School mathematical practices as experiences of identity work: the learning journeys of three students

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This paper illuminates, through stories, how three students' school mathematical practices accumulate over the years into descriptions of identity in relation to school mathematics. The stories are based on semi-structured interviews with students who were successful during lower secondary school from the perspective of their mathematics teachers' reform objectives and according to formal assessments. There was not, however, parallel successes in their personal identifications with the school mathematical enterprise as a whole. The study shows that, from the perspective of the student taking part in school mathematical practices, a sense of belonging to learning communities considered as legitimate is of uttermost importance.

Research has shown that students may become alienated, disaffected or "underachievers" as a result of their participative experiences with school mathematics (Boaler, 1997; Ewing, 2004; Nardi & Steward, 2003). There are students who reject the subject to the extent that they want to exclude mathematics from their adult lives, some because they come to identify themselves as lacking some intrapersonal property (Bishop, 2012; Black, Mendick & Solomon, 2009; Reay & Wiliam, 1999). The study presented in this paper emerged from my initial concern to take students' viewpoints into account within an action research project that involved mathematics teachers in one lower secondary school in the Swedish-speaking part of Finland. Specifically, the paper illuminates, through stories created from interviews, how some successful students, but not others, may end up lacking a sense of belonging to a community of mathematics learners. The stories illuminate the learning journeys of three classmates, Joakim, Kristina and Nette, and span lower secondary school (Years 7–9) and into adulthood.

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The three students were successful from the perspective of their mathematics teachers' reform objectives, but their relationships to school mathe-matics emerged as very different learning journeys over the years. The study presented in this paper is unique in its long-term perspective on students' messages within a Finnish Swedish-speaking reform context.

Local reform of mathematics education

In early 1990s, a group of five mathematics teachers initiated a reform in their mathematics teaching. The teachers described teaching and learning as a "journey", where the teacher and the students travel according to how they think mathematical teaching and learning should be. However, now, as one teacher described the situation, "a crossroad on the journey was reached". The reform was implemented within a three-year action research project (Carr & Kemmis, 1994), which focused on "learning processes and assessment" and reflected many similar reform movements elsewhere (e.g. Black & Atkin, 1996; Simon, 1995) as well as the Finnish national educational policy of the time (Kupari & Haapasalo, 1993; Utbildningsstyrelsen, 1994). The objectives of the reform followed closely the 1994 national curriculum (Utbildningsstyrelsen, 1994) with an emphasis on the development of individual student's mathematical thinking and deep understanding of the subject. Problem-solving was set as the leading teaching principle alongside mathematical-logical requirements. For instance, as a complement to the textbook tasks, the reform teachers introduced two special types of problem-solving activities, "mini-problems" and "projects", both for formal assessment purposes and as tools for learning mathematics in depth.

Social constructivism was taken both as an epistemological lens and as a source of power for the local reform (Björkqvist, 1993). In the reform discourse, the notion constructivist teaching was not a "misnomer", as suggested by Simon (1995, p. 117). On the contrary, it was a fuel in developing the teachers' discourse about restructuring mathematics teaching and learning. In this discourse, students´ individual discovery of mathematical ideas was taken for granted as a cognitive learning goal of great importance, especially in relation to students identified as qualified for and motivated in studying mathematics. By explicitly probing the mathematical thinking of students, the teachers wanted to both encourage the discovery of useful and viable mathematical ideas described as an "active" type of knowledge, and support individual student's emerging sense of control and responsibility for learning to a greater degree than through their traditional teaching. In the midst of Year 9, mathematics teaching

in reform classrooms was differentiated according to the students' academic ambitions. Three new student groups were formed (vocational mathe-matics, upper secondary basic mathematics, and upper secondary advanced mathematics). The teachers summarized their reform intents as follows:

1 increase the amount and quality of interactions in the classroom;

2 spend more teaching time on problem-solving and strategies for problem-solving;

3 make the students more aware of their own responsibility for learning;

4 be more sensitive to the mathematical thinking of individual students;

5 broaden the range of learning/ assessment tools; and

6 develop continuous assessment and support of students' learning.

School mathematical practices

School mathematical practices is defined here as the repertoire of actions and activities that constitute a community of learners of mathematics in school. Participation in school mathematical practices denotes learning in both a social and an individual sense (Wenger, 1998), and as emerging within school mathematical actions and activities both in and out of school.

Participation does not inevitably suggest that a learner identifies himself or herself as a participator and contributor to the forms or contents of actions and activities of the practice. As Boylan (2004) points out, there is a difference between taking part in a practice such as school mathematics and "partaking of it in the sense of sharing the same meaning of the practice" (p. 196). A learner might in one moment be observed as passive, solitary or cautious. Nevertheless, this learner might be highly engaged and aligned (position himself/herself) with actions and activities that constitute the community. Another learner might be observed as active without experiencing his or her participation as a strong engagement (Ewing, 2004; Langer-Osuna, 2011). One student might say how he or she likes to first sit and attend closely to the teacher and then work individually the last 20 minutes of the lesson. Another student might say how boring mathematics is and that he or she listens and writes down what the teacher is saying without any joyful engagement. Both students describe how they

take part in the lesson activities in a rather passive manner. However, the similarities in modes of participation in the practice (attending, listening) seem to latch on to very different experiences, expectations and personal meanings. As the students' messages exemplify, the personal meanings related to actions and activities constituting a school mathe-matical practice may differ significantly between students.

School mathematical practices, as used in this paper, denotes what a school mathematical practice is from the student's perspective at certain moments in time and draws no boundaries between school mathematical actions and activities, and the experiences and expectations of a student participating in these practices.

Mathematics identity

The concept of *mathematics identity* is ubiquitous in mathematics education research (Black et al., 2009). The identity-related studies in mathematics education constitute, however, strong support for the claim that participation in school mathematical practices is a reflexive process that over time both produces and transforms how a student identifies himself or herself in relation to school mathematical actions and activities. The studies reflect variations in research methodologies and perspectives. Some studies take a broad perspective on mathematics identity as having a sociocultural origin (Sfard & Prusak, 2005), as related to teaching cultures and classrooms (Cobb, Gresalfi & Hodge, 2009; Ewing, 2004) and as a long-term effect of the assessments (Reay & Wiliam, 1999) of the daily classroom routines and interactions and how the student is positioned within this discourse (Bishop, 2012; Black, 2004; Langer-Osuna, 2011). Other studies view the mathematics identity as dynamic and shifting micro-identities arising in moments of interaction (Wood, 2013).

There is evidence of the mediating connections between teaching cultures and the participants', especially the teachers', beliefs and actions, and emerging differences in the students' mathematics identities (Langer-Osuna, 2011). Teacher beliefs affect teaching practices (Yero, 2010) and within teaching practices, different "pockets of student experiences" (Star, Smith III & Jansen, 2008, p. 30) are created. Thus, the conclusion by Bishop (2012) that "students in the same class, engaged in the same activity, abiding by the same classroom norms and participant structures can and do enact very different mathematics identities" (p. 43) appears reasonable. In the classroom, a student's success, for example, discovering an anticipated mathematical rule for solving a problem, may be taken by a teacher as evidence of this student's mathematical competence, perhaps even of a stable cognitive capability (Hodgen & Marks, 2009; Ruthven,

1987). In another classroom moment, the teacher may acknowledge the success of a student as a commitment to learning and engagement, but not necessarily to mathe-matical aspects of the discovery or to mathematical competence of the student. Different accounts of the same kind of classroom activity provide different resources for the students' identifications with school mathematics. Possibly students, whose mathematical contributions are repeatedly publicly valued as good or right, learns to respect their own thinking and over time begin to identify positively with mathematics (Bishop, 2012). Other students in the same classroom, on the other hand, may end up feeling marginalized by taking part in activities seen as confusing and that value right answers over understanding (Solomon, 2007). Over time a student's school mathematical practices may thicken into relatively stable, long-term constructions of how the student identifies him/herself in relation to school mathematics (Langer-Osuna, 2011).

In this paper, the notion identity captures how students identify who they are in relation to a school mathematical practice and its reform. I interviewed students to understand the reform process from their points of view; however what they told me about was their *identity work*: their "identity-in-the-making" (George, 2009, p. 201) in relation to school mathematics. This *emergent* character of identity work is different from how the notion appears in the studies by Lutovac and Kaasila (2011) and by Hossain, Mendick and Adler (2013). In these studies, a person's identity work in relation to mathematics is introduced as an *a priori* theoretical construct by the researcher. In short, Lutovac and Kaasila conceptualized identity work as a process of intentionally influencing one's mathematics identity, for instance, by reflecting on negative experiences from the school years. Hossain, Mendick and Adler (2013), who looked at identity from a post-structural standpoint, used the phrase identity work to "emphasize that identity is not essence but active accomplishments, neither fixed nor singular but multiple and fractured" (p. 37). In interviews, they saw how a dominant educational discourse about developing understanding mathematics in depth became part of the identity work of the students. Cast in the reform context of this paper, fostering students' discovery of mathematical ideas would be an analogous example of such a dominant educational discourse.

Methodological considerations

The research reported in this paper is a storytelling case study situated within an interpretive research paradigm (Bassey, 1999, pp. 43–44). Bassey describes storytelling in educational research as giving "predominantly a narrative account of the exploration and analysis of the case, with a strong

sense of a time line" (p. 62). In the form of constructed stories, the paper illuminates three students' learning journeys from their first month in lower secondary school into adult life.

The lower secondary school culture was familiar to me from my earlier professional life. I conducted individual interviews with the teachers at the beginning of the reform work, and I worked closely alongside the teachers as an insider and research companion within the reform process. Using Wenger's (1998) terminology, I was also a broker who introduced elements of students' school mathematical practices as expressed in interviews into the reform process, and who allowed themes for the student interviews to emerge from the discussions at the formal action research meetings. On five occasions during the three reform years, I conducted, on school premises, semi-structured individual interviews (from here on called *school interviews*) with 28 students from the reform classrooms. The students clarified their views and perspectives on timely issues and activities discussed at the action research meetings. They also described and commented on their school mathematical experiences and expectations in relation to actions and activities during recent lessons. At the very end of the reform process, I selected three classmates, Joakim, Kristina and Nette, as key research participants for the following reasons. They were taught by the same reform teacher, Per, for most of their mathematics lessons in lower secondary school. They all participated in five school interviews and their utterances were elaborated and vivid. And finally, my preliminary interpretations of their messages indicated interesting emerging differences in their school mathematical practices. Approximately 10 years after the end of the reform process, I invited Joakim, Kristina and Nette to backward looking individual conversations (from here on called *adult interviews*) that focused not only on their memories of school mathematics teaching and learning, but also on mathematical practices more widely. My preliminary interpretations of their messages in the school interviews were negotiated as well. Concerning Kristina, for instance, I assumed that her school mathematical experiences were accumulating into a very limited view on the purpose of mathematical activities as finding a precise procedure for each task or type of tasks. Her statements as an adult strengthened my interpretation. The adult interviews were conducted on university premises.

Analyses and the construction of learning journeys

The primary empirical material analysed for this paper consists of six audio-taped semi-structured interviews, 12 to 80 minutes long, with each key student, 18 interviews altogether, and the full transcripts. The transcripts were imported into NVivo, computer software for organising and

analysing qualitative material (see NVivo, 2002). The analytic work was done in Swedish and the relevant parts were translated into English by the author of this paper. In the following, the analytical process is briefly described.

First, I compared and contrasted the students' answers to similar questions in the first interview (September, Year 7) and wrote short memos describing each student's initial relationship to school mathematics. To get a deeper sense of each student's emerging messages, I then simultaneously read the transcripts and listened to all of the interviews with each student numerous times. I noted aspects which seemed interesting, confusing or that could have some potential for further analysis. I took both the school mathematical practice and the students' relationship to the practice into account by listening closely to both *what* the student was saying/not saying, and to *how* something was said. I attended to pauses and to words that were often used. The word "task", for instance, appeared in more than 65 utterances from Kristina, but only 15 of these were answers to a task-related interview question. I took this as an indication of her participation in a school mathematical practice where a task discourse continued to dominate across the three reform years (Mellin-Olsen, 2009). Next, I deepened the analysis by interpreting the students' utterances as messages about their sense of belonging (Wenger, 1998) within the actions and activities that constituted their school mathematical practices, and by simultaneously construing expanded texts out of each interview, including the adult interviews. From the perspective of Wenger, a student's sense of belonging is connected to more than engagement with mathematical ideas through participation in a school mathematical practice. It is also about developing ideas through exercising imagination and to imagine one's place in school mathematical practices. In additioin, it is about the degree students align themselves with the actions and activities in the communities (Wenger, 1998, pp. 173–187). Finally, I compressed the expanded texts of the student interviews into short stories to illuminate each student's learning journey. The stories convey messages about how the students' relationship to school mathematics continuously emerged over the years.

Identity work as an emic notion

From the outset I had no intention to categorize or look for connecting explanatory patterns in the interview messages. Neither a student's mathematical identity (e.g. Cobb et al., 2009) nor identity work (Hossain et al., 2013; Lutovac & Kaasila, 2011) were a priori research issues. Through the school interviews, I simply wanted to give the students a chance to make their otherwise unobservable viewpoints heard within the reform

process. School mathematical practices as experiences of identity work emerged as an emic concern at a very late stage in the analytical process, initially as a notion which helped me to understand Joakim's emerging identification with school mathematics. Stake (2000) defines *the emic* as "evolving issues of the actors, the people who belong to the case" (p. 20). Joakim often talked about his relationship to school mathematical moments with references to a learning community inclu-ding himself. For instance, I interpreted Joakim's way of using the personal pronoun *we* in expressions like "*we* go through", "*we take problems* on the blackboard", "*we understand* how to calculate" as an indication of his membership in a local community of mathematics learners where his mathematical knowing was acknowledged as the socially accepted knowledge of a "mathematically able person". Kristina, on the other hand, indicated through messages like "*they* reshape the rule into a form that *you are expected to understand* when *they write* it down on the board, and *we copy*" that she positioned herself in a marginal position in relation to, and did not necessarily identify with, this community. Over the school years, the nature of Nette's identification with school mathematics gradually shifted. She initially conveyed a strong message about wanting to be included in a local community of mathematical learners/discoverers. However, at the end of Year 9, her main message was that mathematics is a dull and uninteresting subject that she dislikes but has to cope with.

Joakim, Kristina and Nette: three successful students?

In the beginning of Year 7, Joakim, Kristina and Nette were mathematically successful and engaged students. They expressed a strong commitment to learning mathematics and to developing the knowing, socially as well as cognitively, that they thought was expected from them and would be useful both in and out of school. Participation in school mathematical activities was expected to, at least partly, afford functional knowledge for use in their everyday lives and futures. All three regarded mathematics as an important, necessary and useful school subject, with some positive aesthetic features. Contrary to findings in other studies (e.g. Ewing, 2004), they were convinced that mathematics need neither be boring to work with, nor impossible to understand and they sensed future success in mathematical activities as within their reach. Nette referred explicitly to a future as a veterinarian, while Joakim and Kristina were generally confident in the significance of learning mathematics. Right through lower secondary school, their awarded grades in different types of assessments, generally continued to be on a high level, very seldom below 8 on a scale from 4 to 10. Before the split in Year 9, all three belonged to a

class referred to as "a group of high-achievers" by their teacher, Per. Per describes how the students in the group willingly explain to each other and help each other. Some boys he refers to as "smart" and "very active", while he describes girls as "silent" and "cautious". Some students in the group he refers to as "weak". In the adult interview Joakim ranked the group as one where "more than half were above 8 on average".

After Year 9, Joakim, Kristina and Nette continued at the same upper secondary school. As a follow up on their choices in Year 9, Joakim studied advanced mathematics courses; Kristina and Nette studied basic mathematics courses. At the time of the adult interviews, they had recently finished their university studies; Joakim in engineering, Kristina in pedagogy and Nette in developmental psychology. Their learning journeys were academically successful but, as the stories below will illuminate, they ended up in very different personal identifications with the school mathematical enterprise as a whole – Joakim never saw mathematics as a problematic subject. Kristina liked mathematics, but realized that it was not for her. Nette started her journey with a positive relationship to mathematics, but increasingly over the years learned to reject the subject and to blame herself for not having "a head" for mathematics.

Joakim: Mathematics was never a problematic subject

Joakim did not always enter the mathematics class "with a smile on his lips". However, he seems to have continued to strategically consider school mathematical work as much from the perspective of usefulness and importance as from the perspective of personal enjoyment.

Both Joakim's own accounts and evaluative remarks by his teacher indicate a continuously successful commitment related to learning mathematics in school. Joakim was in lower secondary school, as it seems, always engaged in the very real issues that concerned him, also when solving closed school mathematical problems of the textbook type. He persistently expresses a sense of being in control over his emerging identity as a learner of mathematics. Also, he relates positively and in a strategically accepting and loyal manner to the teacher and the teaching and assessment practices. His relationship to mathematics in school was relaxed; he was successful and had no reason to believe that his learning policy would work against him in the future.

From Joakim's perspective, the teacher addressed the students in an inclusive and sensitive manner which worked to guarantee his position in a community of learners. He participated in activities where rules he considered to be of a wider legitimacy were negotiated and where his contributions to the construction of the significant rules and their

meanings were adopted. He positions himself both as a rule-producer and a rule-adopter, and he seems to accept and meet the terms of, but not be confused or restrained by, a structured and rule-bound nature of the teaching of mathematical procedures and a need of memorising rules for later use in problem-solving and assessment. Joakim was definitely not disturbed by the rule-following, sometimes monotonous and unchallenging, nature of school mathematics he had experienced. His utterances strongly indicate that he continues to position himself as a member in a community of negotiators where the teacher was included. He continues to expect the community to be a space where his knowing is seen. He locates himself among those who *see* and *are seen* to see important mathematics, who cope with mathematics in school, and who know how "smart" persons legitimately act in the mathematics classroom. Furthermore, he talks about his willingness to abandon a successful problem-solving strategy in favour of "the one the teacher shows" if the teacher's strategy takes him more effectively or rapidly to the correct solution. Some weeks before leaving lower secondary school he finds no reason to doubt that the rules and solutions he is looking for when solving mathematical problems are there "somewhere in [his] head".

Joakim knows about the conceptual resources in mathematics. He knows there are rules to look for. When he finds them, he knows how to apply them. A bigger, but not impossible, problem is to discover order in a mathematical text, to decide what type of task it is. His tolerance for failure is sometimes minimal. However, he also expresses confidence in never being the only one who doesn't see a problem solution and who always understands the teacher's explanations. Thus, he feels no need to worry when he gets stuck. To give up indicates that the tasks are difficult, not that he and others who cannot do them are unable to think, or run the risk of exclusion from the community where the legitimate rules of the mathematical game are constructed. As an adult, Joakim describes himself as one in a group of "tough guys" who coped with the advanced mathematics courses and he refers to the "many wise ones in the group who performed well when they wanted". To *not* succeed in mathematical problem-solving was simply never an issue for him.

Whatever his mathematics teacher did actualize during lower secondary school Joakim always wanted to know as well as possible. He did not, he said, as an adult, "have any immediate need to question the need to know mathematics". His school mathematical practices seldom originated in any need to think differently. To him, school mathematical teaching and learning was a closed and strongly, but not blindly, aligned process of "paper filling" and knowing "the whole paper", his metaphors for the content of school mathematics. In line with his accounts in the

school interviews, he concludes as an adult that his goals for school mathematical knowing were ambitious, but not restricted to "survival", but to secure knowing of the mathematical content of "the filled paper". Mini-problems and projects of the reform practice were informative in the sense that they expanded his view on mathematics, but both were marginal to the contents of the usual school mathematical discourse.

> [...] if you see the content of the teaching of mathematics as an A4-paper you survive on knowing half of that paper. I wanted to know the whole paper. But I never gave so much thought to considering what if you would have crossed those borders a little, even though mini-problems and projects did show a little of what mathematics can be. (Adult interview)

The mathematics Joakim faced in school was seldom of the social and human nature he as an adult experiences in his profession. As an adult and an engineer, he expects mathematics to be a tool "where social issues and your personal decisions matter". In school it was more like "playing around with the models and rules you have learnt". But it was also to excitingly imagine a real need, a pseudo-reality, for the mathematical models and rules and to participate in a stimulating game of success where he was rewarded, in control, and never felt any real risk of being excluded from the community of "smart" persons. His acceptance of mathematics in school was based on experiencing its learning content as secure, hierarchical and absolute, but also on expecting its future meaningfulness in his life and on the support at home, and more importantly, on his confidence in the consistent and challenging support at school. The school support reminded him over and over again about his membership in a community of mathematically able persons.

> When you discovered a more difficult level, the level of great distinction [sw: laudatur nivån] the teacher always showed a [special sign] of support. I felt privileged. We had mini-problems and projects the other classes did not have; something more difficult than according to the curriculum; an image of the class to be proud of. You became like, uups, we are in fact rather able, we who know these things. (Adult interview)

Kristina: I liked mathematics, but it was not for me

Neither a positive relationship to school in general, nor good mathematical performances were enough to positively influence Kristina's sense of agency related to school mathematics during lower secondary school. Her

school mathematical practice was a "fight of existence" enacted on home grounds with family members as well as in the mathematics classroom: to maintain participatory positions not only connected to "who am I in relation to mathematics" but also to "what is mathematics for me". Both at home and in the mathematics classroom she seems continuously to have put her own mathematical actions alongside those of others and to have tacitly asked herself whether or not she belongs to the community of mathematically able persons. In relation to her mother, she positions herself as an "able" person mathematically. But in relation to essential mathematical negotiations during lessons, she locates herself in a marginal position. She was not a contributor and constructor in situations where "finding out the rules" was the focus for the participant's overt attention. She was a person in the margin of the "discovery-of-mathematical-ideas" – activities that were considered significant activities for understanding mathematics by her teacher.

> [...] [the teacher] told us in the beginning of lower secondary about some research project and that it is the right and proper thing that [teachers] should get the pupils to think independently as much as possible and that is why [the teacher] asks us, like, how to calculate things and doesn't tell us how to calculate; he kind of lets us discover an answer by ourselves; I might not discover an answer very often but others do. (December, Year 8)

The act of discovery is a solitary act, but socially situated within a group (she talks about "they") where answers are discovered and mathematical rules are reshaped. She does not consider herself as a knowledge producer. She copies the rules from the board, while the teacher monitors the legitimacy of everybody's work. Copying from the board is a legitimate action. On the other hand, her accounts indicate acceptance of a position among the rule-adopters and among the *we* who copy the rules in order to *later* discover their mathematical and social meaningfulness. However, in the midst of Year 9, and forced by the educational system to actively position herself as, as she said, either "a long or a short mathematician", this acceptance of delayed meaningfulness of hers may have turned into surrendering to the fact that the impossibility of a secure understanding was an ingrained aspect of her mathematical identity. At that time, she realized that she was not and never would be capable of the rapid construction of new meanings required by the constant flow of "new things" during lessons in the group aiming for advanced mathematics courses in upper secondary school.

During the school years and as an adult, Kristina never explicitly opposes her mathematics teachers and their teaching practices. On the

contrary, she clearly expresses confidence in her teachers and aligns strongly with the closed condition of mathematical practice. She argues that the closed condition, which includes learning through the textbook and assessment via textbook-type tasks, as inevitable for coming to know the legitimate mathematics that is negotiated during ordinary lessons. Over the school years, she continues to describe mathematics in a positive manner as an important school subject and as a tool to solve a variety of problems, including projects, in an aesthetically, controlled and orderly fashion. The "boring" aspects she describes as related to, among other things, the lack of time to develop the expected mathematical competence. Mathematics is "fun" in situations where she knows "all the tasks". As an adult she refers to her liking of the orderliness of school mathematics, for instance of the "thick, white and orderly" textbook. She describes the rule-bound activity of solving equations as "fun". She accepts a non-human face of school mathematics characterized by a pressure to perform and by absoluteness. Mathematical activity she describes as either correct or incorrect and resulting from either right or wrong thinking; there is a repeated and constraining rush that contributes to her marginal position during lessons. The tempo with something new "gone through on the board" of almost every lesson demands a constant alertness of her in order to understand mathematics discovered and structured by others. Her core relationship to school mathematical practices as one of "responsible acceptance" and "loyal engagement" is visible in all statements she gives in a negative voice such as the following. In December Year 8 she "would like to avoid mathematics in school altogether because it often is such a boring activity", but she also indicates her strong sense of responsibility. In Year 9, she emphasizes that when taking part in school mathematical activities "you cannot just think of those things that are amusing; mathematics is important, you know". Her loyal engagement is echoed as well in the adult interview where she remembers mathematics in school as a subject she liked very much, because "when you succeed and can keep up with the new things, mathematics is one of the most enjoyable subjects there are in school".

In the school interviews, Kristina described projects and mini-problems as activities with lesser legitimacy and as fundamentally outside the usual school mathematical practices that included "rules and systems", "what we are busy with in the theory booklet", "things to remember, different formulas and ways of calculating". In these usual practices, the security of mathematical knowing was granted cumulatively since, as she states, "some little more can be added all the time, so then it is easier to understand those more difficult tasks". To Kristina such practices seem to have been reconciled during the school years into what can be described as

”creative combinatory practices”. In these practices, mathematical ideas, rules and models must be learned for certainty in advance of, but not through, problem-solving. This problem-solving competency included coping with the constant risk of losing ownership of meaning and being excluded by the instructional tempo together with a confusing and chaotic multitude of rules and models. The dilemma is, according to the adult Kristina, ”that it is not possible to lock creative thinking in boxes as you can do with rules and models”. She describes a competent mathematics learner as constituted by a capability to internalize rules and models offered by a group of significant others to which she did not belong. The legitimate face of mathematics is emerging within ”given rules, types of problems and solution models”; mathematics in school is not a subject ”where you can discuss your way through things”. Mathematical competence means that a student has to be able to construct some sort of ”typology for seeing” out of these rules and models, to understand how to differentiate between types of problems, to remember rules good for each type of problem, and to know how to correctly apply these rules. Finally, as the number of problems grew in an ever-increasing tempo, ”it became chaos and too much”, and Kristina realized that it was not a matter of course that she understands mathematics.

Nette: Mathematics is for others, I don’t have a head for it

A negative trend in Nette’s school mathematical practices was clearly evident through her messages over the school years. Her mathematical experiences became frustrating to such an extent that she, within the first minutes of the adult interview, wanted me to verify her hope that my aim of our discussion was *not* to make her ”calculate mathematics”. She wraps up her negative relationship to school mathematics in a claim of her ”lack of a head” for a difficult and uninteresting subject, much of the legitimate content of which has turned out to be useless and without importance in real life. She offers solving equations as an example of such ”useless mathematics” and as the opposite of ”everyday mathematics”. She further claims that she had to let go of her initial professional plan to become a veterinarian, mainly for the reason that it undoubtedly included mathematics and the need to learn and to know mathematics during the school years. Moreover, she did complete an academic degree in developmental psychology. As an unauthorized substitute teacher with an academic degree in developmental psychology she now ”goes to school again”, and thus in some sense, re-experiences her own frustrating identity work to be and become a mathematics learner. She gets her own frustrations confirmed through the eyes of her students.

In the beginning of Year 7, Nette was strongly committed to take on the responsibility for learning mathematics by discovery in a solitary way. Her evident support of the pedagogical aims of the reform work was clearly visible, and continues to be so over the school years. She is content with a change from a "just doing the pages they gave"-mode of participation at primary school into activities that encourage students to "really think independently". She clearly puts confidence in school mathematics as a practice capable of affording her future successes in situations both in and out of school. Her way of using *we* indicates as well that she looks upon herself as both a rule-producer and a rule-adopter. Nette's strong alignment might, however, have worked as a roadblock for experiencing herself as a competent mathematics student in the long run. From her perspective, it was definitively an unacceptable position if solving mathematical problems meant to remember and to efficiently think about and apply rules and models, *without* an understanding of the mathematical ideas she is applying, including arguments for their use. Over the school years, Nette's interview accounts indicate, however, that the meanings of these mathematical ideas appear more and more strongly to be a matter of construction for *others* within practices where she was not a member, but for her to adopt and understand.

While Nette in the school interviews generally refers in a loyal and respectful manner to the teacher and the teaching practices, she clearly indicates resentment of situations where she was left alone in her struggle to understand the mathematical ideas. This might seem a contradiction as independent thinking was important for her. But, as it is the teacher who "has the rules", it is her wish that the teacher also fulfil an associated and self-evident obligation to *transmit* accurate meanings to students with the help of clear explanations. According to Nette, such teaching practices would grant students the secure knowing which they need to become able mathematical performers and problem-solvers. "The teacher has to see to it that you really understand what he means (...) without you asking for it".

From Nette's perspective, the struggle for understanding was accompanied with feelings of insecurity due to the absolute nature of mathematical knowing. There was no "twilight space" open in legitimate mathematics in school for neither hypothetical thinking nor real collaborative work and investigations. Legitimate school mathematical work she describes as mostly nonsensical and tedious, but on the other hand, it could not *in essence* be of a really investigative, social and playful nature either. She describes the non-human face of legitimate mathematics as inevitable. Mathematics in school became a subject consisting of "numbers and rules [...] a lot of figures and signs to and fro" and always about "rightness or wrongness". A socially important but dull subject

"you cannot avoid" and "just have to put up with" despite a more and more overwhelming disinterest.

Over the school years, Nette expresses disaffection to an ever-increasing extent as well as an emerging awareness of the social constitution of barriers in the participation of students in the classroom. For her mathematics becomes basically a dull, boring, monotonous, useless, meaningless and even painful subject. She acknowledges, however, that the tediousness she experiences may in the eyes of other students be related to situations experienced in an inclusive manner as communicating intelligible ideas and as affording joyfulness. Besides being exclusive to some, "the weak ones", she describes the school mathematical practices as marginal to "the quiet and conscientious ones" and as inclusive to the communities of "geniuses in mathematics" where she as an adult definitely doesn't position herself.

At the end of Year 9, Nette found mathematics lessons "very tedious" and indicates a total resignation and annoyance with such a disempowering practice. The persistence needed for taking part in the usual type of school mathematical activities had simply vanished. Small-talk with friends was her main empowering space for participation during mathematics lessons. As an adult, she still vividly remembers that such interactions were, however, considered disturbing legitimate participation, which was a matter of acceptance of the silent and solitary thinking for correctness in doing tasks "up and down" from "papers (...) super dull textbooks that all look alike; mathematics is right or wrong, if you call this into question, the teacher thinks that you are trying to escape; when you ask the teacher why, you get an explanation that it is just so."

In the adult interview, Nette jokingly tells about her total repression of memories from lower secondary school mathematical practices. Her jokes tell about a sense of neglect and diminishing trust in the teacher and the teaching practices, and about being constrained in a struggle for both making her knowing visible and expanding her problem-solving competencies. She especially remembers two constraints related to access to participation. First, her sense of "stupidity" related to the regrouping of students into "those who knew and those who did not know mathematics" in the midst of Year 9. Second, the disaffection she felt in the spring term of Year 9 when she and her friend grappled with making sense of some mathematical ideas. They wanted to understand and asked for explanations, but were met in a discouraging manner by their teacher. She quotes how the teacher commented on their requests for more explanations with "all the others understand, how come you don't understand". Nette's negativity related to this incident indicates how a comment, maybe thoughtless, from the teacher became reified into disengagement and a silencing barrier for participation from her perspective. In her comparative

accounts as an adult both mathematics as a subject and the student Nette appear to end up on the "wrong side", where to be absorbed and meaningfully engaged in learning and knowing mathematics was an exception, not a normal state in a person's school mathematical practices.

Concluding remarks

Through individual stories created from three students' messages about their sense of belonging (Wenger, 1998) within the actions and activities that constituted their school mathematical practices, this paper has expanded our understanding of the paradox that reform work and good results may go together with bad experiences and an exodus from mathematics (e.g. Bishop, 2012; Boaler, 1997; Ewing, 2004). A bad experience is undoubtedly to be a committed learner and to be assessed with good grades, but nevertheless see oneself as not belonging to, or as existing only on the margins of, the legitimate learning communities in the mathematics classroom.

According to George (2009), a teacher's influence on students' emerging identities related to mathematics has more to do with the pedagogy of the teacher than with the mathematical learning content (p. 204). The teachers of Joakim, Kristina and Nette were no ordinary mathematics teachers. They were teachers in a teacher training school with a strong commitment to restructure a way of teaching they described as "traditional". Over the three reform years, they devoted numerous hours to grappling with restructuring their school mathematical practices in line with the national curriculum and a constructivist epistemology. The teachers' beliefs in the power of constructivism and assessment to transform learning were shared with many other reformers at the time (Black & Atkin, 1996). With no doubt I would argue that they were teachers with good intentions and, as seen from the learning journeys of Joakim, Kristina and Nette, their hard work paid off in many ways. Joakim, Kristina and Nette were successful students from the perspective of the reform objectives: They left lower secondary school with excellent or good grades in mathematics. They were clearly aware of and accepted responsibility for their own learning, an attitude that probably enhanced their academic successes later in life. They wanted to actively take part in the actions and activities of mathematics in school and they especially appreciated project work as a learning and assessment tool. Through project work, they learned that a mathematical practice *could* deviate from the task discourse that continued to dominate the mathematics lessons. Despite all these positive aspects of their school mathematical practices their learning journeys developed very differently over the school years, which strongly affected their futures. There were, as we have seen, significant

consequences to learn, like Kristina, to situate oneself as not-belonging to a community of mathematics learners because of the "non-human" face of the legitimate school mathematical practice, or to learn to align oneself, like Nette, with a community of persons "lacking a mathematical head", or, like Joakim, with a community of "smart" persons. Why did these differences in learning journeys come about? The answer lies, I argue, in the nature of the school mathematical practice as well as in qualities of the emerging relationships between the student and the learning communities considered legitimate.

Kristina's learning journey shows that she responsively accepted and enjoyed school mathematical actions and activities; she aligned both with the reform-related activities as well as with the task discourse (Mellin-Olsen, 2009). In fact, the task discourse afforded her *a sense of security* in learning: mathematics was "fun" in situations where she knew the models and rules for solving each task. However, her compliance with a mathematical practice consisting of a collection of disconnected procedures, problems and solution models, in her own words "given rules, types of problems and solution models", resulted in her becoming more and more reluctant. This type of legitimate mathematical practice announces, she said, "here are the right answers". Kristina turned her back on the practice because it afforded her a narrow frame for creativity and limited social spaciousness. She wished to participate in a school mathematical practice that embraces large problems with mathematically rich activities and is spacious enough to include social relationships.

When I first met Nette, she was one of the few students who undoubtedly imagined a future including mathematics: she wanted to become a veterinarian. Her learning journey shows, however, that her strong trust in the usefulness of taking part in a discourse that continued to be dominated by tasks was slowly diminished. It turned into disaffection and alignment with those lacking some intrapersonal property needed for engagement in mathematical activities. Similar to Kristina and Joakim, Nette was strongly supportive of the reform-related activities in general, and she was explicitly supportive of activities that anticipate students to "really think independently". Furthermore, project work was a mathematical activity welcomed by all three. Also, purely mathematical project work could, in Joakim's words, "show a little of what mathematics can be" as well as afford students legitimate spaces for aesthetics and creativity different from the usual school mathematical practices. But as the projects were marginal to the task discourse, and generally introduced as outside school assignments, project work may in fact have contributed to the alienating experiences seen so clearly in the learning journey of Nette. Over the years, she realized that the school mathematical practice

she considered legitimate did not afford her the same type of thinking as when she became immersed in doing investigative project work. As a result, she identified the task discourse more clearly as not allowing her to become mathematically engaged in order to make sense of and to be interested in mathematical activities.

The learning journey of Joakim is filled with evidence that his membership in a learning community of "we who have found out" was confirmed over and over again. Owing to the reform, he describes himself as a "privileged" student; his school mathematical practices reconciled into a nexus of identification as a mathematically "able" person. This identification was strongly supported by his successful discovery of the mathematical rules and models needed for problem-solving and taking part in playful learning communities for understanding mathematics, including the teacher, where these discoveries emerged. The playfulness was a sign of inclusion and extraordinary challenge, support and concern by his teacher for helping "able" students like himself solve mathematical problems. However, from another student's point of view, signs of playfulness might, as well, have reified as institutional barriers. As the learning journeys of Kristina and Nette illustrate, benign-looking actions that, from the teacher's point of view, were meant as affording inclusiveness might in the moments of teaching for some students have implied the very opposite. For instance, in the gesture used by the teacher to indicate a process of mathematical discovery, mediated from both Kristina's and Nette's perspectives, the knowing connected more, or perhaps solely, to entertainment than to learning legitimate mathematics. Joakim remembered how "the teacher always showed a [special sign] of support". In Joakim's school mathematical practice the gesture communicated alignment within a learning community of "able" students, while Kristina and Nette remembered the gesture as the teacher's joke and as confirming the social bonds of learning communities to which they did not belong.

The teachers' reform intentions included a strong focus on both formal and informal assessment. The teachers acknowledged the power of assessment to increase student engagement and improve learning outcomes. Problem-solving activities and projects were tools both for support of the student's discovery of mathematical ideas and for assessment of the students' mathematical thinking and knowledge development. Strategies for continuous assessment of learning outcomes were on the action research agenda. Soliciting the students' thinking was important. A common question in the teachers' classrooms was, "How do you think here?". The effects of non-verbal and informal assessment acts, such as gestures and gazes (Björklund Boistrup, 2010) on students' learning was, however, never a focus for critical reflection (Carr & Kemmis,

1994). Neither was there any clear focus on *understanding* the students' mathe-matical thinking (Simon, 1995) nor to engage *all* students in collaboratively validating their mathematical ideas, such as discussing and questioning their own thinking as well as the thinking of others (Black, 2004). In the absence of teachers' field notes as well as video-recorded material from the reform classrooms these aspects were in fact impossible to embrace within the action research.

Finally, this article does not intend to disqualify the pedagogy of teachers who undertook a process of action research because they saw a need to restructure their *own* school mathematical practices. Yet, the article illuminates problems that the teachers certainly wanted to avoid concerning the school mathematical practices of *students*. So, you might ask, was the reform work of no use at all? Answering this question would, however, demand students' "non-reform" school mathematical practices to be set alongside the school mathematical practices of Joakim, Kristina and Nette, an approach that lies outside the case study research methodology of the present research.

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