Learning linear relationships through independent use of the mathematics textbook

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This paper discusses the participation of one Grade 8 student in a typical Swedish mathematics class as she tried to learn independently about linear relationships, mainly through the mathematics textbook. Since this set-up has the possibility for every student to progress at different rates, only one student, through data from ten consecutive lessons, was considered. Results from an analysis of this student's use of the textbook showed that the textbook is used independently more as an instrument for learning mathematics rather than an object of learning mathematics. However, further studies are required to determine the extent of learning and the development of mathematical skills.

It is common to see classrooms where textbooks are considered important in the teaching and learning process. The extent of use, however, can vary. In some classrooms textbooks are extensively used, while in others textbooks are just one of many resources accessible to students. To gain further knowledge about learning methods, it would be of interest to investigate the textbook use and its influence on student learning in a classroom where textbooks played a significant role.

Despite the many international studies about mathematics textbooks (e.g. Jamieson-Proctor & Byrne, 2008; Pepin, 2009) the focus has been primarily on the teacher's use of textbooks or features of the textbooks. For example, O'Keeffe (2013) argues that there is limited evidence outlining how students use the textbooks, for most often it is the teacher's perspective that has been investigated. Two studies by Rezat (2006, 2009) form an exception. He designed a model to study students' textbook use, based on the Activity theory developed by Engeström (e.g. 2000). Rezat distinguishes between the role of the textbook as an *object of learning* and as an *instrument of learning*.

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Gallos Cronberg, F. (2016). Learning linear relationships through independent use of the mathematics textbook. *Nordic Studies in Mathematics Education*, 21 (2), 5–22.

With textbook as an *object of learning*, he refers to the interplay between student, teacher and textbook where the aim is to reproduce what is in the textbook and the teacher acts as the mediating artifact between the student and the textbook. On the other hand, when the textbook is considered an *instrument for learning*, the textbook acts as the mediating artifact between the student and the mathematical knowledge. Moreover, in the later study of Rezat (2009) he mentions that if the textbook is used as an instrument for learning then the textbook utilization scheme can be categorized into four types: solving problems, consolidation, acquiring mathematical knowledge and activities associated with interest in mathematics. *Solving problems* refers to activities where a student refers to parts of the textbook to get ideas for doing mathematical tasks or solving problems. When a student is doing textbook tasks in order to improve mathematical skills that have been already discussed in class then this category falls under *consolidation* activities. In the case where a student used parts of the textbook that have not been discussed yet in class then it is categorized as activities related to *acquisition of knowledge*. The last category, *activities associated with interest in mathematics*, refers to a student using parts of the textbook that are found to be interesting.

In the Nordic countries, research on mathematics textbooks has been rather limited, despite the fact that in an international comparison textbooks dominate in the teaching of mathematics (Grevholm, 2011; Valverde et al., 2002). According to Grevholm (2011), studies are required both of how students use textbooks, and of the relation between textbooks and students' learning outcomes. In recent years the research in this area has been growing, to a large extent due to the *Network for research on mathematics textbooks in the Nordic and Baltic countries* (Grevholm, 2011). In the thematic issue of NOMAD (vol. 20, no. 3-4) in total 11 papers are published, all addressing the role of textbooks, and most of them arising from the work in the Nordic network.

In Sweden it is apparent that textbooks are considered the most important feature in mathematics classrooms. To some students and teachers, mathematics is what is written in the textbooks (Brändström, 2005; Johansson, 2003, 2006). Since the 1990s, independent student work has been encouraged in Swedish classrooms (Ståhle, Vinterek & Österlind as cited in Hansson, 2010), and more than half of classroom-time is spent on independent work (Johansson, 2006). In this instructional mode, it is said that the teacher has less responsibility for knowledge generation, because the students are expected to plan and work on their own, on different tasks, independently of other students. From this perspective, doing a study in a Swedish classroom characterized by extensive use of textbooks during independent study time is worth pursuing.

In the *Learner's perspective study* (LPS) (Clarke, 2006), this form of classroom set-up was documented in one Swedish mathematics classroom. It was shown that in this classroom the students mainly carried out mathematical tasks in their textbooks at their own pace. In this case, concerning the chapter on mathematical relationships, the teacher usually gave some short introduction at the beginning of the lessons, after which he moved around to work with individual students. It was decided that a case study should be carried out, based on this type of instruction, focusing on one student throughout a sequence of lessons to seek answers to this question:

> What can be inferred about the role of the textbook in relation to the learning of linear relationships by this particular student?

Looking into details of a student's sense making process while learning about linear relationships through textbook use could illustrate how Rezat's (2006, 2009) framework on analysing textbooks use can be employed. The purpose is to inquire into the dichotomy of textbook use, whether it is used as an *object of learning* or an *instrument for learning*. When it is seen as an instrument for learning, then the different utilization schemes based on Rezat's (2009) four categorizations are employed in the analysis. The findings could contribute to a better understanding of the use of textbooks, especially independent use, and hence could also improve our understanding of what support the teacher could provide, as well as what a textbook could offer in terms of learning certain topics.

Although there are many studies on mathematics textbooks, less research has been conducted that analyses students' use of those textbooks. I am not aware of any study that focuses on just one student using a mathematics textbook. In this respect, this study differs from previous studies on the use of textbooks in mathematics classrooms. Also, Skolverket [The Swedish national agency for education], (2008, 2012) and TIMSS (Mullis, Martin & Foy, 2008) have indicated that mathematical skills, such as ways of reasoning and communicating, require more attention in mathematics classrooms. This could be looked into through an in-depth study of one student using all the available data. In addition, results of the Swedish TIMSS 2007 for year-8 students showed that the students had difficulty with selecting linear relationships that would describe a relationship between x and y from a given table of values (Skolverket, 2008). Furthermore, in the recent TIMSS report (Skolverket, 2012) it is mentioned that only 3 % of the year-8 students that took the test can be considered to have reached a higher level of thinking, meaning the ability to reason, draw conclusions, make generalizations and solve linear equations.

Method

This study has a qualitative research design. Given that this study focused on one student, and the data was drawn from the Swedish LPS team study, a non-emergent case study is employed. A non-emergent case study is suited when the data collection methods and analysis is through qualitative means and data were collected beforehand (Maykut & Morehouse, 1994).

The Swedish LPS team collected data in three different year-8 classrooms. Each teacher was identified by their respective principals as a competent mathematics teacher (Clarke, 2006; Emanuelsson & Sahlström, 2008). The LPS data collection design involved fifteen days of videotaping using three cameras that focused on the teacher, a small group of students and the whole class, as well as interviews with the teacher and some students. Some related documents, such as textbook samples and students' work were gathered. Of the three classrooms, one was selected to be the sample, as the class was found to have used the textbook extensively. Among the fifteen video recordings in this class, only data from lessons 6–15 were used. The first five lessons were regarded as familiarization periods, meaning that the data collected during these sessions were not considered to be of high reliability as the students were still adjusting to the presence of the researchers and the cameras. Purposive sampling was used for the selection of one student out of the twenty-six students in class. Martina (all names in the study have been coded) was selected because she had the most interactions recorded on video. Eight out of ten studied classes contained her recorded utterances, whether intended to her seatmates, the teacher or the whole class, or just thinking aloud. It is evident also from the data that she belonged to a high ability group in class and could even be the best student for result of the test on algebraic expressions and equations during Lesson 9 shows that she missed to answer just one item. Also, she confirmed in the interview that she usually helps her classmates, indicating that she is of good academic standing in class.

This study uses the English translated version of the original Swedish recordings, provided by the Swedish LPS research team. When needed, I referred back to the original Swedish data. In drawing out patterns that emerge on the role of textbook, the use of the Studiocode computer application aided in the organisation and initial coding of the qualitative data.

Theoretical framework

The framework for this study draws on several previous studies (Carlsson, Hake & Åberg, 2002; Cobb, Stephan, McClain & Gravemeijer, 2001; Mullis et al., 2008; Skolverket, 2011). The method used to describe some

features of the textbook is partly taken from a study by Valverde et al. (2002) while the main theoretical model is by Rezat (2006, 2009). Rezat's model is represented by a tetrahedron, where each of the four vertices represents the student, teacher, textbook and the subject of mathematics, respectively.

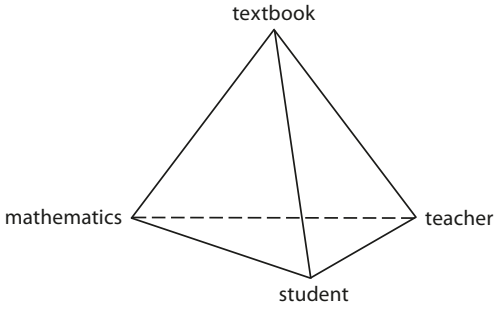


Figure 1. *The tetrahedron model on textbook use (Rezat, 2009, p.1261)*

This model is based on Engeström's (2000) activity theory model, which he claims is drawn initially from Vygotsky's concept of mediation. Rezat (2006) mentiones Engeström's work in his conceptualization of the model shown in figure 1 and also discusses Vygotsky's (1978) triangular model of mediated action. Since Rezat's (2009) model focuses on the analysis of mathematics textbooks use, it is this model that is adopted in this study. In this model, it is said that the use of mathematics textbooks is situated within an activity system composed of the student, teacher, mathematics textbook, and mathematics itself and implies that a way to investigate the use of mathematics textbooks by students is to consider all four vertices of the tetrahedron-model (Rezat, 2006, 2009). Rezat suggests that textbook use is implemented at all faces of the tetrahedron, also the face containing student, teacher and mathematics, because teachers use textbooks in lesson preparation and teaching, therefore his model suggests a multifaceted use of the textbook in the mathematics class. In this study only two faces of the tetrahedron are considered, textbook – student – mathematics, and textbook – student – teacher. From these two faces, the aim is to look at whether the textbook is seen as an instrument for learning or the object of learning, as well as identifying the utilization scheme types.

Furthermore, this study attempts to understand more about textbook use in a natural setting. This means that the student did not have to answer questionnaires or go for an interview specifically on textbook use, rather the student is followed for several lessons and evidence relating to her textbook use is collected. In this way, the possibility of understanding

more about textbook use through this process of data collection and analysis is explored.

The student's textbook with the title *Matte direkt year 8* by Carlsson, Hake and Åberg (2002) and the accompanying teacher's guide, are the primary sources for categorization of goals. These goals were supported by the categorization on cognitive domains for learning used by Cobb et al. (2001) and TIMSS (Mullis et al., 2008), as well as the five mathematical abilities identified by Skolverket (2011); that learners need to develop skills in problem solving, using concepts, performing calculations, reasoning and communicating ideas.

Some features of the textbook

The first edition of the textbook *Matte direkt year 8* and the accompanying teacher's guide are in Swedish so portions that are included in the discussions were translated to English. The pages for the chapter on Relationships mainly contain continuously numbered tasks that are color-coded and grouped according to the level of difficulty and goals. In the basic track (green-coded, 10 pages), where all students are supposed to do all the tasks to attain the chapter's goals, there are 40 tasks to be done individually and 2 tasks to be done in groups. After the basic track, the diagnostic tasks (yellow-coded pages) and the blue and red tracks follow. The results of the diagnostic test are supposed to suggest how the student should proceed, either by choosing the blue-coded exercises that are easy and provide more training, or the red-coded, more difficult ones intended for the more able students. For students who want to work more there are additional tricky questions right after the diagnostic items or in the last page of the chapter. In all, there are over one hundred tasks that a student could potentially work on, with some items having many sub-questions.

The introduction to the chapter on linear relationships enumerates four main goals. It states that after studying the chapter the student "should be able to:

- mark and name points in the coordinate system
- work with proportional relationships, for example comparing prices
- work with relationships that consist of a fixed and a variable part
- interpret different types of linear relationships"

(Carlsson et al., 2002, p. 171)

The same list is also mentioned in the accompanying teacher's guide with suggested pages to attain each goal, such as pages 172–174 (tasks 1–9

and a group game) for the first goal and pages 175–178 (tasks 10–28) for the second goal. It is apparent here that this particular textbook chapter can be more appropriately subdivided into "task item number". This structure diverges from other textbooks that tend to group exercises as "lessons". This was an advantage for carrying out the study as it was relatively simple to describe the mathematical learning goals and skills that the students were expected to acquire.

Focusing on the case of Martina

Drawing on the available data from Lessons 6 to 15, patterns emerge, concerning for example, how Martina used the textbook to support the learning process, how she related it to her previous mathematical knowledge and how the presence of the teacher affected learning.

Martina, the textbook and mathematics

One advantage of looking at only one student is that it enables access to knowledge about how individual learning takes place based on the student's own initiative without intervention by the teacher, and to what degree the object of activity is mathematical knowledge (Rezat, 2006).

It was apparent that Martina started to work independently on the textbook tasks, just as everyone else did, after the teacher had announced that the class should start working on their own. She appeared to take responsibility to go through the textbook tasks at her own pace, though at times she discussed some tasks with her classmates or the teacher. She referred to the answer key provided at the end pages of the textbook to check the accuracy of her answers and sometimes with the teacher. This set-up in the classroom of working independently through textbook tasks appeared to be the norm (Gallos Cronberg & Emanuelsson, 2013).

A study of four continuous pages of her notebook, where she wrote her answers to the textbook tasks, revealed that she appeared to work on most tasks required, and only missed two sub-items. It was also noticeable from the first three pages that every item was marked with an R, indicating all correct answers. Part of the video in Lesson 7 (coded 33:00–34:00) has captured her making these R marks after comparing her answers with those at the back of the textbook. However, there are also examples to show that she provided a different answer than the one provided in the textbook, but still marked this as correct. She did this when marking a task that asks the student to describe how it is possible to see from a graph that two friends on a trip had biked slower on their way back. The answer in the textbook is that *the line is not so steep*, while Martina wrote that *they biked half the distance for the same [length of] time*. Although her

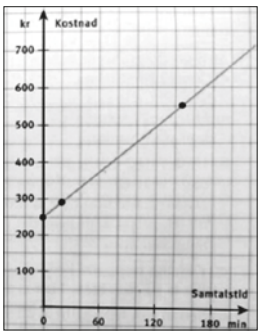
answer is not a generalized description of the graph that could indicate comparison of speeds, it is also correct for she appeared to refer back to the given situation and gave a more concrete description. Here it is apparent that Martina used the textbook as an instrument for learning for she did not primarily reproduce the text in the textbook. Indeed, she came up with her own solution to a certain task, hence, textbook utilization scheme types such as problem solving and consolidating mathematical knowledge and skills are also apparent.

Another indication that she used the textbook as an instrument for learning is by looking into the tasks about telephone subscriptions, shown in figure 2.

With Plingeling you pay only 250 kronor[1] *per month and 2 kronor per minute.*

The advertisement shows how much it costs to ring on a mobile phone. The cost consists of a fixed cost per month and a varying cost that depends on how long the call lasts. The relationship between the costs K and the length of calls x may be written as: $K = 250 + 2x$.
In the graph are points on the line that starts in point (0, 250).

Samtalstid min	Kostnad kr
0	250 + 2 · 0 = 250
20	250 + 2 · 20 = 290
150	250 + 2 · 150 = 550
x	250 + 2 · x



32. Use the relationship $K = 250 + 2x$ or the graph and compute how much you pay for
    a) 100 minutes b) 300 minutes c) 4 hours

33. With the competitor Telesmart, the cost is 150 kronor per month and 3 kronor per minute.
    a) Make a similar table as in the example.
    b) Draw a graph with the same divisions on the axes as in the example. Plot the points and draw the line that shows the relationship between cost and duration of the call.

34. Use the graph and decide how much you pay with Telesmart if you have talked for
    a) 20 minutes b) 100 minutes c) 4 h

35. Draw the line from the example with Plingeling in your graph.
    a) How do these lines differ?
    b) Which subscription rate would you choose? Explain why.

Figure 2. *Textbook tasks 32-35 (translated from Carlsson et al., 2002, p.180)*
Note 1. "kronor" is SEK, the Swedish currency

It was found that Martina was not able to answer all these tasks successfully. The fourth page taken from her notebook showed answers to textbook tasks 34–38, of which tasks 34 and 35 have wrong answers, see figure 3.

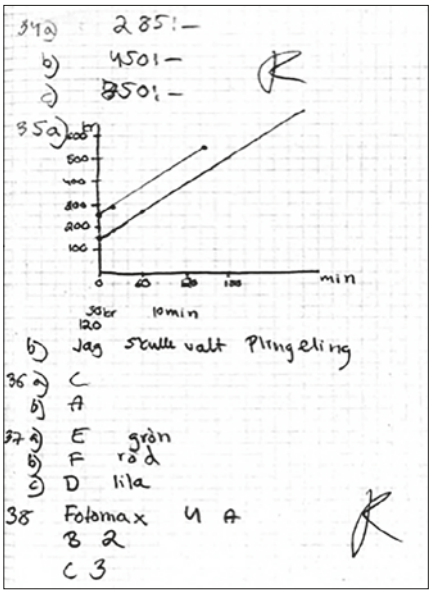


Figure 3. *Martina's notebook page showing answers to some textbook tasks*

Unlike in the first three pages of her notebook where all those items with R marks are indeed correct, in item 34 the marking of R is inaccurate. Her answer to 34a and 34c are wrong. It is possible that she used the situation for Plingeling ($K = 250 + 2x$) instead, and read off from the given graph where she got an answer for costs in kronor (SEK) as 285, 450 and 750 for 20 minutes, 100 minutes and 4 hours, respectively. Unlike the case for 100 minutes that is easier to read off from the graph, the 20 minutes and 4 hours situations require better estimation skills. However, task 34 is similar to task 32 where it requires computing for the telephone cost based on the equation so she could have patterned it from there as well.

As to Martina's answer for task 35, figure 3 shows that she drew the graph inaccurately, for the two lines appeared to be parallel lines when they should not be because of the different coefficients of x indicating different slopes. The equation $K = 150 + 3x$ should have a steeper graph and should intersect with the graph of line $K = 250 + 2x$ at $x = 100$. As the task 35a requires the student to describe how the two lines representing two

subscription rates differ, Martina did not provide this description while the textbook has provided an answer that *the lines start at different points and incline very differently*. For task 35b, her choice was Plingeling ($K = 250 + 2x$) but she gave no reason why. Neither did the textbook. If Martina's attempt to do this task is appropriately approached, the task may serve as a means to learn more about the properties of lines, the meaning of intersection points and some mathematical concepts that could possibly be new to her. In addition, it could also enhance the problem solving, reasoning and communicating skills for the choice of any of these two subscription rates is correct, depending on the reasons provided. Given her choice, which is Plingeling, the fixed cost is higher at 250 SEK. Still it is a good option because if one uses more than 100 hours of phone calls per month it is cheaper. It is in task 35 that the textbook appears to provide a rich opportunity for the student to show that the textbook could be an instrument for learning mathematics, for in this case there is a possibility for the student to understand more about the properties of lines and even get introduced to the next topic which is on systems of equations. Regarding the utilization scheme types as identified by Rezat (2009), given that this task is assumed to be difficult, attempting the task indicates a possibility to do problem solving, consolidate ideas and acquire new content.

Martina, the textbook and the teacher

In analysing the relationship between student, textbook and teacher it is expected that the student is the acting subject and the textbook is the object of the student's activity, while the teacher's role is to mediate between the textbook and the student (Rezat, 2006). In the case of Martina, her ways of carrying out textbook tasks as well as her interactions with the teacher uncovered interesting information, such as her ways of utilizing the textbook and the teacher's interventions. For example, in Lesson 6, it became evident how Martina, the textbook and the teacher's mediation interplayed. An example is in task 24 where she has to find the time it would take to drive 5 mil (1 mil = 10 km), given the graph and equation shown in figure 4.

Martina's conversation with the teacher went as follows:

Martina: Is it like the distance equals, thirty, times ... speed?

Teacher: Thirty, yes.

Martina: Times the time?

Teacher: Yeah, exactly. 'Cause if you take the time then, if you drive for one hour, then you will have driven, thirty kilometers.

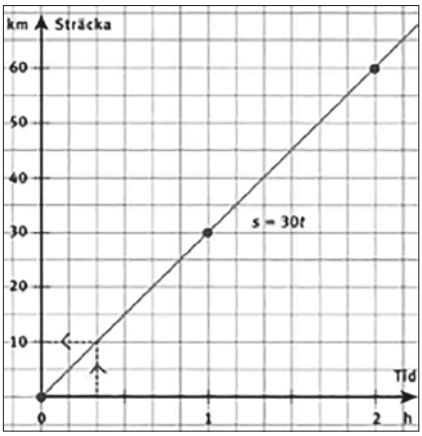


Figure 4. *Graph from textbook (Carlsson et al., 2002, p.178)*

Martina: Times t, yeah, exactly yes. Yes, ahhhhh!

[...]

Martina: Yes but it, it's thirty kilometres per hour that's for sure. Yeah, yeah, yeah, yeah so I get it then ... yeah right V T (writes) ...

Teacher: If you got the distance and the time and you want to find out the speed, then you cover that over ...

Martina: Then I divide that ... ah right okay.

It appears that Martina was trying to confirm the accuracy of her answer by explaining her solution method to the teacher and that the teacher confirms her solution. There is an indication that she used the equation "distance is equal to the product of speed and time". It was not evident, however, why she preferred to use an equation as a tool in solving this task, rather than reading the answer from the graph, which could be simpler for she would just have to read off the x value when y is 50, although she would still also need to convert the distance unit from 5 mil to 50 kilometers. In using the equation, there is a possibility of coming out with a quite complex answer of $1\,^2/_3$ hours or 1.667 hours rather than a more common way of expressing time as 1 hour and 40 minutes that could be directly read off from the graph. There is no evidence though that the teacher suggested other solution methods like using the graph, only that he provided a specific example, that of driving one hour and a tip to solve the equation by covering parts of the equation. This specific example appeared to have helped Martina to confirm her solution. However, tasks 10–23 are similar to task 24. It would have been a way to confirm her answer also by referring back to these

previous tasks and sort of duplicating the process, so she did not need a confirmation from the teacher. Nevertheless, it shows that Martina utilized the textbook more as an instrument for learning through solving tasks and problem. On the other hand, this is where the textbook could be considered as an object of learning for the teacher provided support through a more specific situation, that 30 kilometers could have been driven in an hour. Also, this indicates that the teacher is still considered as an important authority in the learning process.

Another piece of evidence showing the interplay between the textbook, student and teacher was when Martina was working on task 13 about the price of bananas given to be 15 SEK per kilogram. The task was to make a table of values between weight of bananas versus price that would generate three points and then draw a diagram showing the relationship between weight and price. Martina expressed her ideas on how this task could be

Martina: [Calls teacher's name], can you just check to see if I got this right ... I've done some sort of dots here now.

Teacher: Yes, one kilo, fifteen.

Martina: That one kilo fifteen kronor, two kilos thirty, five kilos seventy five ... If it's correct you just do like this?

Teacher: Yes.

Martina: Is it this that's called one of those ... things which you should ...

Teacher: Yes, a proportionality.

Martina: Yes.

Teacher: What's a proportionality then?

Martina: A line like this.

Teacher: Yes...the important thing with a proportionality is it rises the same all the time.

[...]

Martina: So that it means that if you have, if you have the line going here like

Teacher: Yes.

Martina: Can I do a point here if I know that the weight goes here, so you can just draw a line from here to there?

Teacher: Exactly. If you had, if you've got three points then you can draw this one as long as you like.

Martina: Yes, but you only really need one point, I mean.

Teacher: Well, yes, ok, if you know that the price is going up proportionally.

Martina: If you know, I do know, I.

Teacher: Yes, you can do it like that, the advantage is that you can go in, yeah, if I've got one hundred kronor how much can I spend.

Martina: Okay, yeah, okay.

Here Martina is opposing the idea from the book and from the teacher that three points are needed to draw a line for this case. She declared that only one point is enough, as she knows the proportionality constant anyway. It is not evident, however, how she finally found the correct answer, but she appears to have suggested a different solution. Looking at the pages of the book, there was no detailed discussion yet about lines in point-slope form or any discussion related to this. Although she could have derived her solution method from some of the previous tasks on proportional situations such as tasks 11–12 about prices of candies according to the weights in hectograms where the graphs start from point (0, 0). As evident in the transcript above, she described proportionality as "a line like this" and the teacher confirmed this to be correct. As her reference to "a line like this" or "the weight goes here, so you can just draw a line from here to there" was not visible on video, it is more likely she was referring to the line $y = 15x$ for this was the task she was discussing with the teacher. Moreover, there is also a possibility that she referred to one of the three "dots" she mentioned, drew a line through this one point and passing through the origin, disregarding (0, 0) as another point, when actually two points are needed for drawing the line in this way. Notably, the teacher appeared to provide an elaboration of a condition of drawing a line given one point, based on knowledge about the proportionally constant. He did encourage Martina to use her own way of doing the task, although he is not as clear as to why her solution method is advantageous. Here, the interactions between the teacher and the student that were based on student's textbook and the textbook-student-teacher face in the tetrahedron model, showed that the student did the task her own way and this revealed more about her problem solving skills and knowledge acquired than would have been visible if the analysis had been done just on the textbook-student-mathematics relation. Moreover, while the focus was on the student-textbook-teacher aspect, it became more visible that mathematics could not be detached from the discussion where the mathematics and mathematical skills of the student, such as using concepts, problem solving and reasoning and communicating ideas became more evident.

In summary, using the two triangular faces of the tetrahedron model has provided evidence that Martina used the textbook more as an instrument for learning. It also became evident that the teacher-student-mathematics aspect of the tetrahedron model is dealt with indirectly through the conversations between the teacher and the student when they discussed mathematics.

What can be inferred?

Addressing the research question of relating textbook use by the student, to mathematical goals or to the teacher's mediation, showed interesting results. The study showed that the textbook is used independently by the student. It is apparent that the textbook is used more as an instrument for learning mathematics rather than an object of learning mathematics for the mediation of the teacher is less in this form of instruction known as student independent work. The mediation of the teacher appeared to be on confirming the student's answers, providing more specific examples and agreeing to a different solution method from that in the textbook. It becomes visible though that the different utilization scheme types such as problem solving and acquiring new knowledge became visible when the two faces of the tetrahedron model are considered in the analysis. However, it is difficult to pinpoint which of the utilization scheme types dominate, as this student did the tasks following the order in the textbook.

This study shows that learning took place in this kind of classroom set-up where a student could rely mainly on textbook tasks with minimal mediation from the teacher. This also shows what the textbook could offer in terms of learning about linear relationships. As the textbook tasks are assumed to assist students in attaining the four listed goals, and as this particular student had done most of the tasks successfully, it can be concluded that learning about linear relationships has taken place. Indeed, as some researchers (Fujita & Jones, 2003; Valverde et al., 2002) have pointed out, textbooks provide opportunities for students to learn a school subject, and to influence and shape students' mathematical knowledge.

The study also showed that part of Rezat's (2006, 2009) framework could be used as a basis to study textbook use in this kind of classroom set-up. Indeed, to focus only on these two faces of the tetrahedron model he refers to as a quadrilateral structure (see figure 1 to refer to vertices of quadrilateral as that of textbook-mathematics-student-teacher, with the diagonal of the quadrilateral "textbook-student" included) appears to be an appropriate model to look into textbook use seen from the student's perspective. This way of analysing textbook use can reveal about the contrast on the use of textbook as an *instrument for learning* or an *object of learning mathematics*. However, while reflecting on these results, gaps still remain, making it difficult to infer the extent of learning on linear relationships that took place in relation to the stated goals, or the effectiveness of this classroom set-up on independent work. The next section highlights some of these reflections.

Some reflections

There is no doubt that this study has provided some answers to the problems investigated, yet it has also generated some questions that may require more discussions or further studies. One of these pertains to how the textbook was used. It is from Martina's video data, interview and documents that were used to describe how the features of the textbook supports learning and how it is being used. It appears that she used the book extensively as an instrument to access knowledge, by doing most of the textbook tasks with less support from the teacher. It seemed that in her case the book was the primary source for her to learn the basics of linear relationships. However, she belonged to a high ability group in the class. The choice of a student belonging to a high mathematical ability has its limitations for it is unclear what happened to the other students in class. It would be interesting to investigate the progress of an average student, or a student whose first language is not Swedish, or a student who needed more help in using the textbook.

Reflecting on the learning goals as enumerated in the textbook and teacher's guide, it seems unarguable to say that this student has attained all these goals for she was successful in doing most of the tasks. However, it seems too simplistic to assess the extent of learning based on the number of textbook tasks done by a student. Given that some suggestions of the learning goals were stated briefly, it would be difficult to assess if they were attained. Moreover, even if this had been possible, mathematical skills encouraged by Skolverket (2011) such as communicating, problem solving and reasoning, would be even more difficult to assess. This area needs further studies to learn more about the quality and development of thinking that could be associated with the use of the textbook. By further research we could deepen our understanding on what textbooks can offer for students' opportunities to learn, especially in a classroom set-up where independent learning through the use of a textbook dominates.

It is also difficult to assess Martina's abilities in the domain of reasoning or her communication skills, as she did not answer some items requiring these skills, such as task 35. For unknown reasons task 35 was not retained in the revised edition of the textbook. A similar task seems to replace this, that of a situation relating to the work payment per hour but there is no question that requires students to explain or provide reasons for their answers (Carlsson, Hake & Öberg, 2010). If most students have difficulties in doing tasks requiring them to apply their skills in reasoning and communicating, skills that are also considered important, then not including such tasks in the textbook could be debatable. It could be worthwhile to do further studies into what mathematical tasks should

be included in the textbooks, or to look into other areas like teacher responsibility, where the teacher's support on the use of textbooks and the development of skills such as reasoning could be looked into. Teacher responsibility that relates to students' independent use of textbook could be an area that is interesting for further investigation for Hansson (2010) argues that this mode of instruction, that stresses independent work, is problematic. Studies on teachers taking more responsibility for summarizing solutions and ideas, and also encouraging students to share their solutions with the whole class, particularly if their solutions diverge or could give rise to interest and arguments with other students, could be worthwhile to investigate.

Lastly, the study is not meant to generalize mathematics classrooms, particularly concerning the use of textbooks in Sweden, or most Swedish students' modes of learning. Rather, it is anticipated to contribute to the enhancement of understanding of a learning model where an individual student participated as a learner in a class where student independent work and the use of a textbook dominated. Relevant studies in the future could be useful to gain further understanding about mathematics learning and teaching, specifically on the use of textbooks as the main learning resource.

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