Mediating artifacts and interaction in a computer environment

An exploratory study of the acquisition of geometry concepts

Jan Wyndhamn

Nine dyads of twelve-year old students were engaged in collaborative small-group activity in order to find a way to determine the area of a parallelogram via a computer program. The program introduced two models or metaphors of a parallelogram, one changing the perimeter with a constant area ("the deck-of-cards model"), the other changing the area with a constant perimeter ("the frame model"). By changing the form and size of a displayed parallelogram the students had the opportunity to explore and obtain the characteristics of the parallelogram. The interaction between the students and their interaction with the computer were registered for analysis. After twenty minutes work with the computer, only one dyad failed to obtain the idea of the area determinants. The students' discussion, characterized by active interpretation and mutual adoption in combination with the deck-of-cards model as mediating artifact, established a situation in which the students acquired the conception that the area remained invariant through a specific kind of transformation. This knowledge was then used to draw a rectangle with the same area as a given parallelogram.

Introduction

The mathematics classroom is an arena characterized by dynamic interaction. Interactions occur between teacher and students, among students, between the individual and the physical environment and so on (see, e.g. Cobb, Wood, Yackel & McNeal, 1992; Steinbring, 1989; Voigt, 1989; Wyndhamn, 1992). A persistent question is how to describe and make intelligible the relationship between the pupil's external physical and communicative activities and his/her mental and internal activities. Some scientists use the metaphor of tools as a vehicle for describing the interaction between the mind and the world (e.g. Bruner, 1966; Olson, 1976; Vygotsky, 1978). According to the Vygotskian tradition, thinking is not only a matter of representation in the mind, but the dialectical relation

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of a joint activity among people involving real objects, tools and mediational resources. The construct of 'tool-mediated action' is central in the theory of Vygotsky (1986) and his followers (e.g. Zinchenko, 1985). The idea is that external tool-mediated action can be transformed into internal, mental action. Zinchenko argues:

> Internalization is the activity-semiotic transformation not of tools, but of their meanings (ibid., p.102).

And Leont'ev, quoted in Zinchenko (1985), points out that

> the process of internalization is not the transferal of an external activity to a pre-existing, internal 'plane of consciousness'; it is the process in which this internal plane is formed. (ibid., p.107)

Uses of different tools or means in different contexts bring about different cognitive processes (see, e.g. Lave, 1988; Scribner, 1984). A teacher who disregards this fact - and this is easily done - may cause many 'cul-de-sacs' in mathematics education. In this paper, the main focus is on the role of cognitive artifacts in mathematical thinking in relation to understanding the determinants of the area of a parallelogram.

Theoretical background and research questions

The geometrical issue

To find the area of a general parallelogram, it is possible to count the unit squares it contains, but such procedures are tedious and give only approximate results. A general parallelogram can be transformed into a rectangle of the same area by removing a right-angled triangle from one side and replacing it on the other (cf. Wertheimer, 1959).

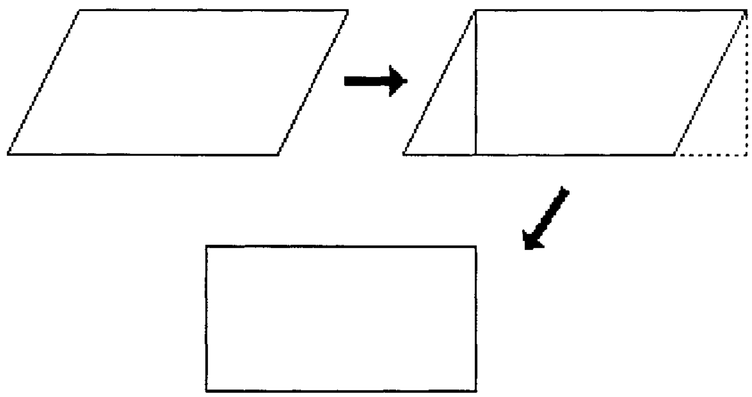

Figure 1. A general parallelogram becomes a rectangle ("the paper-cut model", see reference in the text)

The area of a parallelogram is the product of the length of one side (the base) and the corresponding height. The height is defined as the length of the perpendicular from one side to the opposite side.

In this context, I must mention the so-called Cavalieri's principle published in 1629 (cf. Gellert, Kustner, Hellwich & Kastner, 1977). Cavalieri, who was a student of Galilei, used a metaphor of a pile of equal sheets of very thin material (a simple illustration would be a deck of cards) to show that in spite of different shapes, solids with the same height and with cross-sections of equal area have the same volume.

A natural consequence of this principle is that (in this case) the area of the front side remains invariant. Thus, the area of a parallelogram is a function of its base and corresponding height.

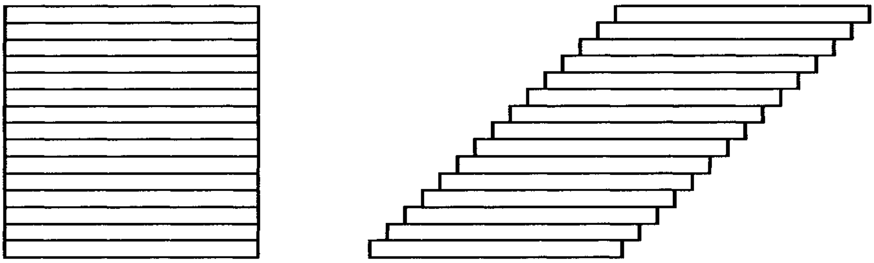

Figure 2. A special case of Cavalieri's principle ("the deck-of-cards model", see reference in the text)

The psychological dilemma

In the previous section, two different ways of conceptualizing the formula for the area of a parallelogram have been mentioned. From now on, I will call these procedures, following Sayeki, Ueno & Nagasaki (1991), "the paper-cut model" (Figure 1) and "the deck-of-cards model" (Figure 3). The paper-cut model has the character of deduction, while the deck-of-cards model has inductive qualities. As long as the number of cards remains constant in the deck-of-cards model, the area observed does not change. No cards have been taken away or added. This fact calls attention to the invariance of the 'thickness' of the deck or, in geometrical terms, the 'height' of the parallelogram.

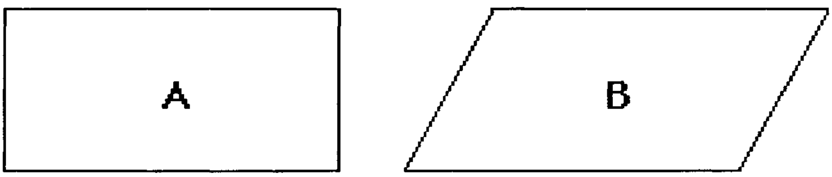

Figure 3. Deforming type 1 ("the deck-of-cards model")

The parallelograms A and B have the same area but different perimeters. It is precisely here that a psychological obstacle arises. The height of the parallelogram is often confused with the length of the shorter side (see, e.g. Hart, 1981). One possible explanation for this might be that a parallelogram can be 'pushed out of shape' quite easily in another way.

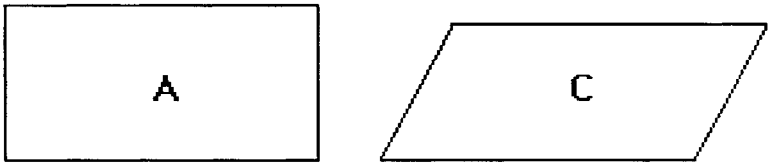

Figure 4. Deforming type 2 ("the hinge model" or "the frame model")

The shape of a parallelogram can be modified, by changing the angles without altering the lengths of its sides. This will be understood by using an analogy of considering four sticks held together by hinges. I will refer to this as "the hinge model" or "the frame model" (Figure 4). The parallelograms A and C have the same perimeter but different areas. In this case as well, it appears as if nothing has been added to or taken away from the original parallelogram.

The psychological dilemma can be formulated as such; the deformed parallelograms B in Figure 3 and C in Figure 4 look alike. Furthermore, many seem to assume on intuitive grounds that conservation of perimeter logically implies conservation of area and vice versa. The difference of the areas is apparent to the eye only when B and C are put next to each other.

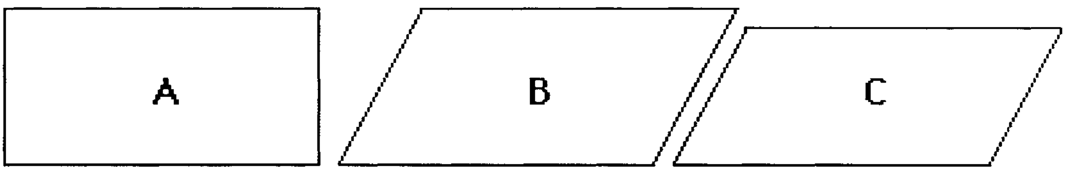

Figure 5. Comparison between the two types of deforming

In Figure 5 the area of the parallelogram C is approximately 90 % of the area of the parallelogram B or the rectangle A. However, the hinge model can be used to show that the rectangle (or parallelogram) can be flattened to an area of zero.

The kernel of the psychological dilemma is that area is an abstract concept, not a tangible one. An area cannot be 'seen', because it is not a direct product of observation, but it must be constructed intellectually. 'If the area decreases where does it go?' is a tricky question, hard to handle for pupils. Theoretical notions and perceptual cues interfere in what is for many a conflicting way. Schoenfeld (1986) shows how difficult it is for students to move from the geometry of observation (the perceptual field) to the geometry of proof (the abstract field).

The didactical problem

The learning of geometry has been analysed by many researchers. The van Hiele theory (van Hiele, 1986) looks at successful thinking and learning as occurring through a necessary sequence of levels which, in abbreviated form, can be described as follows:

Level 0. Visualization. A learner recognizes objects by their global appearance. (Objects as wholes.)

Level 1. Analysis. Learners can recognize properties of figures, but they do not interrelate these properties or the figures. (Objects as bearers of properties.)

Level 2. Informal deduction. A learner can establish relations among the properties of a figure and among the figures themselves. (Objects as bearers of logical relationships.)

Level 3. Deduction. The student understands deduction as means of developing a geometry. (Deductive reasoning in a global sense.)

Level 4. Rigour. Students are able to develop a theory without any concrete interpretations.

Since the area of the parallelogram can be found in a fairly deductive but logical way, the geometrical problem in this study belongs essentially to level 2.

In the van Hiele theory, it is asserted that the advancement through the levels is more governed by the instruction given than by age or maturation. Thus, five sequential phases of learning are proposed: inquiry or information, direct orientation, explication, free orientation, and integration. By the end of the last phase, pupils will have attained a new level of thought. The pupils are ready to repeat the phases of learning at the next level. The van Hiele theory is a "bottom-up" theory or, alternatively, a developmental theory, since the levels and the phases come in order. The processes on lower levels are supposed to precede and be independent of the processes on higher levels.

The didactical problem includes the difficulty of matching the student's level of thinking with the level of instruction. The desired learning and progress may not occur, if the teacher, instructional materials, vocabulary, and so on are on a different level than that of the student.

An inspiring experiment

In a study, Sayeki et al. (1991) showed that working with real cards according to the deck-of-cards model was a superior method to teach pupils to calculate the area of a parallelogram. Traditional instruction, involving a pair of scissors and pieces of paper and following the commonly used paper-cut model, required the teacher's extensive, time-consuming demonstration of the properties of the figures. The students in the experimental group, who were taught by means of the deck-of-cards model, discovered by themselves without difficulty the intended relations among the geometric properties. Post-tests, carried out one week after the instruction, showed a much better retention for the experimental group. However, the study also reveals that even though the students understood how to obtain the area of a parallelogram, they had trouble realizing what is happening when the frame or hinge model is used. The students misjudged the constancy of the area and they were reported to be 'surprised' before they clearly understood this kind of transformation or deforming. In this case, the students worked very practically with the outer cover of a matchbox. A conclusion therefore is that the deck-of-cards model and the frame model are supplementary to each other. The authors emphasize that the students'

> understandings are by no means restricted by the forms of the mediators. Their understandings were mathematically valid, purely "formal" (in a Piagetian sense) and yet deeply and firmly rooted in their everyday intuition of manipulating concrete objects (ibid., p. 241)

and that children

> may acquire through mediational tools, formal operations concepts which are strongly tied to (and therefore intuitively accepted as) the truth "afforded" by reality of objects in the world. (ibid., p. 241)

Concerning the two types of deforming, the crux for the student is to infer what aspects of the shape and the size are invariant through the transformations. In the Japanese experiment, the different kinds of deforming are presented one at a time: first the deck-of-cards model then the flexible frame model. In a concluding task, the pupils in cooperation with the teacher had to discover that the two transformations could coincide when a frame was adjusted to surround the deck of cards. The experiment had to follow this design by necessity, as the mediating tools are not compatible. Then questions arise, what will happen if the two types of deforming can be handled together on one and the same figure? Will there be any qualitative differences in the students' conceptions of the properties of a parallelogram? Will the students grasp the idea of invariance in such a situation? And based on the understanding of these variant and invariant features, will the student obtain the formula for the area of a parallelogram?

Thus, I am primarily interested not in making quantitative comparisons of improvements in learning or remembering, but rather in considering profound changes in mathematical thinking that accompany the usage of different cognitive tools. I am not looking for a tool only as a conceptual amplifier but as a mediator of a basic geometrical structure, or simply a mediating structure, that is a tool as a conceptual reorganizer (cf. Pea, 1987). Can new technology catalyze the mathematical thinking by allowing new forms of mediation?

Mediation in the computer environment

First, let me summarize what has been said so far. The topic is "How to obtain the area of a parallelogram" or formulated differently in Vygotskian terms "How to form, organize or shape an internal plane in the pupil for resourceful mathematical thinking relating to the concept of area". In the perspective outlined here, the pupil has to be an agent and interact or coordinate with an external and relevant structure embodied in a communicative medium. Mediating structures can be embodied in artifacts, arithmetic procedures, heuristics for problem-solving, checklists and so on, but also in methaphors, iconic representations, written language, and systems of mathematical notation (cf. Hutchins, 1986; Pea, 1987). In fact, all symbolic systems can be considered as mediating structures or cognitive tools (cf. Kaput, 1987; Ong, 1982; Säljö, 1992a). All these tools have a common quality: They represent externalized products of thinking, which can be analyzed, reflected upon, and discussed.

The computer environment provides new facilities or utilities and new cognitive tools; computer and screen complement paper and pencil and provide new mediational means and new possibilities for manipulating. The mediating artifact is software designed with specifically fashioned features rendered possible by the external representations, which are manipulable, dynamically linked and simultaneously displayed. The computer itself is a 'practical tool' and the control system for the operations can be seen as a 'meta-mediator'. A student can manipulate, explore and create relationships in the body of information both faster and qualitatively differently than is possible in a paper-and-pencil environment. In this way, some of the unique attributes of the computer are exploited.

Moreover, the computer context offers opportunities to analyse different interactional processes: between pupil and computer and between pupils. It is easy to arrange a situation in which the learner her/himself is in charge of finding a solution, that is a situation without any intervention of an instructor. Of course, the outcome of an activity is important to know, but also the processes by which the result has been achieved. To analyse the process, all the pupil's efforts can be stored in the computer and later scrutinized. Another aspect is pointed out by Sheingold (1987) who writes that

> computer-based activities 'invite' collaborating which can assist accomplishments for children both as individuals and in groups. (ibid. p. 204)

Thus, an analysis of the ways in which pupils talk, in a shared exercise, may uncover critical, significant points whereby the students' thinking is facilitated or reorganized. Are the students talking about the screen output, the underlying geometrical issue or the links between the two? These are crucial issues in the theoretical perspective taken here. Additionally, in a joint activity the conversation or discussion per se can be considered as a potential tool, generating new knowledge and having great impact on the problem-solving process (Säljö, 1992b).

Research questions

The purpose of this study was to shed light on these questions:

(1) How do the different models mentioned influence the thinking of the students about the area of a parallelogram, when the models are presented via a personal computer (PC)?

(2) How do pupils interact with the manipulable representations of parallelograms on a computer screen as mediating tools?

(3) How do pupils express themselves while they are working together in front of the PC?

(4) How do students co-operate in solving a task offered after working with the PC program?

Method

Subjects

Nine pupil-pupil dyads participated in the study. The students were 12 years old. The dyads were selected on a voluntary basis from two classes in the same school. In accordance with their previous academic achievement, each group was composed so that it could be allocated to one of three performance categories: 'high', 'average', and 'low'. The forming of the dyads was done by the class-teachers and there were three dyads on each achievement level. The students were familiar with the formula for the area of a rectangle.

Procedure

Each group - one at a time - was seated in front of a microcomputer (PC) with a mouse-driven pointer. The PC was placed in a special classroom for computer use in the school. The students were informed by the author that after working together with a program concerning parallelograms, they were going to solve a final task jointly. The students were shown two sheets of paper with drawings and were told that they would answer some questions and get some instructions about these papers, later at the end of the session. The program started when one member of the dyad keyed a first name on the keyboard. At the same time, this manoeuvre opened a file where all the trials made by the students were stored.

The computer program was developed*) specifically for this exploratory study and it is divided into four parts. Appendix A gives some illustrations.

Part 1. The Parallelogram Family was presented. The 'members' square and rectangle were shown as moving coloured drawings. The concepts of area and perimeter of a parallelogram were introduced in text and diagram, then the hinge model and the deck-of-cards model followed – in text as well as in colour and in motion. The surrounding text was read by the author. Questions that came up were answered but no more information was given.

Part 2. The students were given instructions on how to change the form and size of the displayed parallelogram. The mouse-driven pointer could be placed in four spots of different colours, and by pressing the two buttons on the mouse, four parameters of the parallelogram could be increased or decreased. They were:

Parameter	indicated by
length with constant angles	red
height with constant angles	green
perimeter with constant area (deck-of-cards model)	yellow
area with constant perimeter (hinge model)	blue

It must be noted that the geometrical terms were not shown explicitly on the screen together with the colours, nor were they mentioned by the author. The students thus had eight ways to change the drawing on the screen. The actual measurements of the area and the perimeter were displayed. The two students in each group could practise as long as they wanted in order to become acquainted with the combinations.

*) by Leif Linderbäck, Linköping

Part 3. This part was the most significant. The screen showed a parallelogram with the measurements (in proper 'computer units') 210 for the area and 62.6 for the perimeter. In five successive tasks, the students were asked to make a parallelogram with the following measurements:

Area	225	100	150	320	100
Perimeter	80	50	100	86	40

When the assigned measurements had been completed, a new task was automatically presented. Every task started with 210 and 62.6. When a correct measurement was found, the numerals changed colour to indicate to the students that their answers were correct. The pupil-computer interactions (items 2.1; 2.2; 2.3; see survey below and Appendix B) were saved on the special file, and the pupil-pupil interaction (item 3.1) was audiotaped and transcribed.

Part 4. Before the students set about the chief target problems, the group was offered the opportunity to revise. The students could repeat whatever they wanted from part 1 and 2 via a special menu. They could, for instance, again look at how the deck-of-cards model transformed a parallelogram.

After the computer-session, it was time for the criterial tasks that had been announced to the students. The papers, which the students saw at the very beginning, were shown again. The first paper with drawings was similar to Figure 3 and Figure 4 in this report, see also Appendix B. The pupils were requested to categorize the two deformings (transformations) in terms of 'the deck-of-cards' and 'hinges' (item 1.1), give reasons for their answers (item 1.2) and specify the properties of the transformations, that is, if the area or the perimeter was constant (item 1.3). On the second paper, a single parallelogram was drawn. The parallelogram had 10 cm and 7 cm sides. The height, corresponding to the longest side, was 6 cm but it was not marked in the drawing. There were no measurements in the drawing, and otherwise the sheet was quite blank without lines or a grid. The students were asked to make a picture of a rectangle with the same area on the paper (item 1.4). They used a pencil and a ruler (graduated in centimetres). The discussion between the students (item 4.1) was audiotaped and transcribed. During and immediately after the sessions, the author took notes.

The connections between the questions in the previous section and the procedure can be further clarified. Question (1) corresponds to the criterial tasks (items 1.1; 1.2; 1.3; 1.4 in Appendix B). The pupils' answers to question (1) are documented on the papers and in the transcripts. Question (2) - items 2.1; 2.2; 2.3 - can be studied through the printouts of the special file containing the attempts from each dyad (see

Appendix C). Question (3) - item 3.1 - and question (4) - item 4.1 - can be answered by analysis of the transcripts and through the author's notes.

Results

Below, the different dyads are labelled: 11, 12, 13 for high achievers; 21, 22, 23 for average ; 31, 32, 33 for low achievers.

The time for a session varied between 26 and 33 minutes. The differences between groups are not statistically interesting.

At an overall level, the participating dyads gave correct answers to item 1.1 – requiring them to identify the two transformations. All dyads, except group 21, specified in item 1.3 the properties of the transformations and solved item 1.4 – the drawing task – correctly. Here is an excerpt from the audiotape and my notes:

> Diana: There is no difference between 'the deck of cards' and 'the hinges'. Area and perimeter are the same all the time. It ought to be so.
> Diana drew a rectangle 10 cm long and 7 cm wide, and Doris did not object.

The most common way of drawing the rectangle in item 1.4 was to draw it somewhere beside the given parallelogram. Two dyads (12 and 31) drew the rectangle over the parallelogram. In doing so, they did not need to use the ruler as a measuring device, but they made use of the constant height and fixed length. Glenn in dyad 31 commented on Gisela's correct drawing by saying:

> The deck of cards lies very tidily (Swedish: jättesnyggt) when you start out.

Carola and Conny in dyad 13 was the only group to place its drawing on the paper at a level with the given drawing. The constancy of height was evident to Conny:

> The deck of cards becomes neither higher nor lower but longer.

In the students' reasons given (item 1.2) for their answers concerning the transformations, three different categories were found:

Reference to:	Examples of utterances:
Appearance	It looks like a deck of cards. The deck of cards is thick and standing more 'upright'. This resembles a deck of cards.
Differences (between transformations)	The hinges press down more than the deck of cards. The hinges put down more. The hinges slope more quickly (The statements are often reinforced by the movements of hands.)
Constancy of height (implicitly expressed)	This is the same ...(pointing to the height). The deck of cards remains ...(showing constant distance between the thumb and the forefinger and simultaneously moving the hand over the drawing).

The responses given by the dyads were not solely from one single category, but rather a mixture. Dyads 13, 23, 31, 32 and 33 answered according to the first category above; dyads 11, 12, 13 and 21 according to the second; dyads 13, 22 and 33 according to the third.

The interaction with the PC (items 2.1; 2.2 and 2.3) can be quantitatively described. Table 1 shows the total number of key punches during part 3 of the computer program.

Table 1. Total number of key punches per dyad

Achievement level of groups:				Sum	Mean
high	70	42	23	135	45
average	39	63	53	155	52
low	81	47	51	179	60
				469	52

The outcome pattern suggests a trend of direction: the higher the achievement level, the lower number of key pressings or trials. But, the within-group variation is considerable and there is no statistical significance between groups. Instead, I would like to present some further qualitative information which accounts for some of the numbers. One member of each of the dyads 13 and 21 (which show low numbers) had "computers at home" and they were used to working or playing with a mouse-driven PC. The members in dyad 11 (with a large number of key pressings) told me this was "the very first time" they had tested a PC. All the other pupils had tried "a little sometimes".

Table 2 shows the total time devoted to the five tasks in part 3, the problem-solving part of the program.

Table 2. Time devoted to problem-solving per dyad. 1 time unit = 30 seconds

Achievement level of groups:				Sum	Mean
high	27	17	10	54	18
average	22	22	21	65	22
low	28	26	21	75	25
				194	22

The pattern from Table 1 reappears. This means that every dyad made 2-3 trials during an interval of 30 seconds. Table 3 shows how the trials were distributed over the four parameters changing the form and size of the parallelogram.

Table 3. Number of key punches altering the parameters

	Achievement level of groups:			
Parameter:	high	average	low	Sum
length	26	45	45	116
height	38	49	42	129
perimeter	37	38	47	122
area	34	23	45	102
	135	155	179	469

The distribution over the parameters is even.

An analysis of the 45 sheets of paper displaying the efforts of the nine dyads to solve the five tasks in part 3 of the program exhibits four categories with respect to the strategy used (item 2.3). The paramount strategy is that after some adjustments primarily of the parameters of length and height, the dyads try to get the desired area, and then via "the deck of cards", that is by changing the perimeter, reach the complementary value. This was the blueprint for success which the dyads 11, 12, 22, 23 and 32 used intentionally in most of the tasks.

A few times, the correct value of the perimeter emerged first by chance and a temporary solution was to change the area by using "the hinges". However, dyad 21 consistently used this solution. This strategy constitutes a second category. Dyad 13 shows a third and more flexible strategy. Carola and Conny in dyad 13 tried constantly to minimize the number of trials. This was the only group which used the graduated axes on the screen output. The parallelogram was temporarily regarded as a rectangle.

Conny: We have got to find a smart solution.

The dyads 31 and 33 used a fourth strategy reminiscent of gambling. Their efforts followed no distinct or well thought-out design. All the same, it worked well for two dyads.

Now over to item 3.1 and the discourse in front of the PC. The content of the discourse is the different kinds of strategies. But which is the form? In analyzing the audiotapes, four categories can be found (an overview is shown below).

The different dyads were not consistent in their manner of expressing themselves. The categories overlap each other in a specific group. The first category was used by 21, 31 and 33; the second by 12, 21, 22, 23, 31, 32 and 33; the third by 12 and 23; the fourth by 11, 13 and 22. Apparently, it is easy to talk with reference to the colours.

Placing the outcome of item 4.1 - data concerning the interaction between the pupils when solving the drawing problem - in a survey as for item 3.1 is not possible. The nine discussions went in different direc-

Designation of category	Characteristic examples of utterances
Demonstrative level	(Pointing at the screen and simultaneously saying:) Take this one. Put the pointer here. Increase that. More there. Try that first.
Visual level	Take the blue spot, I think. Decrease with the yellow. Run with red.
Metaphoric level	Change with the deck of cards. Try the hinges.
Formal level	We try to hit the area first. Change both area and perimeter in one go. Reduce the area. The perimeter is far from correct. Add (Sw.: "plussa").

tions, and each discourse ought to be analyzed separately. I will return to this in the next section in a more elaborated way. However, the most prominent feature is that five dyads used the deck-of-cards model in their argumentation for a specific drawing, namely 12, 13, 22, 23 and 31.

The members of dyad 21 did not talk to each other in a collaborative way; they (as the only group) did not succeed in solving the criterial problems either. As mentioned earlier, one member (Diana) had access to a PC at home. Therefore, I think, she saw herself as superior to Doris. Diana solved most of the tasks alone and Doris said "yes" or was silent. When Diana did not get the right answer quickly enough, she pushed over the mouse to Doris saying: "Now it is your turn." The discussion had no turn-takings or comments. Diana started to draw a rectangle 10 cm x 7 cm and Doris agreed with this solution.

Two other dyads (22 and 23) started in the same manner by measuring the sides 10 cm and 7 cm, and drawing a rectangle 10 cm x 7 cm, but this drawing was corrected during the continued interaction. The pupils asked questions and argued to reach consensus.

Dyad 23.

Frank: Will the sides be the same here (comparing the two drawings on the paper)?
Frida: Do you mean 10 cm and 7 cm?
Frank: They must be shorter straight upwards.
Frida: Yes, but how long?
Frank: It must be ...
Frida: But think of the deck of cards ... These lines must be shorter (pointing to the shorter sides of the rectangle).

Frank: Yes, the measurement here ... between (pointing to the height of the parallelogram).
Frida: Reduce...
Frank: The same space (Sw.: mellanrum) ... 6 cm.
Frida: Mm (draws correctly).

The crucial point is to realize that the height should be the same. No dyad used the word 'height', but they found other expressions to communicate about the issue. Dyad 13 came closest to 'height' by talking about "how high /will the rectangle be/?" All groups, except 21, pointed at their drawings when discussing the problem.

Table 4 in Appendix D summarizes the results of the study. Several relationships can be found, but they are vague and no single one is significant. However, the main result of the study is that the deck-of-cards model seems to be easy to acquire and to apply.

Discussion and conclusions

The present study is about interaction and mediation, phenomena that, in a Vygotskian perspective, concern two different domains: a domain of internal mental activity (e.g. reasoning, learning, perception, attention) and a domain of physical and external objects in the broadest sense of the term (e.g. paper and pencil, abacus, timetable, words written or oral, symbols, gestures). Operations in the first domain are hypothetical, while operations in the other are as a rule observable. These two domains are related through two opposite processes. First, the physical objects are interpreted actively by the individual, or the material evokes thoughts in a more passive and less consciously controlled way. In this process, the material assists but at the same time constrains cognition. All desired operations are perhaps not admissible according to some rules. Second, the mental structures or operations can be projected onto the existing physical material objects, or new cognitive components can be applied to the material (cf. Wyndhamn, 1992). The second process externalizes the individual thinking in order to be communicated or tested. The two processes are repeated as well as mutually dependent. They can be seen as two sides of the same coin, that is making sense of experience.

The interaction mediates the relationship between humans and the world in which they carry out their real-life projects. Pufall (1988) discusses the relation between self and world, where world implies both tangible and ideational reality, and the relation between self and object. In this study, it is important for analytical purposes to maintain this distinction between the relation self-world and the relation self-object.

In Vygotsky's theoretical framework, mediation is achieved through tools ("technical tools") and signs ("psychological tools"). Vygotsky made his most concrete comments on the nature of semiotic (that is sign-based) mediation in connection with natural language, but his list of psychological tools also included:

> various systems of counting; mnemonic techniques; algebraic symbol systems; works of art; writing; schemes, diagrams, maps, and mechanical drawings; all sorts of conventional signs. (Vygotsky, quoted in Wertsch, 1991, p. 91)

A tool has assigned purposes, but many tools do not have meaning. The meaning is created through co-construction in social settings and involves both sense and referents (cf. also Bruner & Haste, 1987). Furthermore, Vygotsky (1986) distinguishes between spontaneous concepts and scientific concepts. Spontaneous concepts arise directly from personal experience and reflect the vivid meaning of that experience. Scientific concepts on the other hand, Vygotsky argues, are learned by means of formal instruction.

So, with this background, what can be said about the 'figure' of the experiment concerning the concept of area ('world') and mediating artifacts ('objects') in the computer environment?

The main interaction - in this study as well as in the Japanese one - is between the cognition of the pupil and the models of a parallelogram as a deck of cards and a frame respectively. The mediating structure is embodied in a 'microworld' (for an analysis of this concept, see, e.g. Hedrén, 1990; Hoyles, 1991). In short, a microworld is a conceptual system and sets a context within which knowledge can be construed. In the Japanese experiment, the medium was physically manipulable and in the present experiment the manipulation is computer-based. The results of the computer version verify the findings of the Japanese study: the two models of a parallelogram are powerful metaphors. They facilitate and support the pupils' understanding of the properties of a parallelogram; they are easy to interpret. After twenty minutes work with the PC, only one single dyad of nine failed to obtain the idea of the area. Thus, it is not a necessary condition for success for these pupils that the objects are physically concrete. The objects may be concrete or computer-displayed. The metaphors in themselves contribute to the creation of conceptions of the properties of the parallelogram.

The original meaning of the Greek word 'metaphor' is "carrying from one place to another". A metaphor can be expressed in forms or phrases such as 'X is Y', 'X is like Y', 'X is as if Y'. Metaphors mediate understanding. Nolder (1991) points out:

> Metaphor makes it possible to talk about X at all /... and/ to relate new concepts systematically to things already understood. /.../ Metaphor extends thought /... and/ compels attention. (ibid., p. 112)

As the results show, the pupils did not talk about the parallelogram (X) and its properties (the tasks or problems per se; the abstract figure; that is the relation self-world) to any great extent. Rather, they seemed to think in the medium or the metaphor (Y) (metaphorical thinking; the drawing is regarded as a deck of cards or a frame; that is the relation self-object). Yet, two dyads (13 and 23) saw the deck-of-cards metaphor as a metaphor.

All group discussions were accompanied by a flow of gestures. The gesture types can be called 'enactive' and 'iconic' in Bruner's well-known terms. A concept or meaning can also be shown or demonstrated through a (virtual) action. Such non-linguistic clues as gestures give information about the significance of what is being said, and they may sometimes replace a missing word. Utterances, gestures and computer display form - in a pragmatic and functional manner - a coherent basis for operational knowing and for reasoning. Or as Dreyfus (1992) puts it in a commentary on Heidegger's discussion of signs:

> A sign's signifying must take place *in a context*, and it signifies, that is it can be a sign, only for those who *dwell* in that context. (ibid., p. 102)

As can be seen, practical and symbolic tools are intertwined in a complex way in the sense-making processes. Therefore, for the continuing discussion, it is productive to distinguish between the epistemic and the ontic aspect of cognition in relation to language (Feldman, 1987).

The epistemic aspect of cognition concerns the means by which we as humans come to know about the world. The mental acts (epistemological processes) operate on objects (concrete operations) or on propositions (formal operations). The ontic aspect of cognition concerns the means by which a situation is first construed. The subject must construe or stipulate what is to be taken as given before, e.g. approaching problem-solving. Giving something ontological status is, according to Feldman, making it 'real'. Ontic stipulations can sometimes be derived from epistemic operations, that is an epistemic process results in an ontic product. Feldman calls this "ontic dumping" (ibid., p. 136).

The process of ontic dumping is at work when the deck-of-cards model, as a strategy or rule for solving a particular problem, becomes a rule in itself for use in any compatible context. The metaphor becomes at best a concept suitable for reasoning about ('think about'; relation self-world), rather than a perhaps step-by-step governed procedure and stereotypical way of knowing in a particular case ('think in'; relation self-object).

Moreover, in her theoretical analysis Feldman finds a striking parallel between the - here, briefly described - cognitive operations and language. With reference to Roman Jakobson and the Prague School of linguistics, she takes up the distinction between the 'given' and the 'new' in a discourse. A required condition for successful dialogue is that the topic (the given) is preserved both by and across speakers. An utterance can refer to the topic, but it can also introduce something new to the discourse. Depending on how an utterance is treated, the discourse can move laterally or onwards or even stop. Feldman continues:

> The same consequences would follow for thinking undertaken in the form of language. For in order that thinking move forward in a progressive way, it too must maintain a clear marking of the cognitively given (ontically stipulated) and the cognitively new (or epistemic). (ibid., p. 137)

Her point is that cognition and language follow each other closely in the same pattern. So, the ontic dumping has its linguistic counterpart in the process when information marked as new in the discourse is being treated as a topic, that is becomes old, given or taken for granted. That is to say, to follow and examine how comments in the discourse construct topics is, at the same time, to follow and watch how processes in the mind build cognitive objects (concepts) to be reasoned about.

The ontic aspect of knowing is determined relative to epistemic operations. Therefore, there are many alternative ways of interpreting a situation, of stipulating, of giving ontic status to what is taken as given for the epistemic plans at hand. That is why people often construct idiosyncratic concepts. However, people can know the same thing, coordinate their actions, if they have a common store of referents, a shared 'ontic dump'.

After this development of some concepts, the following remarks can now be made. Firstly, the results from the discourse in front of the PC can be made understandable. Most of the pupils construct the state of affairs by talking on the demonstrative and visual levels. The demonstrative level contains indexical words – words such as *this, here, first* – accompanied by pointing gestures. The words are completely context-dependent but virtually transparent. The members of the dyad have the same focus for their discussion. Also, the visual level is unproblematic. Shared objects and displays facilitate the process of referential anchoring. Instructions, requests, propositions, questions and so forth on the metaphoric or formal levels have an ontic status demanding convergence and coherence in thought. The results suggest a relationship between the linguistic level and the achievement level of groups, but from a functional and pragmatic aspect, this relationship is indifferent. The language has great flexibility when it comes to establishing shared understanding among participants. On the other hand, the levels reflect disparate addresses for attention concerning the relation self-object and the relation self-world. In the format of this study, the influence of this difference, e.g. retention measured on a post-test, is left unheeded.

Secondly, if the interaction between pupil and computer is regarded as a conversation, the common picture of a pupil who is asking and a computer that is answering, has to be supplemented. The computer - especially in this study - does not give any strict answers, but provides information which has to be construed in the ontic-epistemic or given-new paradigm like an ordinary utterance in a discourse. The different patterns in the pupils' key punches support this suggestion.

Thirdly, the change of medium - from computer-based action to paper-and-pencil drawing - in the final problem challenges the comprehension of the concept of area. In fact, the pupils have to talk in terms of the relation self-world to get shared referents and establish a new here-and-now situation. The personally constructed knowledge through the computer program has to be jointly reconstructed. The discourse in group 23 presented earlier shows how social interaction makes the problem-solving successful through giving comments, asking questions, pointing things out to one another, arguing with and elaborating on each other's ideas. The discourse can be a structuring resource and a scaffolding process (cf. Bruner, 1985) when the situation makes the pupil a participant and a contributor. The discourse can be seen as a potential process of making meaning (Säljö & Wyndhamn, 1990).

Fourthly, what can be said about dyad 21 which was the only group not to succeed in solving or answering the criterial items? The deck-of-cards model and the hinge model did not help Diana and Doris to discriminate between area and perimeter. They were stuck with the psychological dilemma.

Diana: Area and perimeter are the same all the time. It ought to be so.

Of course, looking at this exercise as a learning experience, this outcome is negative. But why does Diana reply as she does? The preceding discussion can be extended to include a possible answer to this question. Diana did not perceive any contradictions between the two models. She was subjectively certain in her own thinking. She applies her self-world relation to the self-object relation when she sees the problem through a 'it-ought-to-be-so' lense. The area problem is subordinated to a personal scheme for reasoning. Diana's beliefs or conceptions were never questioned by the program nor by her partner Doris. As mentioned before, the two girls did not co-operate. Here, I think, a teacher through instruction could cause a cognitive conflict leading to a reflective and reconstructive process. In this way, the results from dyad 21 are positive, they support the general theoretical framework, and emphasize that a particular interaction does not always result in a certain, intended cognitive construction (cf. Pufall, 1988).

To conclude, and as a summary of this study, I will reproduce a part of the discussion (some repetitions are excluded) in dyad 13 from the final task, and make a few remarks on the places labelled with capital letters.

Carola: Think of the deck of cards. A
Conny: (measuring the sides of the parallelogram) ... 10 cm ... 7 cm... 10 times 7 is 70, but ... (now measuring the height). B
Carola: Is it so?
Conny: With the deck of cards ...
Carola: ... it is the same ...(pointing to the height).
Conny: 10 is the same ... 6 cm ... it ought to be 6.
Carola: But ... Does the area decrease now? ... 7 ... C
Conny: No. The perimeter becomes longer ... or shorter.
Carola: Mm. How long is ... (pointing to the height)? This was difficult... D
Conny: Yes, 10 times ... how high ... 10 times 6 is it.
Carola: Mm.
Conny: The deck of cards becomes neither higher or lower ... but longer ... (making gestures).
Carola: Yes... (draws a rectangle using the position of the given parallelogram to get the right height, thus not using the measurement of 6 cm).

A. Carola stipulates the basis of the discussion - a proper metaphor. The pupils then discuss the properties of the parallelogram by referring to the deck of cards. The metaphor monitors and scaffolds the discourse.

B. Conny says more or less automatically "10 times 7 is 70". He is now working metonymically, that is he is searching for another 'name' instead of 'area' in terms of internal relations of the parallelogram. He takes a starting-point in "the area of a rectangle is length times width". However, Conny is aware that the statement must be corrected. Three other groups in the study started in this way, they even drew a rectangle with the sides 10 cm and 7 cm. All these pupils for a longer or shorter period referred to the metonymy "length times width".

C. The question is an example of self-regulation or self-correction of cognition. In fact, precisely here the relation self-world and the relation self-object are compared and scrutinized.

D. Carolas problem is not mathematical but linguistic and/or semantic. I think, she is searching for a proper noun. Conny does not find it either, however, and he tries with an adjective "high". It is notable that the pupils point at a drawing on a sheet of paper oriented in the horizontal plane and still talking about how 'high' something is. This episode illustrates how language objectifies reality. The pupils have to listen to a predecessor of the mathematics culture to get the conventional term or to get their own term confirmed (cf. Bruner & Haste, 1987).

The discourse as a whole exemplifies how important interaction with people and artifacts is concerning learning and problem-solving, but also how the environment for learning is affected by a larger context, that is embedded in a surrounding culture. Geometry (read mathematics) deals with human meanings found in a shared understanding, and it is intelligible only within the context of culture.

References

Bruner, J. (1966). On cognitive growth II. In J.S. Bruner, R.R. Olver, & P.M. Greenfield (Eds.), *Studies in cognitive growth* (pp. 30-67). New York: Wiley.

Bruner, J. (1985). Vygotsky: A theoretical and conceptual perspective. In J.V. Wertsch (Ed.), *Culture, communication and cognition: Vygotskian perspectives* (pp. 21-34). Cambridge: Cambridge University Press.

Bruner, J., & Haste, H. (1987). Introduction. In J. Bruner, & H. Haste (Eds.), *Making sense. The child's construction of the world* (pp. 11-25). London: Methuen.

Cobb, P., Wood, T., Yackel. E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29*(3), 573-604.

Dreyfus, H.L. (1992). *Being-in-the-world. A commentary on Heidegger's Being and Time, Division 1.* Cambridge, MA: The MIT Press.

Feldman, C.F. (1987). Thought from language: the linguistic construction of cognitive representations. In J. Bruner, & H. Haste (Eds.), *Making sense. The child's construction of the world* (pp. 131-146). London and New York: Methuen.

Gellert, H., Kustner, H., Hellwich, M., & Kastner, H. (Eds.) (1977). *The VNR concise encyclopedia of mathematics.* New York: Van Nostrand Reinhold.

Hart, K.M. (Ed.) (1981). *Children's understanding of mathematics: 11-16.* London: John Murray.

Hedrén, R. (1990). *Logoprogrammering på mellanstadiet. En studie av fördelar och nack-delar med användning av Logo i matematikundervisningen under årskurserna 5 och 6 i grundskolan.* [Programming in Logo at intermediate level of compulsory school. A study of advantages and disadvantages of the use of Logo in mathematics instruction in forms 5 and 6 of the Swedish compulsory school.] Linköping Studies in Education. Dissertations No. 28.

Hoyles, C. (1991). Developing mathematical knowledge through microworlds. In A.J. Bishop, S. Mellin-Olsen, & J. van Dormolen (Eds.), *Mathematical knowledge: Its growth through teaching* (pp.147-176). Dordrecht: Kluwer.

Hutchins, E. (1986). Mediation and automatization. *The Quarterly Newsletter of the Laboratory of Comparative Human Cognition, 8*(2), 47-58.

Kaput, J.J. (1987). Representation systems and mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 19-26). Hillsdale, NJ: Lawrence Erlbaum.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life.* Cambridge: Cambridge University Press.

Nolder, R. (1991). Mixing metaphor and mathematics in the secondary classroom. In K. Durkin, & B. Shire (Eds.), *Language in mathematical education. Research and practice* (pp. 105-114). Milton Keynes, PA: Open University Press.

Olson, D.R. (1976). Culture, technology and intellect. In L.B. Resnick (Ed.), *The nature of intelligence*. Hillsdale, NJ: Lawrence Erlbaum.

Ong, W.J. (1982). *Orality and literacy: The technologizing of the word*. New York: Methuen.

Pea, R.D. (1987). Cognitive technologies for mathematics education. In A.H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 89-122). Hillsdale, NJ: Lawrence Erlbaum.

Pufall, P.B. (1988). Function in Piaget's system: Some notes for constructors of microworlds. In G. Foreman, & P.B. Pufall (Eds.), *Constructivism in the computer age* (pp. 15-36). Hillsdale, NJ: Lawrence Erlbaum.

Säljö, R. (1992a). Human growth and the complex society. Notes on the monocultural bias of theories of learning. *Cultural Dynamics, V* (1), 43-56.

Säljö, R. (1992b). Kunskap genom samtal. [Knowledge through discourse.] *Didaktisk tidskrift (2-3)*, 16-29.

Säljö, R., & Wyndhamn, J. (1990). Problem-solving, academic performance and situated reasoning. A study of joint cognitive activity in the formal setting. *British Journal of Educational Psychology, 60*, 245-254.

Sayeki, Y., Ueno, N., & Nagasaka, T. (1991). Mediation as a generative model for obtaining area. *Learning and Instruction, 1*(3), 229-242.

Schoenfeld, A. H. (1986). On having and using geometric knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 225-264). Hillsdale, NJ: Lawrence Erlbaum.

Scribner, S. (1984). Studying working intelligence. In B. Rogoff, & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 9-40). Cambridge, MA: Harvard University Press.

Sheingold, K. (1987). The microcomputer as a symbolic medium. In R.D. Pea, & K. Sheingold (Eds.), *Mirrors of minds: Patterns of experience in educational computing* (pp. 198-208). Norwood, NJ: Ablex.

Steinbring, H. (1989). Routine and meaning in the mathematics classroom. *For the Learning of Mathematics, 8*(1), 24-33.

van Hiele, P.M. (1986). *Structure and insight. A theory of mathematics education*. London: Academic Press.

Voigt, J. (1989). The social constitution of the mathematics province - A microethnographical study in classroom interaction. *The Quarterly Newsletter of the Laboratory of Comparative Human Cognition, 11*(1&2), 27-34.

Vygotsky, L.S. (1978). *Mind in society*. (M. Cole et al., Trans.) Cambridge, MA: Harvard University Press

Vygotsky, L.S. (1986). *Thought and language*. (A. Kozulin, Trans. and Ed.) Cambridge, MA: MIT Press.

Wertheimer, M. (1959). *Productive thinking*. London: Associated Book Publishers.

Wertsch, J.V. (1991). A sociocultural approach to socially shared cognition. In L.B. Resnick, J.M. Levine, & S.D. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 85-100). Washington, DC: American Psychological Association.

Wyndhamn, J. (1992). *Matematisk kompetens och sociokulturell aktivitet. En experimentell studie över hur elever löser problem med datumangivelser*. [Mathematical competence and sociocultural activity. An experimental study of students solving problems concerning date information.] Arbetsrapport [Working paper] 1992:6. University of Linköping, Department of Communication Studies.

Zinchenko, V.P. (1985). Vygotsky's ideas about units for the analysis of mind. In J.V. Wertsch (Ed.), *Culture, communication and cognition: Vygotskian perspectives* (pp. 94-118). Cambridge: Cambridge University Press.

Kommunikationsmöjligheter i datormiljö.

En explorativ studie av begreppsbildning i geometri

Sammanfattning

Nio grupper bestående av två elever (tolv år gamla) ställdes inför uppgifter att genom samarbete och via ett datorprogram finna ett sätt att bestämma arean av en parallellogram. Programmet presenterade två modeller av hur en parallellogram kan uppfattas. Den ena ("helhetsmodellen") visade att arean kan vara konstant medan omkretsen varierar. Den andra ("gångjärnsmodellen") visade att omkretsen kan vara konstant och arean variera. Genom knapptryckningar kunde eleverna lätt ändra form och storlek av en på skärmen visad parallellogram och därmed utforska och upptäcka olika egenskaper hos parallellogrammen. Elevernas samspel med datorn registrerades direkt i datorn och elevernas inbördes diskussion togs upp på ljudband för analys. Efter tjugo minuters arbete med datorn hade alla grupper utom en genomskådat hur olika parametrar inverkar på parallellogrammens area. Samtalet i grupperna och interaktionen med datorprogrammet mejslade fram korrekta föreställningar av vilka geometriska storheter som är invarianta vid olika transformationer. Denna kunskap utnyttjades sedan i en slutuppgift då eleverna uppmanades att rita en rektangel med lika stor area som en snedvinklig parallellogram.

Anm: Detta arbete är en bearbetad version av en artikel som ingår i författarens doktorsavhandling: *Problem-solving revisited. On school mathematics as a situated practice* (Linköping Studies in Arts and Science 98).

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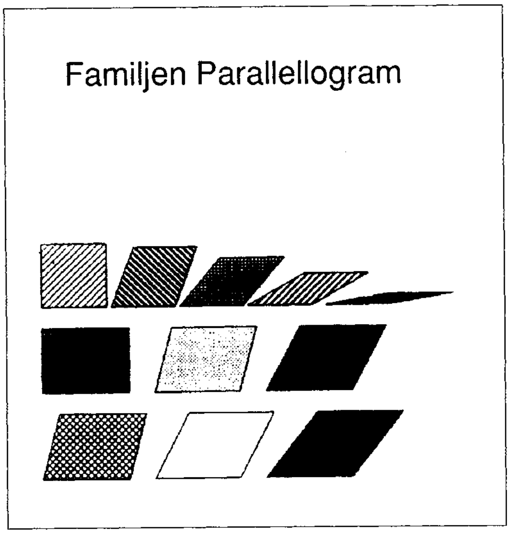
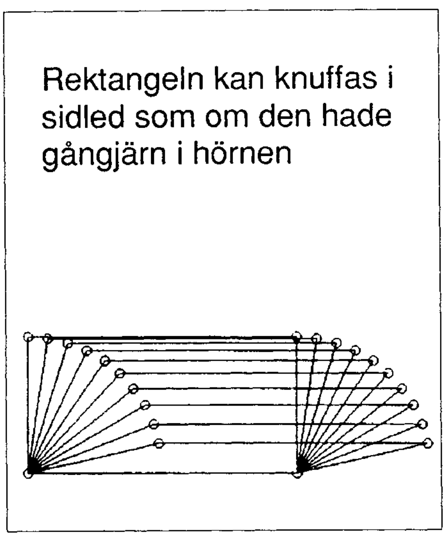
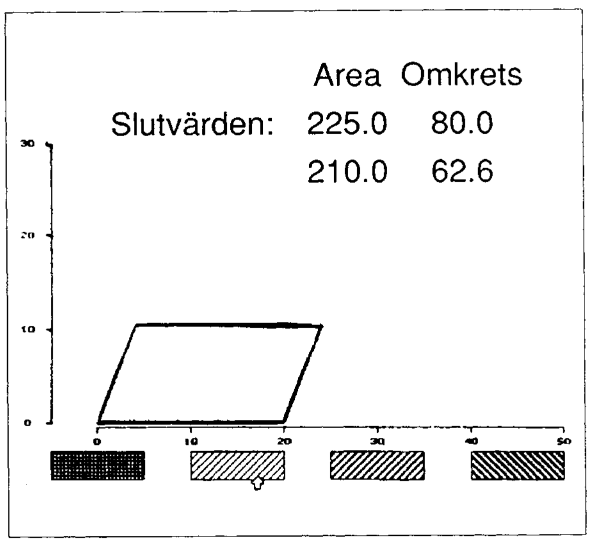
Appendix A



Deck-of-cards model

target values





hinge or frame model

parameter fields with pointer

Appendix B

Items in the study

Direct answers from the students to the criterial tasks:

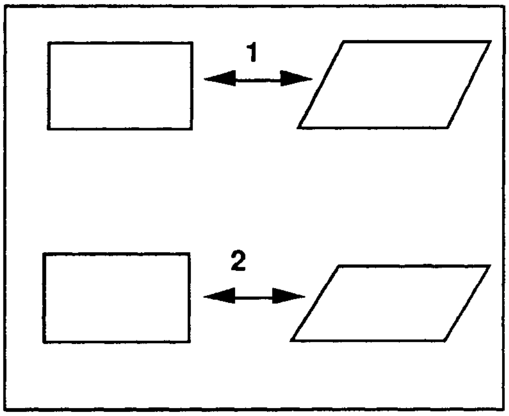
1.1 Look at this paper 1. You see the two transformations that were dealt in the computer program.
Which is the "hinges"-transformation?
Which is the "deck-of-cards"-transformation?

1.2 Give reasons for your answers.

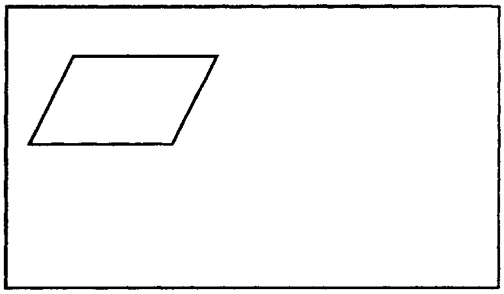
1.3 Say something about the properties of the transformations concerning area and perimeter.

1.4 Here, you see a parallelogram.
Draw on paper 2 a rectangle with the same area.

Paper 1



Paper 2



Answers via analysis of printouts:

2.1 and 2.2 How many trials were necessary and how long time did it take to reach the target values?

2.3 Which strategy is used?

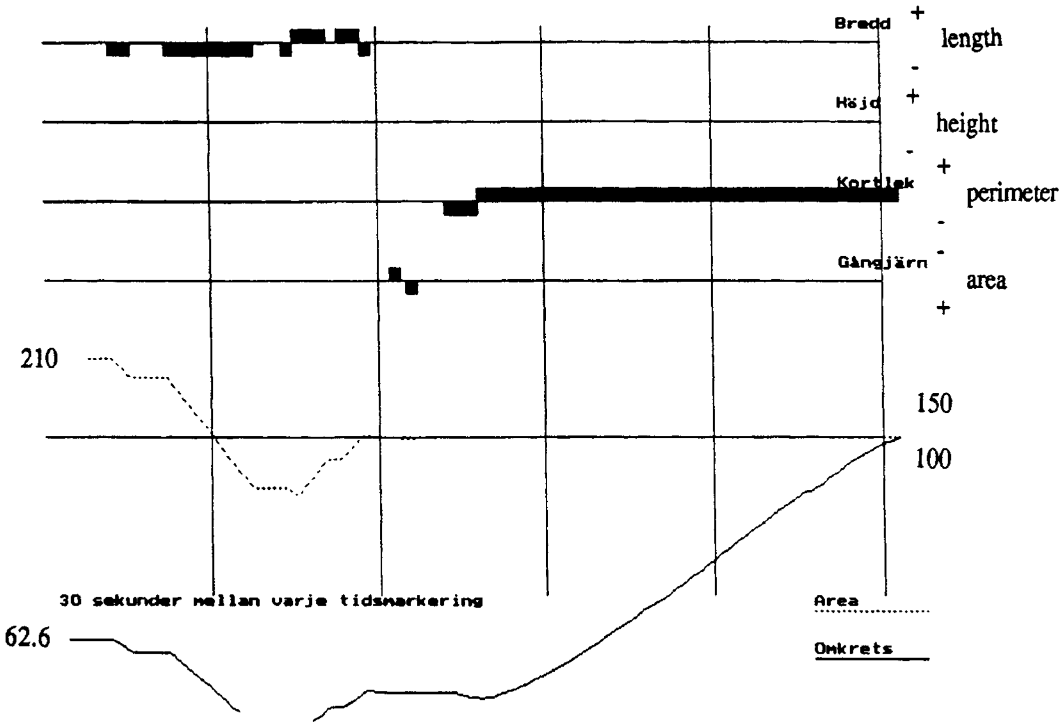
Answers via analysis of transcripts and notes:

3.1 Which is the nature of the discourse in front of the PC?

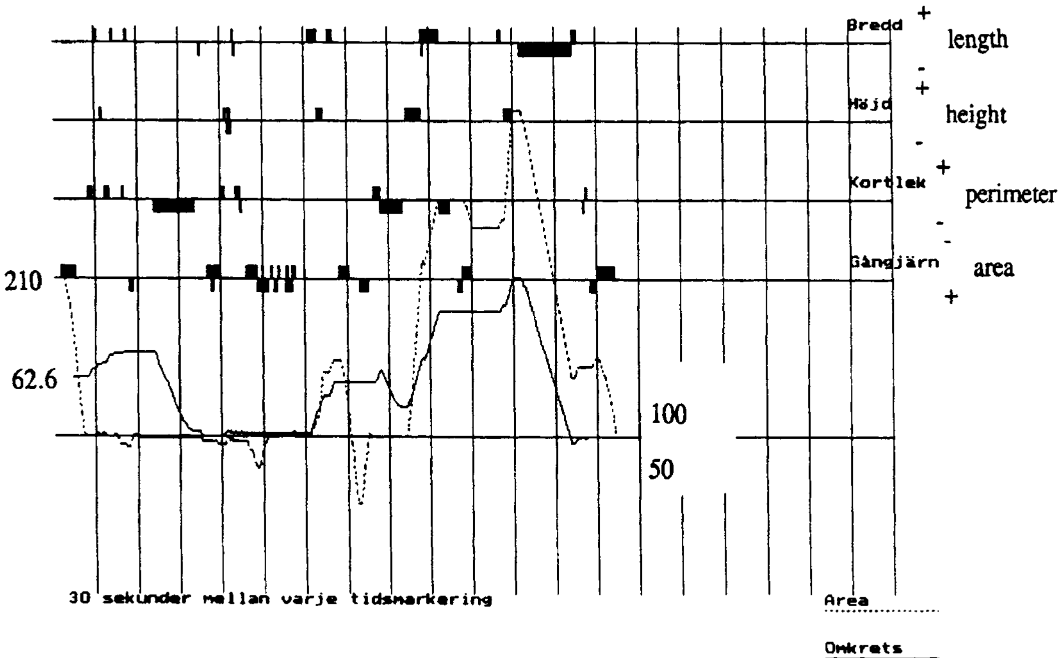
4.1 How to describe the discourse during the problem solving phase of item 1.4?

Appendix C

dyad 23, task 3



dyad 31, task 2



Appendix D

Table 4. A survey over the results: items vs dyads

	Dyad 11	12	13
Item			
1.1	correct	correct	correct
1.2	differences	differences	app./diff./const.
1.3	correct	correct	correct
1.4	correct	correct	correct
2.1	70	42	23
2.2	27	17	10
2.3	deck of cards	deck of cards	flexible
3.1	formal	metaphoric/visual	formal
4.1	pointing	pointing/deck of cards	pointing/deck of cards

	Dyad 21	22	23
Item			
1.1	correct	correct	correct
1.2	differences	constancy	appearance
1.3	wrong	correct	correct
1.4	wrong	correct	correct
2.1	39	63	53
2.2	22	22	21
2.3	hinges	deck of cards	deck of cards
3.1	dem./visual	visual/formal	visual/metaphoric
4.1	no co-operation	exploring rectangle 10 cm x 7 cm / pointing/deck of cards	exploring rectangle 10 cm x 7 cm / pointing/deck of cards

	Dyad 31	32	33
Item			
1.1	correct	correct	correct
1.2	appearance	appearance	constancy/appearance
1.3	correct	correct	correct
1.4	correct	correct	correct
2.1	81	47	51
2.2	28	26	21
2.3	gambling	deck of cards	gambling
3.1	dem./visual	visual	dem./visual
4.1	pointing/deck of cards	pointing	pointing