culture for early mathematics. Further, the main factor that supported such sustainability was perceptions of students' learning. Qualitative and quantitative data supported the position that observations of such learning along LTs motivated teachers to productively face challenges and improve their teaching (Clements et al., 2015).

To check and to extend these results, we present new data from our final fidelity measure as compared to the four prior timepoints. That is, the most recently analyzed data were collected six years after the external intervention, three times the duration of the previous sustainability analyses. At this point, data were available only for 28 teachers (who were not significantly different from the others in demographics, years of experience, or any other variable we checked). Means for each timepoint by item are displayed in table 2. Across the items, the pattern of sustained fidelity to the curriculum can be seen, with one exception. A large decline in use of curriculum components was found for only one area, computer usage. This is undoubtedly due to a lack of school support and an ending of the initial licenses for the computer program associated with the curriculum at the end of the four-year post intervention. Results on other items support the original sustainability findings. For example, teachers scored almost identically as in earlier timepoints on featuring mathematics materials in their classrooms, using everyday math activities, extending and enhancing the written activities, and teaching those activities with fidelity. They encouraged students to actively think, reason, solve problems, or reflect and involved and supported students in discussions of mathematics about the same or even more than at earlier timepoints (e.g. items 33 and 34, which are difficult pedagogical strategies). They maintained or increased their ability to use formative assessment (e.g. item 38).

Table 2. *Fidelity means across general curriculum items*

	Time 1 mean	Time 2 mean	Time 3 mean	Time 4 mean	Time 5 mean	Time 6 mean
General Curriculum						
1. The teacher was within 2 weeks of the scheduled plan (based on the week they started, and adjusted for vacations/field trips/other days not available for mathematics).	.98	.76	.88	.77	1.32	1.32
2. Home activities were sent home with students.	.94	.90	.88	.58	.79	.79
3. Materials were present, including specific math manipulatives, and other materials that can promote mathematical thinking.	3.08	3.08	3.08	3.18	3.07	3.00
4. The teacher uses the curriculum's every day mathematics activities or others like them, involving students in mathematical thinking.	2.89	2.61	2.97	3.17	2.61	2.46
5. The teacher(s) extended the activities in ways that enhanced the quality of the teaching and learning. Some examples follows.	2.44	2.08	2.70	3.10	2.61	2.86
Hands On Center Activities						
6. Teachers posed the tasks in ways that engaged students and maintained involvement.	2.29	2.42	2.76	2.25	2.28	1.88
7. Task was selected by the student.	.67	.77	.63	.80	.42	.68
8. Materials were set up correctly and completely.	.95	.86	.92	.98	.81	.68
9. The teacher set up and introduced the center as written in the curriculum.	2.30	2.39	2.84	2.57	2.19	1.73
10. An adult monitored, guided, and/or participated in the activity as needed.	2.70	2.52	2.70	2.64	2.46	2.19
11. The teacher's classroom management strategies enhanced the quality of the activity and students' mathematical learning.	2.75	2.57	2.77	2.77	2.42	2.35
Whole Group Activities						
12. The teacher displayed an understanding of mathematics concepts, using correct mathematical vocabulary as appropriate, making no significant mathematical mistakes.	3.12	3.02	3.14	3.15	2.82	2.89
13. Materials were set up correctly and completely (if no physical materials, the teacher is well prepared).	3.02	2.96	3.09	3.12	2.93	3.07
14. The teacher began by engaging and focusing students' mathematical thinking.	3.06	2.88	3.18	3.22	2.93	2.96
15. The pace of the activity was appropriate for the developmental levels/needs of the students and the purposes of the activity.	3.00	2.71	3.00	3.10	3.04	2.89
16. The teacher conducted the activity as written in the curriculum.	2.89	2.75	2.88	2.93	2.79	2.78
17. The teacher's classroom management strategies enhanced the quality of the activity and students' mathematical learning.	2.95	2.69	2.82	3.10	2.89	2.82
18. The whole group activity involved mathematical language, including, as appropriate to the activity, a discussion of mathematical ideas or strategies (of any type).	3.05	2.81	3.11	3.19	3.14	3.30
Small Group Activities						
19. The teacher displayed an understanding of mathematics concepts, using correct mathematical vocabulary as appropriate, making no significant mathematical mistakes.	3.08	3.00	3.06	3.12	2.79	2.70
20. Materials were set up correctly and completely.	.90	.96	.98	.95	.82	.86
21. The teacher conducted the activity as written in the curriculum, or made positive adaptations to it (not changes that violated the spirit of the core mathematical activity).	2.92	2.98	3.05	2.97	2.96	2.81
22. The pace of the activity was appropriate for the developmental levels/needs of the students and the purposes of the activity.	2.92	2.83	3.08	3.09	2.89	2.75
23. The entire activity was completed with all the students in the group (if the teacher works with some students on a different day that is acceptable if evidence supports that all students did/will be engaged in the activity).	.95	1.00	.98	.98	.96	.89
24. The teacher's management strategies enhanced the quality of the activity and students' mathematical learning.	2.78	2.69	2.92	3.19	2.71	2.79
25. The teacher promoted and valued effort, persistence and/or concentration.	2.92	2.73	2.84	3.16	3.07	2.86
26. The teacher encouraged students to actively think, reason, solve problems, or reflect, as indicated in the written curriculum.	2.57	2.54	2.77	3.19	2.86	2.75

27. The teacher asked students to share, clarify, or justify their ideas.	2.25	2.42	2.84	3.16	2.61	2.79
28. The teacher facilitated students' responding.	2.24	2.21	2.87	3.14	2.79	2.86
29. The teacher encouraged students to listen to and evaluate others' thinking/ideas.	2.02	1.75	2.33	3.02	2.04	1.96
30. The teacher supported the describer's thinking.	2.25	2.10	2.56	3.11	2.64	2.46
31. The teacher supported the listener's thinking.	1.88	1.48	2.33	2.93	2.04	2.11
32. The teachers support gave just enough assistance.	2.71	2.85	3.00	3.10	2.75	2.86
33. The teacher built on and elaborated students' mathematical ideas and strategies.	1.85	1.62	2.41	2.96	2.43	2.67
34. The teacher went beyond initial solution methods.	1.27	1.48	2.27	2.86	2.21	2.36
35. The teacher encouraged mathematical reflection.	1.78	1.40	2.33	2.91	2.36	2.36
36. The teacher cultivated love of challenge.	1.76	2.13	2.73	2.88	2.68	2.43
37. The teacher observed and listened to students, completing the record sheet (i.e., information on each student's performance; learning trajectory level; comments).	2.82	2.87	2.95	2.74	2.82	2.82
38. The teacher adapted tasks and discussions to accommodate the range of students' abilities and development.	2.65	2.65	2.91	3.03	3.11	3.07
39. The teacher used the "Monitoring Student Progress" help for struggling students or challenge for students who excel as written in the curriculum as needed.	.87	.87	.87	.82	.79	.71
40. Computers were set up correctly and completely	.98	.98	.94	2.78	1.61	1.18
41. Student was "signed in" with her/his correct name.	1.00	.98	.97	.93	.36	.14
42. The teacher introduced the activity, engaging and focusing students' mathematical thinking.	1.91	2.38	2.61	2.41	1.50	.79
43. The teacher or other adult monitored and was available to guide and help students as needed.	2.91	2.88	2.93	2.91	1.86	1.11
44. The teacher's classroom management strategies enhanced the quality of the activity and students' mathematical learning.	2.62	2.65	2.86	2.96	1.82	1.14
45. Observations and records (including computer records) indicated that all or nearly all students will have engaged in the activity by the end of the week.	.81	.98	.86	.76	.37	.27
46. The teacher was actively involved in guiding the activity.	2.23	2.32	2.42	2.65	1.71	1.04
47. The teaching strategies used were appropriate for the developmental levels/needs of the students and the purposes of the activity.	2.46	2.62	2.83	2.87	1.29	.61
48. The teacher had high but realistic mathematical expectations of students.	2.51	2.38	2.83	2.83	1.36	.82
49. The teacher promoted and valued effort, persistence and/or concentration.	2.20	2.24	2.70	2.83	1.61	.82
50. The teacher's support gave "just enough" assistance (e.g. appropriate level of detail, not too little or too much help or information).	2.39	2.54	2.78	2.85	1.29	.86
51. The teacher monitored and/or observed students during the computer activity, taking notes as appropriate.	2.30	2.57	2.60	2.70	1.54	.93
52. The teacher knows how to access computer records of individual students' sessions that are stored on the computer.	.95	.97	.96	.87	.59	.57

Discussion and Implications

The *Technology-enhanced, research-based, instruction, assessment, and professional development* (TRIAD) model was designed to address the challenges of scaling up interventions, especially interventions in mathematics situated in the diverse settings of preschool education. Because research suggests that knowledgeable and responsive adults are the most important feature of successful educational interventions (National Research Council, 2009), we evaluated TRIAD's effects on teachers' practices, whether teachers' practices mediated the effects of the intervention on students' outcomes, and whether these practices were maintained six years after the cessation of project support.

Findings showed that the TRIAD intervention positively affected teachers' practices in mathematics education compared to control teachers (who also received mathematics curricula and professional development, but different from the TRIAD learning trajectories approach). This supports our first hypothesis, illustrated via the paths from "TRIAD PD" to "Knowledge of practices" and to "Teachers' beliefs" in figure 1. The intervention based on children's mathematical learning trajectories, provided a coherent program of teaching and learning, which may have promoted the significant levels of high-quality practice found in this study (Wilson, Mojica & Confrey, 2013).

Beginning with teachers' practice, individual item analysis within the measure of the mathematics environment demonstrated a consistently higher pattern for teachers exposed to the professional development. Building on these individual findings, mediational analysis confirmed that increasing specific aspects of the environment (e.g. the number of activities, quality of the math activities, the number of computers, and the overall quality of mathematics in the classroom) accounted for significant change in student outcome scores over and above the direct influence of exposure to high quality mathematics curriculum. This suggests that these core processes of change serve to support the growth of mathematics learning as well as transfer to expressive oral language. This mediation was similar to, but less than, the mediational impact found in previous research, using the same instrument (Clements & Sarama, 2008). The differences on two individual items were particularly striking, relating to teachers' willingness and ability to listen to students and support students' understanding as they listen to others.

The classroom culture subscore assesses teachers' general approach to mathematics education, indicated by "environment and interaction" variables such as responsiveness to students, use of "teachable moments," and environmental signs of mathematics, as well as "personal attributes of the teacher" variables, including appearing knowledgeable and confident about mathematics as well as showing enjoyment in, curiosity about, and enthusiasm for, teaching mathematics. Teachers within the experimental condition demonstrated increases in the target practices measured within this subscale as compared to control.

The finding involving number of computers suggests that increased use of the Building blocks math software improves math scores. Finally, the mediational impact of the total number of classroom mathematics activities appears to be a simple "more time on task is better" result. However, a separate COEMET measure of total time on task did not mediate the impact, where the number of specific mathematics activities did. Thus, the number of distinct mathematics activities in which students engaged was more important than total time on task in supporting their learning of mathematics (cf. Sylva et al., 2005).

In sum, these results support our second hypothesis, illustrated by the "Classroom instruction" through "Students' cognitions" to "Students' learning" path in figure 1. They also specify which specific practices contribute to that mediation of student outcomes. Those practices relating to teachers' willingness and ability to listen to students and support students' understanding as they listen to others, confirms the work of others regarding the importance of dialogue (Borko, 2004; Fennema et al., 1996; Knapp et al., 1995) as well as the role of the research-based learning trajectories at the core of the TRIAD model. In a similar vein, the mediation of classroom culture on math scores is consistent with the literature supporting the connection between academic performance and general features of the classroom, including signs of mathematical activity and teachers who display both knowledge of and enthusiasm about mathematics and who interact with and respond to students frequently (Clarke & Clarke, 2004; Clements & Sarama, 2007; Fraivillig et al., 1999).

The number of mathematics activities also contributed to the mediation. We hypothesize that students of this age learn more from a variety of activities emphasizing the same level of thinking, as they may learn concepts more readily from generalizing mathematics structures from different problematic situations that require the same mathematical concepts and processes for their solution. Further, such multiple situations may create a greater number of cognitive paths for retrieval. Finally, the mediational role of the computer software confirms separate evaluations of the software as the solitary (unconfounded) component of the intervention (Foster, Anthony, Clements & Sarama, 2016).

Finally, teachers taught the curriculum with increasing fidelity over the following six years (fidelity was collected only in intervention classrooms, so we compared within the same classrooms over time), even though research project staff was no longer able to provide support. They seemed to have internalized the program (Timperley et al., 2007) and to have made sense of the curricular activities involved with whole group, small group, and other components, within an overall structure of LTs that progressed toward a known mathematical goal. By engaging in the initial professional development, and then, becoming empowered by their own knowledge of the trajectories and the practices to support learners through the trajectories, they became progressively more faithful to the intended program, instead of drifting from it as time elapsed and support disappeared, a contrasting negative trend found in other studies (Datnow, 2005; Hargreaves, 2002). This supports the hypothesized path from "Students' learning" to "Teachers' beliefs" in figure 1 (the dotted arrow indicates that a future paper will focus on this path extensively). Further examinations of teacher beliefs and the mechanism of change in relation to changes in practices is needed to confirm this relationship.

One implication, then, is that a coherent model of professional development, curriculum, instruction, and assessment based on LTs may provide the conditions for promoting high-quality instruction as well as sustainability in such practices (Clements & Sarama, 2014b). This may be particularly beneficial in addressing the climate of low expectations in urban schools (Johnson & Fargo, 2010), as teachers increase their understanding of the capacities of all students to learn mathematics.

As teachers come to understand students' probable developmental paths and become adept at anticipating students' strategies and misconceptions, their teaching practices may become more grounded and solidified; this is one way that practices and beliefs may interact. As they notice students' multiple strategies, and probe for the ways in which students' mathematical thinking fits the structure of the trajectory, their teaching practices may become reinforced as student reactions provide positive feedback for their practices (Guskey, 2002). Teachers who demonstrate sustained fidelity of implementation to a program that has demonstrated improved student achievement will have a positive impact on many more students than teachers who implement with fidelity only during treatment. Thus, another general implication is that helping teachers develop the skills and practices necessary to perceive and document their students' learning (which constraints from human subjects research dictates disallowed in this study) may be an effective way to maintain and even increase fidelity of implementation. These positive perceptions of learning may be especially important in motivating teachers to productively face the challenges inherent in fully implementing all aspects of the curriculum. For example, educational technology challenges include limited hardware, hardware and software problems and limited troubleshooting competencies, difficulty scheduling computer use for all students, and inconsistency between computer use and customary practice including contextually constrained choices. Solving problems successfully, such as engaging students productively in technology activities, may engender confidence and risk-taking in future work. Simply, success breeds success, and such changes in practice may lead to positive changes in beliefs (e.g. Showers et al., 1987), again, resulting in co-mutually reinforcing changes in beliefs and practices (Caudle & Moran, 2012) rather than conflicts between them (cf. Einarsdottir, 2003), supporting bidirectional path linking "Knowledge of practices" and "Teachers' beliefs" in figure 1.

The TRIAD model is not simply about a new curriculum or training teachers to use it. Success required complex changes, including a change in instructional structures, pedagogical strategies, and classroom communication and culture (Grubb, 2008). Given the importance of early competence in mathematics (e.g. Duncan et al., 2007; Paris, Morrison & Miller, 2006), the TRIAD implementation described here has implications for practice and policy, as well as research. TRIAD's guidelines should be considered when planning to increase the quality and quantity of preschool mathematics education.

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Julie Sarama

Julie Sarama is Kennedy Endowed Chair in Innovative Learning Technologies, Co-Executive Director of the Marsico Institute, and Professor at the Morgridge College of Education, University of Denver. Her research interests include young children's development of mathematical concepts and competencies, implementation and scale-up of educational reform, professional development models and their influence on student learning, and implementation and effects of software environments.

julie.sarama@du.edu

Douglas H. Clements

Douglas H. Clements is Kennedy Endowed Chair in Early Childhood Learning; Executive Director, Marsico Institute; and Professor at the Morgridge College of Education, University of Denver. His research interests include the learning and teaching of early mathematics and computer applications in mathematics education, creating, using, and evaluating research-based curricula and taking successful curricula to scale using technologies and learning trajectories.

douglas.clements@du.edu

Christopher B. Wolfe

Christopher B. Wolfe is an Assistant Professor of Psychology at Saint Leo University. His research interests include measurement, program evaluation, and early educational development.

christopher.wolfe@saintleo.edu

Mary Elaine Spitler

Mary Elaine Spitler is an independent researcher with interests in early childhood education and professional development.

mspitler@gmail.com