An analysis of two 19th century Norwegian geometry books, and the reactions they caused

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Bernt Michael Holmboe (1795–1850), professor in mathematics, wrote several textbooks on mathematics, and his presentation of geometry was traditional and in conformity with Euclidean ideas. Christopher Hansteen (1784–1873), professor in applied mathematics, wrote a textbook on geometry where he challenged the traditional Euclidean geometry. This paper analyses two very different approaches to basic definitions in the two geometry textbooks written by Holmboe and Hansteen, and what reactions this caused in the contemporary society. The main focus will be on the understanding of basic concepts in geometry, and of parallel lines and Euclid's parallel postulate.

Towards the end of the 18th century, a great effort was made to establish mathematics as a school subject in the higher education in Norway, and a school reform that was introduced around year 1800 re-introduced proper teaching in mathematics. The mathematical community in Norway at that time was small, and all the participants necessarily became significant members of the community. This was a time of considerable development in the subject of mathematics, and that also influenced the debate about mathematics education.

The aim of this paper is to describe and compare the content of two school textbooks in geometry, and the reactions these books caused in the society and in media. The purpose of such descriptions is to give present day teachers insight in this important part of Norwegian school history and history of mathematics education. The different views in subject matter and in didactical perspective in these two geometry books was made public in a bitter and emotional controversy in the newspapers,

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which also had an impact on the future textbooks in geometry used in the learned schools of Norway. A relevant question would be: *What can we learn today from textbooks and didactical debates in early 19th century?*

There is normally not one approach in methodology that is used in conducting historical research like this research, but a set of steps may be followed. The method used in this study is a literature review with a thorough study of the books and documents, and a content analysis using a hermeneutic approach. Hermeneutics has been influential in the general formulation of interpretivism as an epistemology, and the central idea is that the analyst of a text must seek to bring out the meanings of the text from the perspective of the authors. Mathematical textbooks published almost two centuries ago are sensitive to the context within which they were produced, and therefore a hermeneutic approach seems correct (Christiansen, 2008).

Cathedral schools[1] were schools from the medieval time that were connected to cathedrals, and they were meant to give a theological education to future priests. All cathedral schools were turned into *latin schools* or *grammar schools,*[2] when the reformation was introduced in Norway in 1539, and it was mandatory for every town to have one. The new Latin schools, together with the old cathedral schools, constituted the so-called *learned schools*[3]. Most of these Latin schools were, however, of very poor quality, so in reality, the higher education in 1814, preceding the university, was only four cathedral schools with a total of 200 pupils, in addition to some that had private tuition. By a governmental decree in 1809, the pupils started at the learned schools at the age of 9–10 years, and the duration was normally eight years consisting of four two-year grades, and each day at school was seven hours – four before noon and three after. The learned schools gave a classic education, and a higher education in scientific subjects at the same standard as the learned schools could be achieved at the Military Academy. This school admitted pupils from the age of 12–14 years. Several intermediate schools4 were established in smaller towns after 1814, these were learned schools without the upper two-year grade (Andersen, 1914).

The University of Christiania[5], established in 1811, was in function from 1813, and the only use of mathematics in the beginning was for the "examen philologico-philosophicum" – a preparatory exam for other subjects at the university. The lectures in mathematics were on trigonometry, stereometry, basic algebra, and later applied mathematics after Christopher Hansteen's appointment. The university qualifying examination[6] was arranged by the university.

The Elements (Euclid, 1956), collected by the Greek mathematician Euclid of Alexandria (approx. 325–265 BC) has for more than two thousand years been a model for rigour in mathematics, as well as containing knowledge of mathematics gathered in a deductive way. A well-known, and probably the most disputed, of the axioms in the Elements is the parallel postulate. The parallel postulate was for a long time accepted as obviously true, but some asserted that it was too complicated to be admitted as an axiom, and it ought to be a theorem. From Antiquity, several attempts have been made to prove it, but all without success. In the early nineteenth century, these attempts led to the discovery of non-Euclidean geometry.

The comprehension of the concepts of geometry have changed considerably, and I will in this paper discuss the use and understanding of the basic concepts, with special focus on *parallel lines*, in two Norwegian textbooks in geometry from 1827 and 1835, the former written by Bernt Michael Holmboe (1795–1850) and the latter by Christopher Hansteen (1784–1873), both professors at the University of Christiania. I am interested in who, and what ideas, influenced Holmboe and Hansteen when they wrote their textbooks. These textbooks were written for use in the learned schools of Norway.

Issues addressed in this paper, and earlier research about the textbooks of Bernt Michael Holmboe, may be found in Christiansen (2009, 2010, 2012a,b) and Bjarnadóttir et al. (2013). All translations from Norwegian and Danish-Norwegian to English are made by the author.

Background

Gray (2008, p. 83–84) says that "The Elements is a highly organized, deductive body of knowledge. It is divided into a number of distinct themes, but each theme has a complex theoretical structure". Book One of the thirteen books in the Elements (Euclid, 1956) starts with a number of definitions. Then follows some postulates and common notions, which we today would call axioms, and form the proofs for the following theorems. The traditional definition of parallel lines in Euclid's Elements states that "Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction" (Euclid, 1956, p. 202), and the fifth postulate, the so-called parallel postulate, states "That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles" (Euclid, 1956, p. 202).

There are several equivalent substitutes for the parallel postulate, and it is asserted that a parallel line to a given line does exist, and that it is unique. If we remove the parallel postulate and everything depending on it, we get a so-called ”neutral” geometry, or the ”core” of the Elements. There have been numerous attempts to prove the parallel postulate, but they have failed, mostly for using arguments that are equivalent to the parallel postulate.

Carl Friedrich Gauss (1777–1855) was probably the first mathematician to doubt the self-evidence of the parallel postulate, and to conceive an idea of the possibility of a non-Euclidean geometry. He did, however, write little and published nothing on the subject. His ideas have been deducted from his correspondence and posthumous works (Ewald, 2005).

Janos Bolyai (1802–1860) and Nikolai Lobachevsky (1792–1856) tried independently of each other to investigate the independence of the parallel postulate from the Elements, and their works led to what was later called non-Euclidean geometry. Further references on the Euclid’s parallel postulate and various attempts to prove it may be found in Gray (2008), Greenberg (2008) and Ewald (2005).

The textbooks by Holmboe and Hansteen

Bernt Michael Holmboe (1795–1850) was born in southern Norway. He worked from 1818 to 1826 as teacher at Christiania Cathedral School, then as a lecturer at the university from 1826 until 1834, and after that as a professor. Holmboe wrote textbooks in Arithmetic, Geometry, Stereometry, Trigonometry and Higher Mathematics. These were the textbooks in mathematics that were predominantly used in the learned schools in Norway between 1825 and 1860, a decade after Holmboe’s death. He was probably one of the most influential persons in the development of school mathematics in the first half of the 19th century in Norway. His ways of presenting the subject matter was in many ways very traditional, and they were challenged by his colleague and former mentor, Christopher Hansteen. As teacher at Christiania Kathedralskole, Holmboe earned his reputation as Niels Henrik Abel’s teacher in mathematics. After 1826, Holmboe also held a position as teacher in mathematics at the military academy (Christiansen, 2009, 2010, 2012a,b).

Christopher Hansteen (1784–1873) was born in Christiania in Norway. He was first a law student in Copenhagen, but became interested in the natural sciences when he met the physicist H. C. Ørsted. He became a teacher in applied mathematics at the university in Christiania in 1814, and he was professor from 1816 to 1861. Hansteen was very productive, and wrote about terrestrial magnetism, northern lights, meteorology, astronomy, mechanics, etc. He was a well-known scientist, and received further international recognition after an expedition to

Figure 1. *Bernt Michael Holmboe (left) and Christopher Hansteen (right)*

Siberia in 1828–30 to study the geomagnetism. In 1835, Hansteen wrote a textbook in geometry where he challenged the traditional Euclidean geometry (Christiansen, 2012a, b).

Holmboe (1827, 1833)

The textbook in basic geometry (Holmboe, 1827) starts with several definitions of basic concepts. The very first definition describes geometry as a science about the coherent magnitudes. Coherent magnitudes are the space with all available dimensions and time. According to Solvang (2001), Holmboe's way of organizing the subject matter was influenced by Adrien-Marie Legendre's (1752–1833) introduction to geometry (Legendre, 1817). The geometry of Legendre is constructed mainly the same way as Euclid, and starts with a long list of what he calls *explanations,* similar to what Euclid calls *definitions*.

Lærebog
i
Mathematiken.
Anden Deel,
indeholdende
Begyndelsesgrundene til Geometrien,
af
B. Holmboe,
Med 10 Tabeller i Steentryk.
Christiania.
Trykt og forlagt af Jacob Lehmann.
1827.

Figur 2. *Lærebog i Mathematiken (Holmboe, 1827)*

The first definition in Legendre (1817) defines geometry as a science which has for its objects the measure of extension. Extension has three dimensions, length, breadth and thickness. With reference to classification of coherent magnitudes in space and time, Holmboe classifies geometry in two parts:

1. The real geometry defined by the relations between the various magnitudes in space, without considering their changes in time.

2. Mechanics, defined by the changes the magnitudes goes through in time. All changes in a magnitude through time are called motion, and it is conditioned by force.

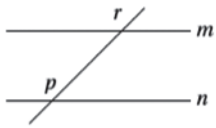
It is postulated that the space stretches indefinitely.[7]

Holmboe advises the teacher to show moderation in the review of proofs, and to show examples using numbers before the examination of the proof. This practical advice contradicts the structure of his textbooks, which is strictly Euclidean. There are few exercises and numerical examples, and the notion of construction means to elucidate the concept, not to use compass and ruler. Holmboe does not give any detailed instructions on how to use ruler and compass in this book, nor does he mention geometric locus. Instead he writes about elucidative[8], or abstract, objects, magnitudes and concepts. His idea may have been that mathematics teaching should educate the students with respect to formal logic by encouraging them to think and draw conclusions.

The fundamental statements of the straight line is that a straight line may be prolonged infinitely, one may always draw one straight line between two points, and one may never draw more than one straight line between two points. The part of the straight line that lies between the two points is the shortest of all lines drawn between the points, and it is called the *distance*[9] between the points.

Two of the chapters are called ”About two straight lines intersected by a transversal”[10] and ”About parallel lines”[11]. The first of these chapters gives a thorough description of all pairs of angles this situation produces. This chapter is followed by the consequences of two corresponding angles being equal, and vice versa. These situations have the consequence that the corresponding angles are equal.

The chapter ”About parallel lines” has a theorem with proof which states that when two straight lines are intersected by a transversal, such that an outside angle is equal to its corresponding interior angle, that is $\angle r = \angle p$ in figure 3, then the two straight lines cannot intersect no matter how far they are prolonged in both directions (Holmboe, 1827, p. 45). The structure of the proof is that if the two lines cross on one side



$$\begin{aligned} \angle r = \angle p &\implies m \parallel n \\ \angle r > \angle p &\implies m \nparallel n \\ m \parallel n &\implies \angle r = \angle p \end{aligned}$$

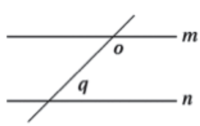
Figure 3. *"About parallel lines"*

of the transversal, then the two lines and the transversal form a triangle, where $\angle r$ is an outside angle. Holmboe has already demonstrated that an outside angle of a triangle is always greater than any of its interior angles, so therefore $\angle r > \angle p$, which contradicts the condition. This proof is followed by Holmboe's definition of parallel lines:

> Two straight lines in the same plane that do not intersect when prolonged indefinitely to both sides, are parallel to each other, or the one is parallel to the other. [12] (Holmboe, 1827, p. 46)

In two following theorems, using the same situation of two straight lines intersected by a transversal, he demonstrates first that if the outside angle is greater than the interior, $\angle r > \angle p$, then the two straight lines are *not* parallel. He next proves that *if* the two lines are parallel, then $\angle r = \angle p$. This last proof is done by assuming that $\angle r \neq \angle p$, and showing that the lines then are not parallel.

In a following corollary he then states that if two lines are parallel, and intersected by a transversal, then the sum of the two interior angles equals $2R$. This is a consequence of the previous theorem that proves that $\angle r = \angle p$. This is followed by another corollary stating that if the sum of the two interior angles is not equal to $2R$, then the two lines are not parallel (figure 4).



$$\begin{aligned} m \parallel n &\implies \angle o + \angle q = 2R \\ \angle o + \angle q \neq 2R &\implies m \nparallel n \end{aligned}$$

Figure 4. *"About parallel lines"*

These two corollaries carry many characteristics of corresponding angles in the original text. It is the last one mentioned here that has the same wording as Euclid's parallel postulate, but it is not emphasized in any way. Holmboe is in his textbook very true to the ideas of the Elements in the way of introducing and presenting the subject matter, but without ever referring to or even mentioning Euclid.

Holmboe's textbook in geometry came in a total of four editions, but only the two first were published in Holmboe's lifetime. There are very few differences from the first edition to the second, and none concerning the concepts discussed in this paper.

Hansteen (1835)

In 1835, Christopher Hansteen published a textbook in basic geometry (Hansteen, 1835), which in many ways challenged Holmboe's textbooks. Hansteen's book was 278 pages, which is a lot more than is expected of a textbook in elementary geometry. The author is intentionally trying to tear down the walls that existed between the classical geometry on one side, and the newer analytical geometry and the infinitesimal geometry on the other. The basis of the textbook is real life, with references to artifacts like corkscrews, stove pipes and hourglasses. The presentation of the subject matter is very unlike Euclid's Elements. The style is narrative and written in the first person, sometimes very lengthy, and there are many numerical examples. Hansteen tried to expand Euclid's definition of straight lines and of parallel lines, and Euclid's parallel postulate.

Hansteen's textbook contains a comprehensive preface which also contains definitions of fundamental concepts. The first concept to be defined is the straight line (Hansteen, 1835: III–IV), which is also, according to Hansteen (1835, p. III–IV), "the foundation of geometry"[13]. It is of great importance that this concept is clearly defined, especially in a science that demands a consistent and logic practice. Hansteen presents five different ways a straight line may be defined:

Figur 5. *Lærebog i Plangeometrie (Hansteen, (1835)*

- ”A straight line is a line which lies evenly with the points on itself” from Euclid (1956). Close to this is also Baron Wolff ’s definition stating that ”a line is straight when a part is similar to the whole”.[14]

- Archimedes, and most French geometers after him, defined the straight line as ”the shortest trajectory between two points”.

- Some geometers regard the straight line as a hereditary concept that only needs to be mentioned to be understood, and defines a straight line as ”those things which is known to be a straight line”.[15]

- Abraham Kästner says that ”a straight line is that whose points all bear against one trace”[16], and he adds that ”no one will learn to know the straight line from an explanation, and no one needs to; but one may say something about it, that guides the attention to a closer attention to what makes it a straight line”.

- Finally, others say that ”when a point moves continually in the same direction, then its trajectory is a straight line”.

According to Hansteen, after such definitions, all geometers introduce a postulate which states that ”one may create a straight line between two given points, and prolong such a given straight line in any direction in both directions as one pleases”. Hansteen makes noteworthy objections to such a postulate by asking with what tool such a prolonging shall be made, and how to make sure that the line made by such a tool is homogeneous, or that it satisfies the demands made in the various definitions of a straight line.

Hansteen elaborates towards a definition where he lets lines be produced by the movement of a point, and there are two kinds. One kind has the quality that when two points of a part of the line are placed on two arbitrary points on the whole line, then all points of the part of the line will coincide with points in the whole line – analogously, if we let a part of a line move along the whole line, and the part always fits with the whole line. Such lines are called *homogeneous lines*[17] , and there are two types – *straight* and *curved* lines. A homogeneous line has the same curvature all over it, and all perpendiculars of any plane homogeneous line will, when duly prolonged, either intersect in one point, or do not intersect at all. There are in other words only two types of homogeneous lines in a plane – the straight line and the circle. When a point moves from one place to another in a space, then it describes a line. If this line is straight, it is called the direction of the motion. From the concept of the straight line we may derive the concept of the plane, and from these definitions we may prove that a line is straight when all the points in the line remain unchanged in the same position as the line is rotated around two arbitrary points on the line, and that a straight line between two points is shorter than any curved or broken lines between those two points. These two statements are not axioms, but theorems.

Hansteen writes that it is more proper that a craftsman, or a "mechanical artist", derives the rules for his practice from the definitions and theorems of the geometry, than that theoretical geometers shall direct their concepts and definitions towards this practice. The carpenter's planer and the metalworker's file are tools that are suitable for producing homogeneous planes and lines, and the geometer should not neglect to acquire the theoretical principles on which these methods are based. A *ruler* is described as a tool – made of wood or metal – by which one may produce straight lines in a plane.

The cause for the much discussed controversy Hansteen's textbook made was the handling of parallel lines. Hansteen states very clearly that the Euclidean definition of parallel straight lines, embraced by nearly all geometers, has all the logical errors a definition can have. He states correctly that parallel lines are defined, according to Euclid, by a *negative* quality, and not a *positive*. He continues by stating that the quality by which the parallel lines are defined is *outside all experience and test*, as it points towards the infinite. Euclid's definition may also not be used on curved lines, which may also be parallel – according to Hansteen (1835, p. 28); "No one will hesitate in declaring two concentric circles reciprocally parallel". There is a definition stating that if two lines in a plane never intersect, no matter how far they are prolonged in any direction, do not make an angle. There is, however, no mentioning that these lines are parallel.

Hansteen argued for an understanding of parallel lines where one lets a perpendicular to any kind of line move along this line in such a way that it always is a perpendicular. Any point on this perpendicular then describes a line, where any point's shortest distance to the original line is always the same. Consequently, Hansteen has this definition of parallel lines "Any line that is being described by a point on the perpendicular to a given line, when it moves along the same with an unaltered angle, is said to be parallel to the directrix"[18] (Hansteen, 1835, p. 59), where the characteristics of a line, parallel to another, are:

– It always cuts off equal parts of all its perpendiculars.

– Any perpendicular to one of these lines is also a perpendicular to the other.

A parallel to a straight line has in addition the following characteristics:

- The parallel is also a straight line.
- As these straight lines never intersect, they form no angle with each other.
- If the parallel lines are intersected by a transversal, then the alternate interior angles are equal, the corresponding angles are equal, and the consecutive interior angles equals $2R$.

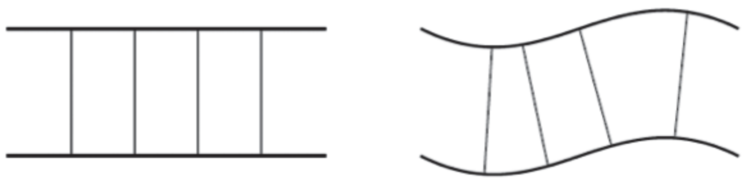


Figure 6. *Parallel lines*

The definition of parallel lines given by Hansteen is exactly the same as a definition given by ibn al-Haytham (965–1039) with the not insignificant difference that Hansteen does not restrict his definition to be valid for straight lines only (Greenberg, 2008; Katz, 2009).

By following these properties of parallel lines, Hansteen transforms Euclid's disputed axiom into a corollary which he proves (Hansteen, 1835, p. 70). If two straight lines, *KB*[19] and *CD* are intersected by a transversal *EF* in such a way that the sum of the two interior angles $\angle x$ and $\angle y$ is less than $2R$, then the two lines must necessarily cross when prolonged in the directions *JB*[20] and *GD* (figure 7). This is demonstrated by showing that $\angle x = \angle m + \angle n$ together with the premise $\angle x + \angle y < 2R$ gives that $\angle \mathrm{m} + \angle \mathrm{n} + \angle \mathrm{y} < 2R$. Since $\angle \mathrm{n} + \angle \mathrm{y} = 1R$, then must $\angle \mathrm{m} < 1R$, and the two straight lines *KB* and *CD* must cross.

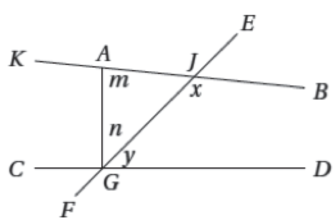


Figure 7. *Illustration to proof*

The logical fallacy in this proof is the first assumption that $\angle x = \angle m + \angle n$. This assumption presupposes that the sum of the angles in a triangle equals $2R$, which is equivalent to the parallel postulate that Hansteen is trying to prove.

Hansteen lets lines and planes be produced by the motion of points and lines, because this method gives the clearest conception of a line's direction in any point. One may easily imagine that a point in motion has a certain bearing in any place of its trajectory. Hansteen admits that some geometers object to this method since motion involved *time* and *power*, two concepts that are irrelevant to geometry, but belong in mechanics. Hansteen states that the motion of an immaterial point requires no power, and that we are only elucidating a motion in our minds (Hansteen, 1835, p. XII). The perpendicular in a point of a curve requires smoothness and differentiability, and one may easily find examples of curves where a parallel according to Hansteen's definition will cross both itself and the given curve.

He also claims that if two parallel lines are intersected by a transversal, and the sum of two interior – or exterior – angles equals $2R$, means nothing more than that the sum of two adjoining angles equals $2R$.

Hansteen's textbook was published in one edition only, and one reason may be that it contained much subject matter outside the school curriculum. He explains that because of a limited production of textbooks in Norway, he has added subject matter that is outside the curriculum of the learned schools, but that should be of interest for students that want to prepare themselves for a study of higher mathematics. It is also worthwhile to mention, as a curiosity, that Hansteen in his textbook introduces and describes the *metre* as a new unit of length (Hansteen, 1835, p. 81).

The controversy

Holmboe's textbooks were more or less controlling the Norwegian market for textbooks in mathematics in the first half of the 19th century. Hansteen's textbook challenged Holmboe's textbooks, and was the cause of a bitter controversy between the two professors in mathematics.

A newspaper polemic between Holmboe and Hansteen about Hansteen's textbook in geometry took place in *Morgenbladet* from December 1835 to January 1836, and in *Den Constitutionelle* from June to September 1836.[21] The core of the debate that followed was whether one in mathematics education should let utilitarian considerations overrule logical deduction and theoretical thinking. Hansteen declared that proofs should not be used in the elementary teaching before it was necessary for the students. This, he said, invited the students to memorize without understanding. To this, Holmboe replied that you either have to prove all or nothing, as half a proof is worse than no proof.

The polemics between Holmboe and Hansteen have later been called the ”dispute about parallelism” and they both published booklets where they justified their views (Holmboe, 1836; Hansteen, 1836).

The main article on the 5th of December, 1835, was written by Holmboe and called ”On Professor Hansteen's new understanding of parallel lines”[22]. It was a review of Hansteen's textbook and it was very critical of Hansteen's definitions of straight and parallel lines. Ten days later there was an unsigned article titled ”Concerning Professor B. Holmboe's article in Morgenbladet: 'On Professor Hansteen's new understanding of parallel lines' ”[23]. The author praised Holmboe for his ”touch of thoroughness”, but he continued that it was too much to expect from a man who had too long occupied himself with obsolete knowledge to be an impartial judge of new knowledge. Hansteen's signed reply to Holmboe's article was published on the 18th of December. He stated that Holmboe had reviewed his textbook in a very unseemly manner, that Holmboe considered Hansteen's textbook dangerous, and that teachers should be warned against it, so that young people would not be led astray from the rigour of pure and orthodox geometry into heresy and delusion. A short declaration from Hansteen appeared a week later, where he admitted that he probably never would agree with Holmboe about what a good mathematics textbook should be, and that he would publish a booklet the following week. Then there was a short notice signed by Hansteen, dated 18th of January 1836, titled ”To the purchasers of my textbook in geometry”[24], where he admitted that some explanations in his textbook might be simplified. He had therefore produced some new pages that by the end of the week would be available at the publisher, free of charge, to the purchasers of the book.

There was an unsigned paragraph in Den Constitutionelle on the 15th of June, 1836, indicating that a professor Jürgensen of Copenhagen had written a review of Hansteen's textbook in the *Monthly Journal for Literature*[25]. This review took no part in the controversy, but asserted the intention of making Hansteen's textbook known in Denmark. Three weeks later there was an article signed by Hansteen, titled ”On the teaching of mathematics in the schools”[26], where he indicated that the reviewer, that is professor Jürgensen, had been unfortunate with his review. Holmboe now rejoined the fray. In an article he opposed Hansteen by asserting that Hansteen claimed that the only controversies that had been proposed against his textbook in geometry was mainly the question of whether one is allowed to define a concept before one can prove its existence and possibilities. Holmboe wrote that this was not the case. Hansteen now wrote a long and final article, titled ”Farewell to Professor Holmboe”[27]. Hansteen ended his article with an anecdotal remark about Frederick

II of Prussia complaining over the difficulties of being at war with the Russians – ”not only did you have to shoot them, you also had to knock them over with your rifle butt”, meaning that you not only had to kill them once, you had to kill them twice. Hansteen concluded that he would leave Holmboe standing upright until he got tired – he would not take the trouble of knocking him down. This was the last newspaper article from Hansteen in this matter. Holmboe replied that he was surprised that Hansteen continued these polemics, even though he long time ago said that he would not. Holmboe also asserted that Hansteen had not read the booklet he published after the controversy in Morgenbladet. (Morgenbladet, 1835; Den Constitutionelle, 1836).

Summary of the controversy

Both Holmboe and Hansteen published booklets where they justified their views. Hansteen wrote a booklet (Hansteen, 1836) titled ”Investigation of Mr. Professor B. Holmboe's review of my Plane Geometry, Morgenbladet no. 339, 5th of Dec. 1835” [28], dated 26th of December 1835, which means that it was written towards the end of the period the polemics were active in Morgenbladet. Holmboe's name appears only in the title, later he is only referred to as ”the reviewer”. In addition to defending his own textbook, Hansteen also criticized Holmboe's arguments in the review, and he attacked Holmboe's textbook in geometry (Holmboe, 1827).

Hansteen's booklet is organized in five sections, labeled A to E where he focuses on five complaints from Holmboe's review.

A. *Absence of contingency proof.* [29] Holmboe's complaint is that Hansteen uses the attributes of lines and surfaces before he defines them. Hansteen starts his textbook by classifying lines as *homogeneous* [30] or *heterogeneous* [31]. Hansteen blames Holmboe for not respecting authorities like Newton and Laplace, and he attacks the definitions of basic concepts in Holmboe's textbook in geometry. Hansteen justifies his presentation of the subject matter by the fact that his book had been used for half a year at the Christiania Kathedralskole.

B. *Definition of a straight line.* Hansteen is accused of not using accurate descriptions and terms, and Hansteen argues with the fact that the textbook is written for children, and their only previous knowledge is their language, and names of concepts from their everyday life. Therefore one has to use a language that stimulates the imagination.

C. *"A circle is a circle"*. A vital error, according to Holmboe, is that Hansteen states that there exist only two homogeneous lines in a plane, the *straight line* and the *circle*, at a stage where it is not properly defined.

D. *Theory of parallelism*. The definition of parallel lines in Hansteen's textbook states that *a line parallel to another has the characteristics that it cuts equal parts of its perpendiculars*. This relates to straight as well as curved lines, and it follows that they will never cross no matter how long you extend them. [32] This definition is, according to Holmboe, not generally correct, as parallel curved lines may cross one another according to Hansteen's definition.

E. *Euclidean definition of parallel lines*. Hansteen states that it is better for a concept to be defined by a positive property than by a negative one, and parallel lines are by Euclid defined by a property that lies beyond our experience, and it refers our minds towards the infinite. He also attacks Holmboe's statement that "to construct is to elucidate the specified concepts of the definition of a magnitude" [33], and he finds it paradoxical that thorough knowledge of geometry does not assume the use of compass and ruler. How may such a mental construction elucidate the shape of a curved line, if it is defined by an equation between its coordinates, he asks. He also claims to have met students that didn't know one end of a compass from the other. Holmboe calls the use of compass and ruler an *insignificant requirement* [34] which should not be included in a textbook, and he claims that he has not found these instruments mentioned in textbooks by Lacroix, Legendre, Kästner, Wolff or Vega. Only the textbooks by Hansteen and Thomas Bugge mention the use of compass and ruler.

Towards the end of his booklet, Hansteen recommends that a new edition of Lindrup's textbook [35] should be made, if one wants easily understood textbooks in arithmetic and geometry that does not frighten students away from studies in mathematics.

Holmboe responded by writing a booklet (Holmboe, 1836) titled "Retort provoked by Mr. Professor Hansteen's enlightenment of my review of his textbook in geometry, containing: 1) Defense of the review containing proofs collected by a continued review of his textbook. 2) Refutation of his attack on my textbook in mathematics" [36], and this was dated the 8th of March 1836. It was written in the period between the two polemics in Morgenbladet and Den Constitutionelle. Throughout the booklet, Hansteen is referred to as "the author". Holmboe's booklet is structured in the same five sections as Hansteen's, and Section D is – not surprisingly – the most comprehensive. Holmboe shows a wide knowledge of the subject matter by quoting Klügel's definition of curved parallel lines from 1763, in addition to the textbook "Theorie des lignes courbes" by Lacroix. The latter does not call curved lines parallel. Holmboe admits that Hansteen is correct in his objection against Euclid's definition of parallel lines, that it declares a property that is beyond all experience, in the sense that the definition appears before it is proven that two straight lines in a plane could have such a location that they will never cross if they are prolonged indefinitely. Holmboe is very clear in adding that Hansteen's theory of parallel lines is in obvious conflict with the existing theory, which states that a curved line at a certain point is parallel to another curved line at a certain point, only if the tangents through each of the two points are parallel. The better part of Holmboe's booklet is a defense against the attacks made by Hansteen on his textbooks, and Holmboe constantly refers to Legendre and his definitions.

Some concluding remarks

Holmboe is in his textbook very true to Euclid in his presentation of the subject matter, without ever mentioning his name, and parallel lines are dealt with in a very thorough way. The difference between the two textbooks was rooted in whether in mathematics education one should present the subject matter in a traditional Euclidean way or not. There was an ongoing debate about the use of Euclidean ideas in textbooks in geometry, and when Hansteen published his textbook in geometry, it was evidently a controversial issue and his textbook was seen as an attack on the Euclidean textbooks.

It took many years for the ideas of the non-Euclidean geometry to be accepted by the mathematical community, and it was with the works of von Helmholtz and others that the meaning of these new ideas became accepted. Various models of non-Euclidean geometry in Euclidean space were introduced, trying to convince that the non-Euclidean geometries were as valid as the Euclidean from a logical standpoint, and to emphasize that the question of the "truth" of Euclidean geometry in the real world no longer had an obvious answer (Katz, 2009).

The first half of the 19th century was in many ways a turning point for higher education in mathematics in Norway. The position of mathematics as a school subject was strengthened through school reforms at the turn of the century, and the first university was established in Norway in 1811. Bernt Michael Holmboe's textbooks in mathematics were the ones that were predominantly used in the learned schools at that time. His textbooks were, as we have seen, not without opposition – an opposition addressing the use of proofs in elementary mathematics, and whether the introduction of geometry should be in a traditional Euclidean way, using logical deductions and theoretical thinking – as in the case of Holmboe – versus a more "informal" way using everyday language and terms.

Hansteen encouraged using a language that stimulated the pupils imagination, and he also used a definition of parallel lines, which – according to Holmboe – was not generally correct.

The issues addressed in the newspaper polemics were both the mathematical topics of geometry, and didactical issues – how geometry should be presented to the pupils. Today we find it impressing that a debate like this reached as far as the public press, and that it was given so much attention and space in the papers. This shows, more than anything else does, the position professors at the university had in the society.

Hansteen states in the preface of his textbook that there is no lack of good geometry books in the Danish-Norwegian language, but they are all very true to Euclid. It is Hansteen's stated intention to differ from not only Euclid, but also other textbooks. The world of mathematics had been through a development towards strong demands on rigour in definitions and methods (Christiansen, 2010), and Hansteen turned against this in his way of presenting the subject matter. Hansteen concretized the mathematical objects, and talked about straight lines as something one could make with a ruler, while the mathematical objects for Holmboe were something one had to elucidate in one's mind, and not to construct. For Holmboe, the mathematical correctness was the most important, while Hansteen had, what we would call today, a much more didactical approach.

Hansteen's textbook was only published in one edition, but in addition to being untraditional, Hansteen's textbook also contained much subject matter outside the school curriculum. Holmboe's textbook in geometry was published first in 1827 and in a new edition in 1833. After Holmboe's death in 1850, Jens Odén edited new publications in 1851 and 1857. Even if Hansteen's way of presenting the subject matter would be closer to how we today view didactics, tradition was stronger and Holmboe's books were used in the learned schools until they were replaced by textbooks by Ole Jacob Broch about a decade after Holmboe's death (Christiansen, 2009).

The pupils at the learned schools where normally somewhere between 12 and 20, and a newspaper debate about school mathematics today would probably be completely different.

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Notes

1 Katedralskoler.

2 Latinskoler.

3 Lærde skoler.

4 Middelskoler.

5 The capitol city of Norway, Oslo, was called Christiania from 1624 until January 1st, 1925. The spelling was changed to Kristiania in 1877 in documents from the government, and in 1897 in documents from the city (Kunnskapsforlaget, 2006).

6 Examen artium.

7 Geometrie er en Videnskab om de sammenhængende Størrelser. Sammenhengende Størrelser ere Rummet med enhver deri forekommende Udstrækning og Tiden. Med Hensyn til de sammenhængende Størrelsers Inddeling i Rum og Tid, inddeles Geometrien i 2 Dele. (1) Den egentlige Geometri, der bestemmer de i Rummet forekommende Størrelsers Forhold til hinanden uden Hensyn til deres Forandring i Tiden. (2) Mekanik, der bestemmer de Forandringer, som Størrelserne undergaae i Tiden. Anm. Enhver Forandring, som en Størrelse i Tiden undergaaer, kalles Bevægelse, hvis betingelse kaldes Kraft. Fordringssætning. Rummet maa tænkes udstrakt i det Uendelige. (Holmboe, 1827, p. 1)

8 Anskueliggjørende.

9 Affstanden.

10 Om to rette Linier, som overskjæres af en tredie.

11 Om parallele Linier.

12 To rette Linier i samme Plan, som til begge Sider forlængede i det Uendelige ikke skjære hinanden, siges at være parallele med hinanden, eller den ene at være parallel med den anden.

13 Geometriens Grundvold.

14 Linae recta est, cujus pars quæcunque est toti similis.

15 Qvæ linea recta dicatur notum est.

16 En ret Linie er den, hvis Punkter alle ligge hen mod een Egn.

17 Eensartede Linier.

18 A fixed line used in describing a curve.

19 KB is misprinted as HB in the original text.

20 JB is printed as IB in the original text, which was common.

21 See Morgenbladet (1835) and Den Constitutionelle (1836). The newspaper Morgenbladet was established in 1819, and was until 1857 a substantial voice for the opposition against the establishment, both literary and political. It was also the first daily newspaper in Norway, and it exists now as a weekly newspaper with a liberal, radical and intellectual profile. Den Constitutionelle existed as a daily newspaper in Norway between 1836 and 1847. The idea was to establish a newspaper on a considerably higher intellectual level than Morgenbladet. Den Constitutionelle made high demands on the journalistic content, and it introduced daily editorials (Kunnskapsforlaget, 2006).

22 Om Professor Hansteens nye Parallellære.

23 Angående Professor B. Holmboes i Morgenbladet No. 339, 1835, indrykkede Stykke: "Om Professor Hansteens nye Parallellære".

24 Til Eierne af min Lærebog i Geometrie.

25 Maanedsskrift for Literatur.

26 Om den mathematiske Underviisning i Skolerne.

27 Afsked til Professor Holmboe.

28 Belysning af Hr. Professor B. Holmboes Anmeldelse af min Plangeometrie, Morgenbladet No. 339, 5 Dec. 1835.

29 Forsømmelse af Muelighetsbeviset.

30 Eensartede.

31 Ueensartede.

32 Den almindelige Charakter for en Linie, som er parallel med en anden, er altsaa: At den overalt affskjærer ligestore Stykker af dennes Normaler; hvoraf altsaa følger for alleslags parallele Linier, saavel rette som krumme, at de, i hvor langt de end forlænges, aldrig kunne skjære hinanden.

33 At construere er at anskue det ved en Størrelses Definition fastsatte Begreb.

34 Uvæsentlig fordring.

35 The Danish teacher of mathematics, Hans Christian Linderup (1763–1809) published a textbook in basic mathematics in 1807.

36 Gjenmæle fremkaldt ved Hr. Professor Hansteens Belysning af min Anmeldelse af hans Lærebog i Geometrien, indeholdende: 1) Forsvar for anmeldelsen med Beviser hentede ved en fortsat Recention over hans Lærebog. 2) Gjendrivelse af hans Angreb paa min Lærebog i Mathematiken.

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