Knowledge used when orchestrating mathematical discourses – doing, guiding and requesting

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There is a need to understand more about which types of knowledge teachers use when orchestrating mathematical discourses. This article combines models for mathematical knowledge for teaching with a recent framework that describes the actions that teachers typically use during classroom discourses in mathematics. By looking into what knowledge each action demands from the teacher, three areas related to mathematical knowledge for teaching are described: doing, guiding and requesting. Doing describes different ways the teachers are doing the mathematical work themselves. Guiding describes how the teachers help, while leaving most of the work to the students. Requesting describes different ways teachers asked the students to explain or contribute to the discourse.

It seems to be well established that student talk and active participation is important for students' learning of mathematics (Franke, Kazemi & Battey, 2007; Walshaw & Anthony, 2008). However, when teachers orchestrate discourses in the classroom, there are many different ways that they can engage students. Within the IRE pattern (*initiation – response – evaluation*) (Mehan, 1979), the student contributions are typically limited to responses to teachers' initiatives. Another method is to focus more on having students present and share their work, but this also may be limited to "show and tell" (Ball, 2002), with no real discussion. It is not enough to just make students talk; how they contribute and share their thoughts, for example, by addressing details (Franke et al., 2007), and how they engage in discussions, for example, by taking the initiative and by evaluating each other, is important. These methods are all aimed at engaging students cognitively, which is a necessary condition in order to increase their level of achievement (Wiliam, 2007).

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The research literature has described several ways that teachers can orchestrate discourses in order to include students' contributions, for example, by enlightening, extending and supporting students (Fraivillig, Murphy & Fuson, 1999), by challenging, advocating and reformulating (Alrø & Skovsmose, 2002), and by enlightening details and requesting justification (Drageset, 2014).

Given that orchestrating mathematical discourse is an important part of students' learning, and that the different ways that this is enacted provide students with qualitatively different opportunities to learn, one should look closer into what this demands from teachers. It is important to find out more about the different types of knowledge that might be beneficial in order to create and orchestrate a productive mathematical discourse in the classroom. However, although there are manifold frameworks that describe mathematical knowledge for teaching (for example, Ball, Thames & Phelps, 2008; Chick, Pham & Baker, 2006; Kilpatrick, Swafford & Findell, 2001; Niss, 2001; Shulman, 1986), little attention is given to the knowledge that is needed to orchestrate mathematical discourse. This article addresses this gap by asking: Which types of mathematical knowledge do teachers use when orchestrating a classroom discourse?

Classroom discourse

In a review of research literature concerning the teacher's role in classroom discourse, Walshaw and Anthony (2008) find that effective instructional practices demand that students engage in mathematical discussions. This is in line with the findings from the review conducted by Franke et al. (2007), in which it is emphasized that the development of mathematical understanding requires students to present solutions to problems, talk about a variety of mathematical representations, make explicit generalizations, explain solutions and prove why solutions work.

Teachers who sustain discourse merely to continue the conversation, perhaps in an effort to allow everyone to speak equally, and who often accept all answers, do not necessarily help students to advance their thinking (Walshaw & Anthony, 2008). Merely making your thinking available to others is insufficient, because too much is normally unsaid. The manner in which we make our thoughts available seems to be crucial (Kieran, 2002), which shifts the focus from students' discussions per se to the content of students' discussions (Walshaw & Anthony, 2008). Consequently, details matter, and "one of the most powerful pedagogical moves a teacher can make is one that supports making detail explicit in mathematical talk" (Franke et al., 2007, p. 232). However, it is not sufficient to move the entire class forward mathematically if the discourse is limited to the students taking turns sharing their solution strategies, with no filtering or highlighting (Stein, Engle, Smith & Hughes, 2008). The hallmark is that the teacher actively uses students' ideas and work to lead them toward more powerful, efficient and accurate mathematical thinking. Instead of just letting students "show and tell" different results and methods, Ball (2001) emphasizes the active use of students' contributions by using them and by making them available for the rest of the class to utilize.

Although there is an increasing agreement that students' contributions must play an important role in classroom communication, there is also a need to understand more about how this can be achieved. Carpenter et al. (1999) suggest using a careful selection and sequencing of student strategies. Stein et al. (2008) suggest a similar strategy as part of a model that specifies five key practices for a teacher to use student responses more effectively in discussions: anticipating, monitoring, selecting, sequencing and connecting. This model directs attention towards how students' thinking about mathematical content can be used to create reflection and learning. According to Wiliam (2007), engaging students cognitively is a necessary condition in order to maximize mathematics achievement.

An important question for this article is how teachers actually orchestrate discourses and how this can be conceptualized. A study by Fraivillig et al. (1999) of one skillful first grade teacher provides one example. The study resulted in a framework called "Advancing children's thinking" (ACT), which describes how teaching could be conducted to lead students towards more powerful, efficient and accurate mathematical thinking. The framework has three components: eliciting children's solution methods, supporting children's conceptual understanding, and extending children's mathematical thinking. While the eliciting and supporting components focus on the assessment and facilitation of mathematics with which the students are familiar, the extending component is focused on the further development of the students' thinking. Another detailed study of discourse in mathematics classrooms is presented by Alrø and Skovsmose (2002). As part of the inquiry-cooperation model, they identify eight communicative features: Getting into contact, locating, identifying, advocating, thinking aloud, reformulating, challenging and evaluating. These features were present both in the student-student interaction and in the teacher-student interaction.

The redirecting, progressing and focusing framework (Drageset, 2014) adds to the map of concepts and frameworks that describe mathematical classroom discourse. It is similar to the frameworks of Fraivillig et al. (1999) and Alrø and Skovsmose (2002) in its detailed description of classroom interaction. However, while Fraivillig et al. (1999) and Alrø and Skovsmose (2002) describe situations, the redirecting, progressing and focusing framework (Drageset, 2014) goes one step further and describes individual teacher comments related to how these actions use student comments to work with the mathematics.

Table 1. *The redirecting, progressing and focusing actions framework (Drageset, 2014)*

Redirecting actions	Progressing actions	Focusing actions
Put aside	Demonstration	Enlighten detail
Advising a new strategy	Simplification	Justification
Correcting question	Closed progress details	Apply to similar problems
	Open progress initiatives	Request assessment from other students
		Notice
		Recap

Redirecting actions are typically those in which a teacher tries to change the student's approach, either by putting the student's suggestion aside without further comment, by advising the student to use another strategy or by using correcting questions. Progressing actions are typically what the teacher does to help the student arrive at an answer. Sometimes, the teacher demonstrates the entire process without involving the students; at other times, the teacher gives hints that simplify the task. A third approach is to divide the task into small steps and ask one question for each step (closed progress details), and a fourth approach is to ask open questions and see what the students are able to accomplish. The focusing actions are typically requests for student contributions or teacher comments that tend to stop progress for shorter or longer periods in order to look deeper into some important detail. This includes questions about how a solution was reached (enlighten detail), questions about why a solution is correct (justification), requests for students to apply the same method or reasoning to a similar task, and requests for assessments from other students. Focusing actions also include teachers pointing out important ideas or methods during the solution process (notice) or giving an overview with an emphasis on important ideas and methods (recap).

Such frameworks and their concepts provide details about how teachers might orchestrate discourses, how they do or do not make students talk, what the students are and are not encouraged to discuss, where the authority is placed and who and how suggestions and ideas are evaluated.

One illustrative example is that, according to Stein et al. (2008), the hallmark of second generation practice is that teachers actively use students' ideas. Fraivillig et al. (1999) describes how this can be done in three ways: eliciting, extending and supporting. Another example is that Ball (2002) emphasizes that it is not enough to merely "show and tell" when encouraging students to talk. Ideas such as challenge, advocate and reformulate (Alrø & Skovsmose, 2002) and enlighten detail, justification, notice and correcting questions (Drageset, 2014) make it possible to conceptualize how teachers act when they develop a discourse into something more than just "show and tell." The main contribution from such frameworks is that they enable us to conceptualize classroom interactions in more detail.

Mathematical knowledge for teaching

The manifold models that depict mathematical knowledge for teaching illustrate several ways to describe the knowledge a teacher needs in order to teach mathematics. The starting point for much of the debate is the article "Those who understand: knowledge growth in teaching" by Shulman (1986). Arguing against the cleavage between subject matter and pedagogy, Shulman (1986) states that mere content knowledge is likely to be as useless pedagogically as content-free skill and calls for a blend of these two aspects that "pay[s] as much attention to the content aspects of teaching as we have recently devoted to the elements of [the] teaching process" (Shulman, 1986, p. 8). The main new idea from Shulman's article (1986) is pedagogical content knowledge (PCK); this section will discuss how PCK can be further detailed, and how PCK relates to subject matter knowledge (SMK) and communication.

Ball et al. (2008) divides subject matter knowledge and pedagogical content knowledge into sub-categories that describe the mathematical knowledge needed for teaching. The main new concept is specialized content knowledge (SCK), which is defined as "the mathematical knowledge and skill uniquely needed by teachers in the conduct of their work [...] and therefore not commonly needed for purposes other than teaching" (Ball et al., 2008, p. 34). It is also described as mathematics in its decompressed or unpacked form, and it is purely mathematical in the sense that it does not require knowledge of students or teaching. SCK is, in its nature, rather similar to a profound understanding of fundamental mathematics (Ma, 1999), which is something more than sound conceptual understanding; it is also the awareness of conceptual structure, so that the teacher is able to "reveal and represent connections among mathematical concepts and procedures to the students" (Ma, 1999, p. 124).

The core of profound understanding of fundamental mathematics is an awareness of the simple, but powerful, ideas of mathematics, so that they can be revisited and reinforced repeatedly, related to different content.

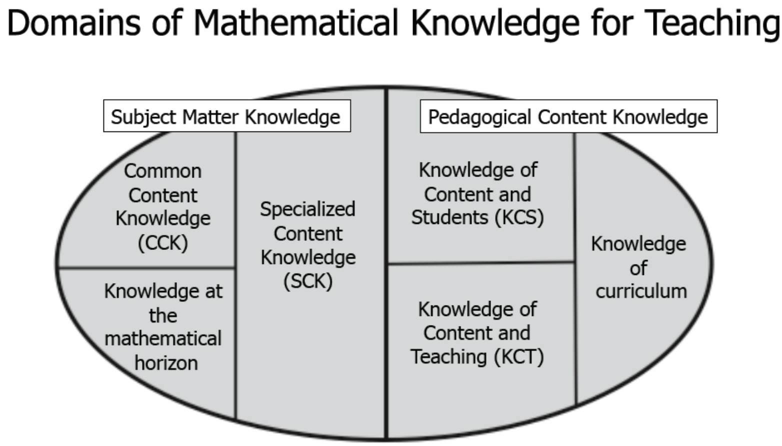

Figure 1. *Content knowledge for teaching (adapted from Ball et al., 2008)*

The content knowledge for teaching framework (Ball et al., 2008) divides pedagogical content knowledge (PCK) into three domains: knowledge of content and student, knowledge of content and teaching, and knowledge of curriculum (see figure 1). Another detailing of PCK is suggested by Chick et al. (2006) and Chick (2007): clearly PCK, content knowledge in the pedagogical context, and pedagogical knowledge in a content context. The difference between them is that Ball et al. (2008) divides PCK related to the knowledge needed to teach, while Chick et al. (2006) divides PCK related to its origin (from either content or pedagogy).

Other researchers also offer detailed frameworks of subject matter knowledge and pedagogical content knowledge without building on Shulman's (1986) ideas. For example, Niss and Højgaard Jensen (2002) suggest both a framework of "mathematical teacher competency" (curriculum competency, teaching competency, the uncovering of learning competency, assessment competency, collaboration competency and professional development competency) and a framework of "mathematical competency." The former relates to PCK, while the latter relates to SMK. Similarly, Kilpatrick et al. (2001) suggests two frameworks: one that describes mathematical proficiency and another that describes proficient teaching of mathematics. The former relates to SMK, the latter to PCK, and both use the same five concepts: conceptual understanding, fluency, strategic competence, adaptive reasoning, and productive disposition. While, for example, fluency does not have the same meaning within mathematical proficiency (SMK) and proficient teaching of mathematics (PCK), the idea is related.

Knowledge about how to orchestrate a mathematical discourse is not explicitly mentioned in any of these concepts or frameworks, although it is included in several places as elements among others. For example, both knowledge of content and teaching (Ball et al., 2008) and teaching competency (Niss & Højgaard Jensen, 2002) include knowledge about how to lead mathematical discussions in the classroom. This illustrates that there is a need to know more about how different types of teacher actions during classroom discourse can be related to mathematical knowledge for teaching, or more precisely: What kinds of knowledge are needed in order to facilitate a productive discourse?

By using the existing frameworks that describe mathematical discourse, it becomes possible to conceptualize the teachers' actions. The idea of this article is to use such a conceptualization in order to inspect the types of mathematical knowledge that the different types of teacher actions might demand. To do this, the redirecting, progressing and focusing framework (Drageset, 2014) has been chosen because it goes into more detail than the others, as it describes individual teacher comments, while other frameworks describe situations and sequences.

Data and method

This study is part of a project in which 356 teachers completed a test and a questionnaire from which two knowledge constructs ("common content knowledge" and "specialized content knowledge") and two belief constructs ("rules" and "reasoning") were established (Drageset, 2009, 2010). Then, 12 general teachers with diverse profiles related to the knowledge and beliefs constructs were selected for further study. They were contacted and informed why they were selected and what was wanted from them, and five of the 12 accepted. In order to describe the teaching of the five teachers, their practices were studied. These five teachers teach in upper primary school (grades five to seven, students aged ten to 13). All of their mathematics teaching was filmed for one week from the inception of the topic of fractions. The camera followed the teacher, who also wore a microphone that captured all of the dialogues in which the teacher participated.

The main data analysis in this article comes from the development of the redirecting, progressing and focusing actions framework (Drageset, 2014), which was developed by inspecting one teacher comment at a time and grouping them according to how the teacher used student comments to work with mathematics. This work gradually developed categories that describe 13 types of teacher actions in three superordinate groups (see table 1). The analysis is a conversation analysis, as it describes the discourse itself and is not a window into something else.

In the next section, the categories from the redirecting, progressing and focusing framework will be re-visited in order to describe the mathematical knowledge for teaching that each of them requires, especially related to the knowledge needed to orchestrate the discourse in productive ways. For example, to request a justification of a student solution, the teacher only needs to ask "why?". However, knowing when this question provides meaning, when this question is important in order to help the student and the entire class to understand, and when such questions are superfluous, requires mathematical knowledge for teaching of some sort.

The analysis that was conducted to achieve this is based on a deep revisiting of each category by looking through its comments and describing different actions that seemed to relate to knowledge in different ways. Next, categories that seemed to require similar types of mathematical knowledge for teaching were put together in groups. This included re-organizing the categories from the redirecting, progressing and focusing framework that related to the mathematical knowledge for teaching required.

Findings

The findings will be presented in three groups: guiding, doing, and requesting. For an overview of the categories, and how they are related to these three groups and the original framework, see table 2 at the end of this section. The examples have been selected from all five practices.

Guiding

The teacher writes several fractions on the blackboard and asks which of them have a value larger than one half, and this exchange follows.

Excerpt 1

T: Tell me one that is larger than one half.

S: Five six ... five sixths.

T: You have found that five sixths is larger than one half [writes 5/6]. Why do you know with certainty that this is larger than one half?

S: Because three ... is one half and ... [interrupted]

T: Yes. Because three sixths is one half [writes 3/6=1/2]. Right? When we have the half in the numerator of the denominator, then we have one half. Then, we can find all that have more than the half in the numerator.

In the last comment, the teacher emphasizes a key idea for the class by pointing out that when the numerator is half the value of the denominator, the fraction equals one half, and how this can be used to find fractions that are larger than one half. Then, the discourse continues, using the idea to consider fractions. This is a form of guiding during a discourse in which the teacher picks up and emphasizes an idea from a student, sometimes clarifying in order to help other students understand, and at other times, generalizing in order to help students see the potential or the importance of the idea. Also, the teachers occasionally suggested that the students should use another approach, or indicated that the approach that was used was wrong, using correcting questions. Suggesting another strategy is another example of guiding students, but instead of using the students' ideas, the teacher develops his own alternatives. A third way of guiding students is by putting their suggestions aside, and thereby, guiding students away from the current strategy. A fourth way is by asking correcting questions that act as indirect requests for students to try another approach. The commonality between pointing out, suggesting, putting aside and correcting questions is that the teacher guides, but leaves the work to the student.

Doing

At other times, the teacher is also involved in the solution process. In the following excerpt, the teacher is helping a student with the task of drawing the entire figure when $\frac{1}{4}$ is provided (see figure 2).

Figure 2. *The task was to draw the entire figure when one fourth of it was provided*

Excerpt 2

S: But how should I draw that one then?

T: Yes, it is one fourth. You have to draw the three other fourths. First, one fourth. You can see at once where that has to be.

S: [no answer]

T: It has to be exactly the same size as that one [pointing out the figure of one fourth]. Where is it? Draw with your pencil, just show me.

[four turns without progress is omitted]

T: But this one, the white part here [pointing out the other half of the two squares]. Isn't this exactly the same?

S: Yes.

T: Aha. Then you have ... if you draw ... like this, then you have another fourth, true? Then you have two fourths. And you have used two squares on two fourths, how many squares do you have to use for the two other fourths?

S: ... two.

T: Yes. Here it is in fact ... if I now just draw obliquely like this [divides the two squares to the right in the same way as the two original squares], then I have one fourth, one fourth, one fourth and one fourth [finishes the drawing and points out each fourth]. That is the whole. Do you understand now?

S: Mmm.

T: Good.

In this case, the teacher tries, in two slightly different ways, to give the student an idea of what to do, but the student does not produce an answer or a sensible approach. Then, the teacher provides more help and ends up doing the entire task. In such cases, the teachers either demonstrated a solution process without using students, or gave hints, so that the complexity was reduced and the students therefore were able to contribute. In other cases, the teacher was deeply involved in the solution process without demonstrating or changing the task. In the following excerpt, the task is to determine the largest of one third of twenty-four and one fourth of twenty. One student suggests one third of twenty-four. The teacher asks why, and this exchange follows.

Excerpt 3

S1: Because one third is larger than one fourth.

T: Yes, but you have, you do not have the same amount on both.

S2: One third of twenty-four is much more.

T: But, how much is one third of twenty-four?

S2: ... seven?

S3: It is eight.

T: It is eight. And then, one fourth of twenty, how much is that?

S4: It is ...

S2: Five.

T: It is five, yes. And do you know what is the largest, then?

S2: Eight.

After first asking about the final answer, the teacher changes strategy, splits it into several smaller tasks and asks for answers to each of them. One aim of this strategy might be to ensure that every student is able to follow the line of thought by leading them through every important step. The result is that the teacher takes control of the process and probably reduces the complexity of the task for the students.

When a teacher demonstrates, simplifies or divides the task into small and basic steps, it is the teacher who does the main work of solving the task. When demonstrating, the teacher does both the process and the calculations; when the teacher gives hints (simplifies), it changes the task, so that it becomes more available for student contributions, and when dividing it up into steps (closed progress details), the teacher controls the process and lets the students do the calculations. The commonality in these three types of orchestrating is that the teacher maintains control and decides every step towards the solution, and by this, the teachers do the majority of the mathematical work themselves. At other times, the teacher has to clarify how a solution was reached:

Excerpt 4

S2: Because six plus six equals twelve.

T: Hm? Because?

S2: Six plus six equals twelve and five plus five is ten.

T: Yes. Because if I double this one [writes 5/6 and "x 2" behind the numerator] and double this one (writes ("x 2" behind the denominator), then we come here (writes 10/12). So ... then we might say that this one is correct.

In this case, the student does not produce an explanation that is precise enough for the teacher, so the teacher recapitulates by adding some information that might clarify why five sixths equals ten twelfths for the other students. This action of recapitulation, either with emphasis on existing details or on added information to provide clarification, was found quite frequently when students gave explanations that were partial, messy or imprecise. In these cases, the teacher typically needs to provide clarification for the rest of the class, and sometimes must do the rest of the work.

Requesting

In addition to guiding and doing the work themselves, the teachers also requested the students to contribute in different ways. One example is this.

Excerpt 5

S: It is possible to simplify two sixteenths.

T: Yes, great. Tell me how you did that.

Another example of a request for a student contribution is this one:

Excerpt 6

T: You have found that five sixths equals ten twelfths [writes 5/6 = 10/12 at the blackboard]. Is this correct, what S1 says? S2, is this correct?

S2: Yes.

T: Why?

Both excerpts five and six are examples of requests for explanations, but they are asking for different types of explanations. By asking how it was done, the teacher asks for a description of how the student arrived at a solution, which enlightens all of the steps for others to understand or assess the process. By asking why something is correct, the teacher requests a justification, which is normally more mathematically demanding. In addition to asking students to explain, two of the teachers, a few times, asked the students to apply the method or rule that they just established on similar tasks. Asking students why, how, and to apply enables the teacher to emphasize important concepts or methods by requesting student contributions. In addition, a few times, two of the teachers asked other students to assess a student's suggestion or solution.

Discussion and conclusion

Guiding, doing and requesting describes three different ways that teachers involve themselves when orchestrating a mathematical discourse, and each of these three broad categories demand different types of knowledge (see table 2). All three might belong to what Niss and Højgaard Jensen (2002) call teaching competency, and they can be seen as a further detailing of it.

Guiding describes actions in which the teacher either redirects (advises a new strategy, puts a student suggestion aside, asks correcting questions), or points out important elements (notice), and then leaves the student to do the work. In order to do so, the teacher, at a minimum, needs to know

Table 2. *Overview of the categories related to Guiding, Doing, Requesting and the original framework (Redirection, Progressing, Focusing)*

	Redirecting	Progressing	Focusing
Guiding	Put aside Advising a new strategy Correcting questions		Notice
Doing		Demonstration Simplification Closed progress details	Recap
Requesting		Open progress initiatives	Enlighten detail Justification Apply to similar problems Request assessment from other students

the mathematics with which the student is working, and the teacher must be able to see at least one possible solution strategy. Such knowledge is described by Ball et al. (2008) as common content knowledge, but it might also be described in a broader way as mathematical competency (Niss & Højgaard Jensen, 2002) or mathematical proficiency (Kilpatrick et al., 2001) in the content with which the student is working. Guiding has similarities with supporting children's conceptual understanding (Fraivillig et al., 1999), but guiding might be seen as a narrower category.

Doing describes actions in which the teacher either does nearly all of the work (demonstration, simplification, closed progress details) or contributes in a crucial way (recap). When the teacher recapitulates the solution, clarifying the key steps and what is important to remember from the process, this builds on the discourse, and it is not the sole work of the teacher. However, usually such a recapitulation was observed when the solution process was messy or unclear, and it was crucial for the teacher to contribute with a clarification. At other times, the teacher also controlled the process. One such way is to demonstrate the entire solution without any student input, while another way is to simplify by giving hints or adding information. A third way is to divide the solution process into small steps and ask rather basic and closed questions. These three actions involve no real student contribution, but might serve an important role as illustrations of the exemplary solving processes. The demand on the teacher in such cases is mainly that the teacher must be capable of solving the tasks, or have the actual common content knowledge, as Ball et al. (2008) describes it. Doing is also related to the concept of fluency (Kilpatrick et al., 2001), both mathematical proficiency (doing the solution process in a fluent way) and the proficient teaching of mathematics (for example, by using teaching tools and concrete materials fluently).

Requesting describes actions in which the teacher either asks for more details about how the solution was reached (enlighten details) or why it is correct (justification), asks the students to apply (apply to similar problems) or assess (requests assessment from other students), or initiates progress by asking an open question (open progress initiatives). While doing and guiding primarily relates to the mathematical content and the solution process, requesting is different. For example, while asking why (requesting justification) is easy, it is rather difficult to know when to ask why and when not to. Justification is related to proof and argumentation, which are elements of a sound mathematical knowledge (see, for example, Kilpatrick et al., 2001; Niss & Højgaard Jensen, 2002). Similarly, while making details visible for students is one of the most powerful moves a mathematics teacher can perform (Franke et al., 2007), one cannot ask for every detail all the time. Requesting is also aimed at challenging and advocating (Alrø & Skovsmose, 2002) and ways to involve students in real discussions about content, and not just talk for its own sake. While doing and guiding typically places the authority with the teachers, it is possible to change this by requesting students to explain how and why, and to assess and apply. In this way, requesting describes tools that might contribute toward moving the authority from the teacher or the textbook and towards mathematical arguments.

An important challenge is how guiding, doing and requesting might be used in a purposeful way for students' learning. Guiding is not only aimed at pointing out a direction, it is also aimed at when and how this can be done to help students build their mathematical knowledge. In a similar way, doing is not only aimed at helping the students find an answer, but when and how to give hints, split a problem up into smaller steps, clarify or just demonstrate, so that it helps students to learn mathematics. While requesting seems to describe more precise tools that can be used to include students' contributions in discussions and to create a discussion that focuses on details, arguments and proof, it is still important to have balance. One cannot use all contributions or justify all suggestions. Sometimes, students need advice, demonstrations and hints. The knowledge of how to purposefully orchestrate mathematics discourses rests on a deep knowledge of the mathematics content that is discussed in order to purposefully choose which suggestions to use, which ideas to highlight, and which ideas to let go. This deep knowledge of content might be described as specialized content knowledge (Ball et al., 2008) or profound understanding of fundamental mathematics (Ma, 1999), which is a knowledge that is both purely mathematical and, at the same time, strongly related to the profession of teaching.

Even a teacher that knows how to guide and support students towards solutions and understanding, knows how to do the mathematics in order to exemplify, and knows the mathematics well enough to request further details or justifications, still faces additional challenges. An important aspect of orchestrating mathematical discourse in purposeful ways is when to use which tool, when to guide, do and request, and how to do this. Such knowledge relates to both the content and the students that are involved, and a thorough knowledge of the thinking and reasoning of both the typical and the individual students is necessary in order to choose among and balance the tools purposefully. Ball et al. (2008) describes this as knowledge of content and students, and it is also possible to describe this as an example of ”clearly PCK” (Chick et al., 2006), as it is impossible to separate this knowledge as content or pedagogy.

As with any categorization, there are several border problems. One of the most problematic was correcting questions, which might belong to both guiding and requesting. Any question might be seen as a request, and correcting questions is, in its nature, both a correction (redirection) and a question (request). The decision to brand correcting questions as mainly guiding was based on a determination that the questions often are more for advice than an open request, in which one direction is pointed out as correct. Another problematic issue is that both simplification and closed progress details involve guiding, either by giving hints or by asking one question for each step until the student arrives at the solution that the teacher wants. The reason for including these in the doing category is that, in both cases, the teacher is doing the majority of the work, only leaving basic tasks for the students, while guiding typically leaves nearly all of the work to the students. Also, at a superordinate level, guiding and requesting might be seen as quite similar, as most requests can be interpreted as guiding. However, the difference is that guiding is aimed at helping the student towards a solution, while requesting is aimed at understanding the content. In addition to these border issues, there might also be a fourth category that consists of apply, notice and recap. The common feature among these three is that they all use student comments, and they build on and develop students' thinking. Such a category has similarities with extending children's mathematical thinking (Fraivillig et al., 1999).

An approach for future research could be to use tools such as guiding, doing and requesting and examine how different uses of them affect discourse, and possibly, the learning results as well. After all, it is fair to assume that what students are working with, they will learn. Requesting more explanations of details, more arguments, and more frequent justifications will change what the students work with, and probably, also what and how they learn.

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