Mathematics teachers' knowledge-sharing on the Internet: pedagogical message in instruction materials

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This article reports on a study of teacher-shared documents containing mathematical tasks published on the Internet. The aim was to identify the goals, methods and pedagogical justifications presented in the documents and what was needed to solve the tasks. Content analysis was used to define their pedagogical message. The results show that the documents mainly involve content goals for younger pupils that are not consistent with the explicit descriptions. The conceptual goals are communicated to a great extent, but are not supported by task features. The reasons for why the tasks given are expected to lead to a certain goal are very often implicit, and, as a result, the content of the documents and the quality of the tasks are somewhat unclear to other teachers.

There are social network sites where mathematics teachers can publish instruction material they believe colleagues would benefit from in their daily work. Such documents are thus easily accessible (e.g. YouTube, lektion.se, Del og bruk), and the Internet has become a medium for (Swedish) teachers' everyday knowledge-sharing (e.g. Olofsson, 2008; Pepin, Gueudet & Trouche, 2013; Ruthven, 2016; Skolverket [Swedish National Agency for Education], 2009; Van Acker, Vermeulen, Kreijns, Lutgerink & van Buuren, 2014). In addition to the mathematical task itself, these documents most often include guidelines: descriptions of the tasks, and the intended setting. Hence, it is possible for researchers to study the pedagogical message embedded in the mathematical tasks and guidelines posted to fellow teachers and intended pupils on the Internet.

The entire range of activities on social network sites has expanded in the past decade. Studies of these kinds of communities focus to a great extent on the sites as communities of practice and online teacher

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education (e.g. Bissessar, 2014; de Carvalho Borba & Llinares, 2012; Manca & Ranieri, 2014; Pepin et al., 2013; Rutherford, 2010; van Bommel & Liljekvist, 2015), thus analysing, for instance, teachers' use of online curricular material or chat conversations in forums (e.g. Hew & Hara, 2007; Landqvist & Teigland, 2005; Manca, Ranieri & Fini, 2012; Olofsson, 2010; Triggs & John, 2004). The content-focused discussions often relate to ICT as a tool for learning (e.g. Vavasseur & MacGregor, 2008). However, the mathematics instruction materials published on the Internet have not been the objects of extensive mathematics education research. In this study, a document uploaded to the Internet that includes tasks, worksheets, guidelines and descriptions is called a teacher-shared document. The term "task" is chosen as an overall description of a variety of exercises and problems designed for an (certain) activity aiming to develop knowledge about a specific mathematical idea (cf. Margolinas, 2013).

When planning lessons and constructing tasks, teachers engage in pedagogical issues such as sensitivity to pupils, mathematical challenges and the management of learning. Creating a challenging learning environment that provides a chance for students to dig into mathematical ideas, to discuss and to reason is the main object (Potari & Jaworski, 2002). The teacher rearranges knowledge in order to isolate certain notions, concepts and properties, and to separate them from the complex network of mathematical activity that provides origin, meaning and motivation, so as to transpose it into the classroom (Brousseau, 1997; Brousseau & Gibel, 2005). Further, van Bommel (2014) finds in her study that it is important to carefully describe elements of the lesson (i.e., models for explanation, curricular documents, pupils' preconceptions, related materials and exercises) in order to accurately connect these elements and outline lesson goals explicitly and in detail in a planning document. To outline and, as in teacher-shared documents, communicate mathematical tasks, the uploading teacher to some extent considers the intended pupils' pre-knowledge, suggests a pedagogical setting and addresses content goals and competence goals. The teacher-shared documents hence contain a message on teaching issues, that is, goals, methods, pedagogical justifications, and so forth.

Research question

This study explores the subject-specific pedagogical messages that are communicated in teacher-shared documents containing mathematical tasks in terms of goals (what the pupil is supposed to learn), methods (how the task is to be implemented) and justifications (why the implementation of the task will lead to the learning goals).

In order to analyse the teacher-shared documents, a conceptual framework was created and a procedure of analysis was developed. The framework and the analysis are outlined in the following sections.

Conceptual framework

The conceptual framework was created to define the meaning of mathematical content and conceptual goals in Swedish curricular materials, such as the teacher-shared documents, in relation to relevant literature and the Swedish syllabus for compulsory school (Skolverket, 2011). First the meaning of *content goals* and *competence goals* will be outlined, followed by the *mathematical competences* (i.e., problem solving, mathematical reasoning, and communicating mathematics) that may be relevant to consider besides the content goals when analysing the mathematical tasks.

Content goals and competence goals

The Swedish syllabus specifies content goals and competence goals in terms of *core content* and *abilities* (Skolverket, 2011). The content goals in this framework are hence described in terms of the core content of the syllabus.[1] The competence goals are formulated as overarching abilities in relation to core content and expressed in the behavioural terms of formulating, analysing and choosing appropriate methods and strategies when working with mathematical tasks. The hypothetical learning progression is specified by means of the level of knowledge demands in grades 3, 6 and 9 (Skolverket, 2011). The pedagogical message of content goals in the teacher-shared documents is thus related to the intended progress. Still, this is not sufficiently detailed. Concepts related to each of the core content areas were examined (i.e., an extensive literature review was conducted based on Lester, 2007, especially Part IV) in order to determine, for example, what the accurate "connections" are and what kind of conceptual understanding is required of each core content area. In the analysis section the procedure of unitizing the concepts is developed further.

The competence goals are here framed by the extent to which a task gives pupils opportunities to develop mathematical abilities. The requirements of conceptual understanding are, as mentioned above, analysed in relation to core content area, and the three (intertwined) competence goals examined in this study are: 1) problem solving, 2) using and applying mathematical reasoning and 3) communicating in the language of mathematics. These aspects of the competence goals are outlined in the sections below.

Problem solving

In the Swedish syllabus, problem solving is presented both as a goal for instruction in itself, that is, as a core content area, and as a method to assess pupils' abilities. In this study, however, the tasks address not only problem solving but also other core content areas, such as understanding and the use of numbers, geometry, etcetera.

While problem solving as a means of learning mathematics (Lester, 1983) is "embedded in, and linked to, the content and context of the situation, rather than existing as a stand-alone process or skill" (Lesh & Zawojewski, 2007, p. 767, authors' emphasis), the features of the teacher-shared documents linked to pupils' possibilities to learn – and not the actual learning situation in the classroom – can be described. It is well known that a problem-solving task for one pupil might be a routine exercise for another (e.g. Lithner, 2008; Schoenfeld, 1985; Taflin, 2007). Thus, if a specific task is a problem or not for a specific pupil depends on how challenging it is for that pupil to carry out. The Swedish syllabus states, however, that mathematics teaching should aim at developing students' abilities to "formulate and solve problems using mathematics and also assess selected strategies and methods" (Skolverket, p. 59). Hence, tasks offering opportunities to investigate and explore (e.g. non-routine tasks, exploration of concepts, processes, relations etc., or when different strategies are endorsed when solving the task) are, in this study considered an invitation to the pupil to engage in and experience problem-solving strategies and methods (cf. Schoenfeld, 1992).

Mathematical reasoning

The issue of how to frame reasoning is important in examining the teacher-shared documents, since it is a pronounced competence goal in the Swedish syllabus. However, mathematical reasoning is not well-defined in the syllabus, which is why a conceptual framework (Lithner, 2008) is used in this study to describe aspects of the tasks that give opportunities to use and apply reasoning.

The analytical tool used to identify reasoning opportunities is *Creative mathematically-founded reasoning* (CMR). It is defined in Lithner (2008) in terms of three aspects: 1) novelty, that is, a reasoning sequence that is new to the pupil is created or re-created if forgotten, 2) the arguments used in the reasoning sequence motivate the strategy used and 3) the reasoning is anchored in intrinsic task properties. CMR is distinguished from imitative reasoning, which is characterized as following routine procedures either by memorizing, as recalling facts, or by algorithmic reasoning, for example, using a set of rules. In a series of studies, Lithner et al. (e.g. Bergqvist, Lithner & Sumpter, 2008; Boesen et al., 2010; Lithner, 2003; Jonsson, Norqvist, Liljekvist & Lithner, 2014) show that pupils, given an algorithmic solution method to a task, "will probably mainly apply AR [algorithmic reasoning] to solve the task without considering the underlying meaning of the concepts, representations or connections" (Lithner, 2015, p. 501).

Lithner (2008) points out that "CMR does not, as problem solving, have to be a challenge. The definition also includes elementary reasoning." (p. 266). The CMR task properties can be an extensive part of the task or just a small part (Boesen, Lithner & Palm, 2010). Reasoning is here viewed as a broader concept than a mathematical proof, since the requirements in school tasks are different and the logical rigour is of less importance. The reasoning sequence starts in the task and ends with an answer (Lithner, 2008). However, the study object (regarding reasoning) in this study is the task and the intended answer.

Communicating mathematics

The study object is teacher-shared documents and not the actual classroom setting. Thus the focus is on pupils' opportunities to communicate in the language of mathematics when working on the tasks (e.g. Niss, 2003). The intended pedagogical setting (e.g. group work) is opportunity to discuss and give an account of arguments, that is, activities put forward in the syllabus (Skolverket, 2011). Further, the requests that the pupils' answers to the tasks should be given in a certain form, for instance, showing and telling, or writing a story, are seen as a way to address the communicative competence goals. This means that the focus is on determining whether there are opportunities to communicate in a mathematical subject, whether the learner can make a genuine contribution and whether there can be an interaction (Pririe & Schwartzenberger, 1988). Bartolini-Bussi (1990) calls for studies to determine whether the communication occurs before, within and/or after working with the tasks. However, this is not within the scope of the present study.

The mathematics pedagogical message in the teacher-shared documents is hence framed in terms of the syllabus and the conceptual framework. The analysis procedure is outlined in detail in the sections below.

Analysis procedure

The analysis procedure, based on content analysis (Krippendorff, 2004), contains steps where inferences are made, such as what is required to solve the task. The interpretative approach in the coding procedure gives the researcher opportunities to explore latent content, as well as manifest content (Bryman, 2001). It is a matter of probing "beneath the surface in order to ask deeper questions about what is happening" (p. 188). Hence, the development of "a theoretically valid protocol" (Rourke & Anderson, 2004, p. 8) is central in the data-making procedure. In a first step, features relevant for identifying the requirements of the tasks were defined via an extensive literature review (e.g. Lester, 2007). Second, the features were ordered and described in groups and subgroups. A pilot study was conducted, and the coding schedule was then revised, that is, complemented due to gaps in descriptions and themes.

Colleagues in the research group tested the coding schedule and coding manual in order to establish stability and reproducibility when generating data from the teacher-shared documents (cf., Bryman, 2001). Since the coding was a one-person endeavour over a period of time, the matter of intra-coder reliability was addressed in a procedure of re-analysing documents to establish consistency of coding over time. The procedure was then well within the scope of the method originally developed to describe the content of communication (cf. Rourke & Anderson, 2004), that is, the pedagogical message. The reliability and validity of the data-making can be judged if the procedure is made transparent (Krippendorff, 2004; Rourke & Anderson, 2004). The coding procedure is therefore thoroughly described in the sections below.

Coding

The coding schedule and the coding manual were outlined in order to unitize the documents (Krippendorff, 2004) and to obtain a quantitative overview of the content. The coding is explained in table 1. The units either describe a categorical feature, such as intended age of the pupils working with the tasks (henceforth: intended grades), or a

Table 1. *Descriptions of the codes*

Code	Description
1	The feature is described within the teacher-shared document as it occurs downloaded from the Internet
0	No description of the feature within the teacher-shared document
[text]	A text segment in the teacher-shared document regarding a specific feature is collected (e.g., wordings on competence goals)
[number]	Numerical information is available, such as rating, on the specific site
X	Not relevant

thematic feature, such as manifest pedagogical justifications discernible as wordings related to syllabus.

The coding schedule

The coding schedule comprises five groups: background variables, intentions (i.e., goals and justifications), type of tasks, abstraction level and, in addition, non-relevant claims on learning outcome related to instruction, goals and other task features (e.g. missing connections in the document or contradictions related to the chosen topic etc.). The groups were divided into subgroups.[2] These were divided into 131 coding labels. Table 2 shows the coding schedule for the five groups, the corresponding subgroups and the possible coding labels. Examples of coding labels are shown in table 3.

Examples

In this section the analysis procedure is demonstrated through two examples of teacher-shared documents, A and B, and an excerpt of the coding manual is then presented in table 3. The table shows the corresponding

Table 2. *Coding schedule*

Group	Subgroup description	Code
1. Background	a. features according to the web site database tool (e.g. grades) b. topic, as noted in the instruction, concepts related to topic c. estimated time on task, didactical setting (i.e. individual, pairs, groups, whole class)	1, 0, [text], [number]
2. Intentions	a. wordings on content goals b. wordings on competence goals c. justifications on learning goals to i fellow teacher, ii pupils, iii parents	1, 0, [text]
3. Type of tasks	a. what kinds of tasks are in the documents; i sequence of facts, ii algorithms, rules to follow, no explanations, iii algorithms and explanations, iv exploratory b. assessment tasks c. word problems d. digitally interactive task e. recommended aids and tools (i.e. paper-pencil, measuring equipment, calculator, computer etc.) f. claims on answers (i.e. straightforward answer, sentence, product etc. "Product" is then unitized, that is, constructions, stories etc.)	1, 0
4. Abstraction level	a. kinds of representations in the student worksheet (e.g., verbal, informal or formal pictures, numerical or symbolic) b. number domain c. mathematical notation and mathematical terminology (i.e. accuracy and relevance for intended grade)	1, 0, [text]
5. Non relevant claim	non relevant claims on learning outcome due to missing links (connections) or unsuitable 'things' (e.g., related to the topic) within instruction, goals or other task features	1, 0, [text]

coding instruction to each code displayed. The coding for A and B are in the right-hand columns.

A. Description of document 46036

Intended grade: 2–6. *Topic*: numbers. *Setting*: pair, part of lesson.
Instruction to a fellow teacher: A game using the number system.
Worksheet: It consists of a colourful game plan with a staircase. The staircase has 15 steps and the start and stop are different numbers (determined by the teacher).

The playing rules are given on a separate sheet and intended for several versions of the game plan:

> You need [three 10 sided] dice and markers. You start by throwing the dice, and then you decide which number can be built, for example, ones, tens, tenths etc. Then place it somewhere on the staircase to form a kind of number line. If it is not possible to write a new number, put a cross on one of the steps. The person with the fewest cross marks is the winner. Good luck.
>
> (Excerpt from document 46036, my translation)

B. Description of document 79007

Intended grade: 7–9. *Topic*: numbers. *Setting*: individual, whole lesson.
Instruction to a fellow teacher:

> By means of easy mental arithmetic, get pupils to see that the equal sign means "is equal to" and not "becomes". (My translation)

Worksheet: The pupils are supposed to work through 37 tasks, which are ordered into three levels. The first 20 tasks are like the following:

Which number is missing?
7 · 3 = 17 + __ [Task 79007 17]

The next part contains 10 tasks, which are more open:

Write two numbers to complete the equality.
250 · 4 = ___ + ___ [Task 79007 29]

In the third level of tasks, there are 7 open statements like the following:

Create your own tasks so that the equality is true.
__ / __ + __ = __ + __ + __ [Task 79007 34]

Analysis and re-analysis

The next step in the analysis was to make inferences as to what is required to solve the tasks. It was straightforward to unitize the content goals via the coding manual and conceptual understanding. However, further analysis was needed to determine requirements in terms of other abilities, that is, to use and apply reasoning, and to communicate in the language of mathematics. Documents coded under 3.a1.3 *Algorithms with explanations* and 3.a1.4 *Exploratory* (see table 3) were re-analysed in order to explore the requirements related to reasoning and communicating mathematics.

As stated earlier, mathematical reasoning is seen as a requirement if a new reasoning sequence must be constructed in order to solve the task, or if a forgotten sequence must be recreated (Lithner, 2008). In order to give a full account of how reasoning is dealt with in the analysis, we now provide further clarification of it.

If the pupil tasks do not contain a fully described solution method, reasoning is required. However, this is not to say that "guess and try" is an accurate method for solving problems. For reasoning to occur, the pupil has to give an account of the chosen strategies and anchor their reasoning in intrinsic mathematical properties. Since the intended grades for the task often encompassed several school years (e.g. a teacher can label the teacher-shared document as suitable for grades 1–9), it is not straightforward to decide the novelty of the tasks (for the pupil). In other studies, mathematical tasks were analysed in terms of similarity with textbook tasks (e.g. Palm, Boesen & Lithner, 2011). However, this approach was unpractical since the documents referred to several school levels. The probable novelty of tasks was hence analysed via the progress described in the syllabus.

In worksheet A above, reasoning is required since the solving method is not fully described, and the pupil needs to use strategies anchored in mathematical properties (i.e., the number system). However, if the number system is an established concept, it becomes a trivial task. The document is therefore coded CMR for the lower of the intended grades (the document is labelled, by the teacher, as suitable for grades 2–6). Worksheet B is also coded CMR for the lower of the intended grades. There is no fully described solving method and, in order to solve the tasks, the pupil needs to anchor strategies in mathematical properties (e.g. prioritising, counting rules).

The requirements to use the mathematical forms of expression in communicating mathematics when solving the tasks were identified, that is, the type of answers requested and the type of pedagogical setting outlined in the document. A re-analysis was made of documents coded with sentence, product[3], etcetera, and of documents coded collaborative setting. Hence, the requirements for communicative abilities were verified or not verified.

Here is an example: in document A above, the intended pedagogical setting is "pair", but there are no expectations expressed about the pupils

Table 3. *Excerpt from the coding manual*

Group	Subgroup	Coding label	Coding instruction: Mark if the document...	A	B
1	1.b1 Understanding and use of numbers	1.b1.1 Number concepts	deal with the meaning and size of numbers (e.g., the number system, relations within/between numbers, ordering of numbers etc.)	1	1
		1.b1.2 Multiple representations	deal with equivalent expressions and representations of numbers (e.g., iconic, symbolic etc.)	1	0
		1.b1.3 Operations	deal with operations on numbers	0	1
		1.b1.4 Computing/counting strategies	deal with formulating and applying, judging and evaluating computing and counting strategies	0	1
1	1.b4 Algebra, relationships and change	1.b4.1 Early algebra (EA) notations	contain pre-algebraic notation, such as correct and varied arithmetical notation as 3+_=4+2 etc., or if the pupils are expected to use it in solving procedure/answer	0	1
		1.b4.2 EA tables	contain representations in (simple) tables, or if the pupils are expected to use it in solving procedure/answer	0	0
		1.b4.3 EA graphs	contain representations of (simple) graphs, or if the pupils are expected to use it in solving procedure/answer	0	0
		1.b4.4 EA natural language	supports general solution ideas, but does not contain expressed demands in algebraic terminology	0	1
3	3.a1 Kind of task	3.a1.1 Sequence of facts	contains at least one task assessing memorization	0	0
		3.a1.2 Algorithms without explanations	contain at least one task assessing procedure without making connections to intrinsic properties of the task	0	0
		3.a1.3 Algorithms with explanations	contain at least one task assessing procedures with connections to intrinsic properties of the task	0	0
		3.a1.4 Exploratory	contains at least one task assessing non-algorithmic thinking (e.g., non-routine task, exploration of concepts, processes, relations etc., different strategies are endorsed when solving the task)	1	1
4	4.c1 Mathematical notation	4.c1.1 Numerical	contains numerical notation (Note! number domain is coded in 4.b)	1	1
		4.c1.2 Variables and constants	contain variables etc.	0	0
		4.c1.3 Symbols in "simple" operations	contain symbols (i.e., + - ·/ > =)	0	1
		4.c1.4 Symbols in "advanced" operations	contain symbols (e.g., √ , exponents etc.)	0	0
4	4.c2 Mathematical terminology	4.c2.1 Correct	has mathematically correct instructions	1	1
		4.c2.2 Relevant	has relevant terminology for the intended grades; note: it can be partly incorrect	1	1
		4.c2.3 Type of incorrectness	has incorrect terminology. Describe and copy text segment	X	X

actually explaining to each other why they put a number in a specific place on the staircase. Hence, the competence goal to communicate in the language of mathematics when engaging in the task is not required in the worksheet

Data

The data in this study were assembled from mathematics teachers' knowledge-sharing on a social network site where no formal review process is at hand. A pilot study was conducted in the autumn of 2011, where documents on different types of social network sites and web sites [4] were briefly explored. The instruction material on the Internet is [electronic] documents with information in both the written form such as illustrations and figures, and as video or interactive programs.

The sample is a subset of existing instruction material from one Swedish social network site (n ≈ 3300, January 2012). The search key was documents labelled grades 4–6 or 7–9 by the publishing teacher. The documents were subsequently selected from a specific starting date in January 2012 and "backwards in time" until 52 documents had been collected within each search key. When there was an overlap between the labelled grades, the document was only counted once. The sample gives a representative subset of the documents on the social network site as the documents were randomly published (cf. Bryman, 2001; Cohen et al., 2000). In all, 84 electronic documents containing instructions to fellow teachers, lesson plans, worksheets and other instruction material, including at least one mathematical task, were further analysed. The documents contained about 900 tasks altogether. Sixteen documents were excluded from this study, as they did not contain mathematical tasks.

Results

The mathematics pedagogical message in the teacher-shared documents is communicated through the information to the downloading teacher, and intended pupils. The information contains the mathematical tasks and explanations and advice on how to implement them, such as instructions for how to think and what to do when engaging in the task and, to some extent, descriptions of goals. The results are derived from the analysis of the pedagogical message concerning goals, methods and justifications. Further, results of what is required to solve the tasks are shown. An overview of the results is shown in table 4. The excerpts in the result section were translated by the author (in an uncensored and unedited form) with the document id-number shown in brackets.

Content goals and competence goals

Content goals related to the core content area of *Understanding and use of numbers* appear in 68 % (n = 57) of the 84 documents. Goals related to *Geometry* appear in 18 % (15) documents, to *Probability and statistics* in 6 % (5) documents and to *Algebra, relationships and changes* in 8 % (7) documents. Explicit wordings on competence goals are more rare, as these are found in 16 % (13) documents (see table 4). The abilities mentioned in the documents are *Communication* in ten documents and *Reasoning* and *Conceptual understanding* in four each. *Problem solving* is an explicit goal in three documents, although one of them does not contain problem-solving tasks.

Pedagogical setting

The pedagogical setting is collaborative, such as pair or group discussions, in 49 % (41) of the 84 documents, and 20 of these 41 documents contain problem-solving tasks:

> A simple excel file with diagram that shows $y = kx + m$ can be changed to k and m to see how the graph changes. should (sic!) be two pupils at each computer in order to invite discussion. You can probably write some questions to be answered in the "simulation".
>
> (79002, to fellow teachers)

The remaining 43 documents contain individual tasks, and the teacher recommends a follow-up discussion in the whole class in nine of these documents.

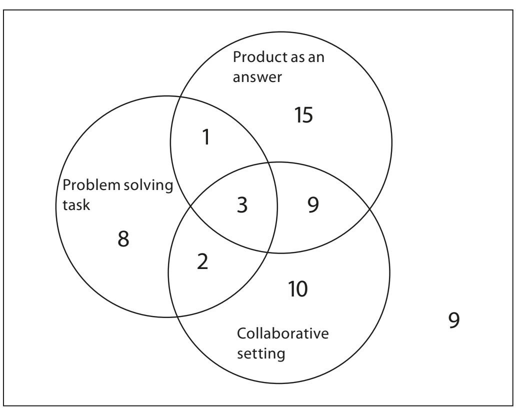

Figure 1. *Distribution of intended implementation in documents labelled "Understanding and use of numbers" (n=57)*

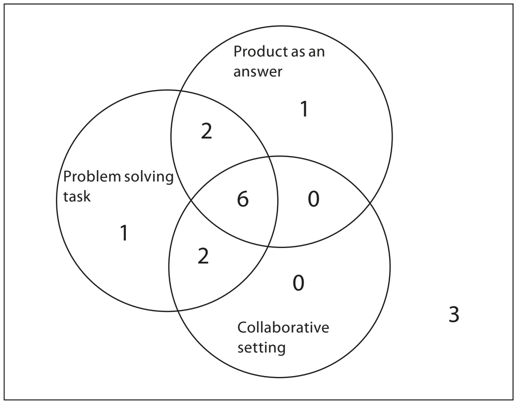

Figure 2. *Distribution of intended implementation in documents labelled "Geometry" (n=15)*

In 52 % (44) of the 84 documents, at least one of the tasks requests the answer to be "a product", for instance, drawing a sketch or writing a story in order to show or tell. For example, in documents labelled *Geometry*, nine of 15 documents contain tasks aiming at drawing or constructing models and 4 of 5 in *Probability and statistics* involve making an investigation and then presenting it to classmates.

The methods of intended implementation in terms of collaborative setting, a product as an answer, and problem solving as the means of learning mathematics are intertwined. This is shown in figures 1 and 2.

Inaccuracies in the instructions

In 14 % (12) of the documents, there are inaccuracies that might confuse pupils and hinder implementation, such as spelling mistakes, layout problems or errors in mathematical notation and language, for instance, using the wrong sign, such as decimal point, or mixing the words "number" and "digit".

Justifications given for use of the suggested task

The descriptions given to fellow teachers contain, explicitly or implicitly, arguments for why a task addresses a curricular goal. Justifications related to the syllabus are explicitly at hand in 8 % (7) of the 84 documents, such as, "I think it is important to follow up [the tasks] especially in order to assess the problem solving strategies." (46063). In 60 % (50) of the documents, curricular goals are implicitly stated, such as justifications related to a specific pedagogical setting, and in terms of core content and/or assessed abilities:

> I made two worksheets for this activity. It has to do with number sense, multiplication and division, numbers ending with zero etc. My pupils worked in pairs and discussed and gave arguments for their solutions. The calculator finally gave the answer – but there can be several solutions! (46077)

The instructions in the remaining 27 documents are more rudimentary, for example, "Activity in pairs on rational numbers" (46081) or "Some tips" (46001;79049). While these instructions can be detailed in terms of how to use or carry out the task, they may still not contain any explicit or implicit reasons for why the task is expected to lead to a specific content goal and/or competence goal fulfilment. The instructions contain teacher jargon, such as "The tough corner is in blue" (46018), when the teacher refers to a multiplication table and the calculations from six times six to nine times nine.

Justifications given to pupils for carrying out the suggested task

Half of the teacher-shared documents contain instructions to the pupils. The instructions are foremost on how to carry out the task such as "Draw lines between [certain objects] and then count [how many lines]"(46016) or "Don't forget to show your thinking process!"(46005). In 12 % (10) of the 84 documents, the instructions to the pupils are detailed and related to goals in the syllabus and about how to perform the task. These tasks contain strategic instructions such as "You can often solve a mathematical task through making an appropriate sketch. Draw a sketch for each task and then figure out how to solve it [mathematically]" (79029).

Connections in the documents

In 19 % (16) of the 84 documents, the description to fellow teachers is not coordinated with the actual task as, for example, in "This is a task, a game, about the number system" (79004). In the worksheet, however, it was not apparent in what way the pupils could capture the notion of place value, for instance. In seven of these 16 documents, the meanings of "concept" and "word" are mixed up, for example: "[...] the pupils work with the concepts of addition" (79016). The task is about learning terminology related to addition by rote and is not anchored in (suitable) mathematical ideas about addition; hence no conceptual understanding can be assessed.

Requirements for solving the tasks

In the study, 98 % (82) of the 84 teacher-shared documents contain core content and require abilities related to *Understanding and use of numbers*. In 21 of the documents, the requirements are related to *Geometry* and, in eight documents, *Probability and statistics*. Further, in 39 % (33) of the documents, core content and abilities related to *Algebra, relationships and changes*, foremost in early algebra, are present (see table 4). In 57 % (48) of the 84 documents, the tasks are within the natural number domain. None of the documents deals with negative numbers, and just a few with fractions.

Table 4. *Overview of the results: distribution of goals, intended implementations and requirements in the teacher-shared documents*

Core content area	Explicit goals					Intended implementation		Requirements			Overarching requirements	
	Core content	Abilities				Collaborative setting	Product as answer	Competence goals			Content goals	Concepts
		PS	Co	Re	Cu			PS	Co	Re		
Understanding and use of numbers	57	1	5	3	3	24	28	15	10	10	82	41
Geometry	15	1	5	1	1	8	9	11	1	0	21	9
Probability and statistics	5	0	0	0	0	4	4	4	2	1	8	17
Algebra, relationships and changes	7	1	0	0	0	5	3	7	2	2	33*	7
Total	**84**	**3**	**10**	**4**	**4**	**49**	**52**	**37**	**15**	**13**	-	-
Problem solving **						20	37	37				

Notes. Abbreviations: Problem solving (PS) Communication (Co), Reasoning (Re), and Conceptual understanding (CU)
* Early algebra. ** Overarching core content area.

Requirements of abilities

The requirements in the documents differ in the explicit wordings (see table 4). For instance, there are 37 documents containing problem-solving tasks, although only three documents contain problem-solving goals. In 13 of these 37 documents, the requirements also involved creative mathematical reasoning for the 4–6 grade pupils. Another example is the 57 documents labelled *Understanding and use of numbers*. The competence goal requirements are as follows (explicit goal in brackets): problem solving 15 (1), communication 10 (5), reasoning 10 (3) and conceptual understanding 41 (3).

Conceptual understanding is explicitly mentioned in four documents, although the results give another picture of the actual requirements (see table 4). For instance, 17 documents require conceptual understanding of statistics (e.g. tables, diagrams etc.) to solve the tasks.

As mentioned above, the intended implementation is collaborative to a great extent (49 %). However, the results show that this intention does not automatically lead to requirements for communication in the language of mathematics. In 18 % (15) of the documents, the requirements for mathematical communication abilities are sharper; that is, the pupils need to use mathematical expressions (at a grade-relevant level) to deal with the tasks. In an individual setting, 13 documents have a communicative requirement in this respect.

Progression related to core content and syllabus levels

The progression in the Swedish syllabus is indicated by means of the level of knowledge demands in grades 3, 6 and 9 (Skolverket, 2011). The labelling of the documents covers one, two or three levels. In the 42 documents covering only one of the syllabus knowledge levels, 52 % (22) contain core content requirements only for the lower of the intended grades. Further, if a teacher-shared document is intended for a larger span of grades, that is, covering two or three of the syllabus levels, the analysis shows that the requirements are merely content goals for the lowest of the intended grades in 79 % (33) of these 42 documents. There are exceptions, however, as three documents contain tasks that can be described as "rich problems" (e.g. Taflin, 2007), where all the three syllabus levels can be addressed.

Conclusion

In this study, the method used reveals that the pedagogical message in the teacher-shared documents consists of two parts: *explicit message* and *implicit message*. The explicit message is the outspoken communicative content in the teacher-shared document that clearly states what the pupils are supposed to learn, that is, goals (learning outcomes in the form of content goals, competence goals), methods (how the task is to be implemented) and justifications (why the implementation of the task will lead to the learning goals), as described in the documents as comments to the task. The implicit message, on the other hand, is shown in the underlying assumptions embedded in a teacher-shared document, for instance, the levels of demand inferred from the mathematical content, and the task design and layout of the teacher-shared document (e.g. the inherent requirements in terms of content knowledge and abilities to solve the task). An implicit message also emerges when instructions to fellow teachers are brief or when there are contradictions between the guidelines and the task design. The inconsistencies between what is manifest (explicit message) and what is latent (implicit message) in the teacher-shared documents indicate a pedagogical problem that could be discussed as two interrelated aspects. In this kind of pedagogical communication, there may be a risk of disguising the problems in the task constructions, on the one hand, as well as of under-communicating the potentials of a task, on the other.

Discussion

The focus of this study is on instruction material on social network sites containing mathematical tasks. In addition to the mathematical tasks, the material includes descriptions of the tasks, and the intended setting, hence containing a message on mathematics pedagogical issues, that is, goals, methods, pedagogical justifications, and so on. In the analysis procedure, the following inferences were made: the designing teacher considers the target pupils' pre-knowledge, suggests a pedagogical setting and addresses content goals and competence goals when communicating the mathematical tasks.

The results show that the methods of implementation are intertwined in the documents. In about half of the documents, it is suggested that the task should be performed in a collaborative setting. This is a different message than the message given by Swedish mathematics textbooks. Further, the intention is rather commonly to have the pupils produce ”things”, when solving the tasks, in order to show and tell. The mathematics pedagogical message in these teacher-shared documents thus aims at communicative competence goals and to give pupils opportunities to develop their ability to discuss and argue for their choices and strategies (e.g. Niss, 2003; Pririe & Schwartzenberger, 1988). However, the results suggest that it takes more than labelling a task as ”collaborative” to gene-rate communication in the language of mathematics. The task itself (or the instruction) needs to be more precise, that is, opportunities to communicate in such a way that the learner can make a genuine within-subject contribution (Pririe & Schwartzenberger, 1988).

Further, this study shows inconsistencies between what is explicitly stated and what is hidden or implicit in the teacher-shared documents. In the sections below two aspects of this are discussed more in detail.

Discrepancies in the message: intended grades and goals

We should, of course, be aware of the genre in which the teacher-shared documents are published. There are no formal requirements or demands of coherence, etcetera as in, for instance, peer-reviewed documents. Teachers most certainly take the contextual frame for granted when sharing these kinds of documents and hence do not communicate pedagogical issues in detail (cf. Hew & Hara, 2007). It is not reasonable to expect teachers to carefully consider the subject pedagogical message in already existing documents on the site in order to maintain consistency in all of the documents shared on this particular site, or, for that matter, to expect teachers to consider what areas in the curriculum that have already been covered in documents shared by others.

Although teachers share whatever interests them on social network sites, it is also a matter of turning the teacher-shared document into a resource for others (Gueudet & Trouche, 2012; Hew & Hara, 2007; van Bommel & Liljekvist, 2015). This means that for a teacher-shared document to fulfill its potential as part of teachers' everyday sharing of knowledge, it needs to be user-friendly in terms of a coherent pedagogical message. There seems to be a lack of a mathematics-specific pedagogical language that can effectively clarify the intention, level and goal of a certain task. The result is that the constructor of a teachers-shared document does not detect inconsistencies in his/her own task design and planning document. If there were a practice of placing tasks in a communicative pedagogical framework, the constructor would more easily detect such deficiencies.

The teacher-shared documents contained, as expected, content goals and competence goals related to the syllabus core content, mostly the category of *Understanding and use of numbers*. As the documents in the study were chosen to address core content in grade 4 and above, the expectation was to find tasks on, for instance, fractions or negative numbers, but the prevalent number domain was natural numbers. This means that many of the documents do not address the relevant core content area for the intended grades, as explicitly communicated in the labelling of the document. This pattern is also seen when looking into the competence goals. For instance, the requirements of problem solving or reasoning are, with some exceptions, for the lower of the intended grades.

This pedagogical message is also shown in the fact that the teacher-shared documents are often labelled for grades overarching two or three knowledge levels in the Swedish syllabus: grades 3, 6 and/or 9. But the tasks in the documents analysed do not, apart from a few exceptions, reflect such breadth and depth as in the syllabus. This means that there is an inconsistency between the explicit message (e.g. the labelling and description to fellow teachers) and the implicit message (e.g. the mathematical content and requirements in the worksheet).

It would be interesting to further study the reason why so many teachers choose this ”wide labelling”, as the method chosen does not give that kind of answers (Krippendorff, 2004). We could argue in terms of (lack of) teachers' mathematical, pedagogical and subject-specific pedagogy knowledge. There are empirical studies supporting this explanation (cf. Henningsen & Stein, 1997; van Bommel, 2014), but we can also elaborate on other explanations: Is it a rational decision based on hands-on experience of the combined roles of a task designer and a task user? Do teachers refrain from interfering with a fellow teacher's pedagogical decisions due to the expected wide range of mathematical understandings in Swedish classrooms? Considering pupils' pre-knowledge is one aspect raised by Potari and Jaworski (2002) in their framework for describing the complexity of teaching development.

Discrepancies in the message: hidden resources

The reasons given as to why the tasks shared are expected to lead to a certain goal are very often implicit or rudimentary and, as a result, the content of the documents is somewhat imperceptible to other tea-chers. For instance, the competence goals for conceptual understanding are hardly mentioned in the documents, while the results show quite a number of documents that contain tasks requiring this ability. The same pattern is repeated regarding the competence goals of mathematical reasoning and problem solving. It is well known that teachers tend to simplify mathematical tasks when implementing them in a classroom setting (cf. Henningsen & Stein, 1997). In this study it seems rather that the teachers cannot, or choose not to, express and describe the pedagogical qualities that their documents contain.

If this result indicates a lack of knowledge or a lack of a functional or effective subject-specific pedagogical language, it also indicates a problem for teachers to strategically plan their courses, to evaluate the usefulness of certain tasks and also to communicate with their pupils. This should not be seen as a criticism of the teachers who share their instruction material on the Internet, or of the phenomenon as such. On the contrary, these fora are likely to open channels for intercollegiate learning. However, they also show that the lack of subject-specific communicative pedagogical tools, which probably are not limited to the Internet, has consequences for the learning opportunities that pupils are offered.

Final comments

Conducting research on the Internet is challenging due to the ”Internet time” (Karpf, 2012), that is, the rapidly changing context and content, and the code-based modifications: ”Standard practices within the social sciences are not well suited to such a rapidly changing medium. Traditionally, major research endeavours move at a glacial pace [...] between conception and publication” (p.640). The method used (content analysis) does not give answers to the ”Why-question” (Krippendorff, 2004). However, as it is a method for studying manifest and latent messages in the media, it is useful when analysing teacher-shared documents in social network sites. The sites studied in the first pilot and in the main study are all still available and active. This makes it possible to do a follow up study in order to see how the teacher-shared documents develop, for instance, as the social network sites become a more frequent part of teachers' professional development (see, for example, van Bommel & Liljekvist, 2015, and Liljekvist, van Bommel & Olin-Scheller, in press).

The teacher-shared Internet documents are interesting to study in order to see what teachers choose to share and communicate. We cannot take for granted that the ambition of the documents is to mirror the syllabus in all its parts, but we can expect the documents to reflect a need for ”different” mathematical tasks and pedagogical settings. The combined roles of the teacher as a task designer and a task user in the Internet community offer a kind of professional communication as the teacher-shared documents do contain pedagogical messages about goals, methods and justifications (cf. Pepin et al., 2013).

The bottom-up design in this kind of communities is an important aspect, and, hence, ”preserving the bottom-up design possibilities pleads for avoiding the assessment by experts, or by an institutional authority” (Gueudet, 2015, s. 238). Nevertheless, the results imply a need for teachers to address the competence needed to clearly construct and communicate instruction material, perhaps together with teacher educators, researchers and other stakeholder, and to construct tasks and assess specific content and relevant competence goals in order to convey a more consistent pedagogical message. These questions have to do with developing and maintaining a professional language (e.g. Grevholm, 2010) and considering the teacher-shared documents as combined resources and scheme of utilisation central to the teachers' professional development (Gueudet & Trouche, 2012). The teacher-shared documents have their pros and cons, but, if the pedagogical message was clearer, the fellow teacher could more easily reflect on how relevant the tasks would be in the process of planning lessons, that is, in Brousseaus' terms: rearranging mathematical knowledge in order to isolate certain notions, concepts and properties in order to arrange for the pupils to learn mathematics.

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Notes

1 Understanding and use of numbers, Geometry, Probability and statistics, Algebra, relationships and changes, and Problem solving.

2 This report does not include subgroups such as topic-specific subgroups in geometry, statistics and probability, or digitally interactive tasks, since there are few such tasks in the sample.

3 That is, constructions, stories and so on.

4 Instruction material is present in some well-established social communities such as "YouTube" and "Facebook". There were also sites more specific to education, such as "Del og bruk" and "Lektion.se". In addition, there was instruction material on blogs or on personal homepages, and also material on corporation homepages, that is, publishing firms, and on official sites and NGOs.

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