The first foci of elementary school students dealing with prognosis tasks in interviews

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The nature of stochastics is not only characterized by its relationship as a model of the real phenomena described by it as well as by its usage to find hypotheses to be tested in reality, but also by its peculiar characteristic of modeling the relation between model and real phenomena. Stochastic prognoses can be one key concept for elementary school stochastics to implement the fundamental idea of the specific nature of stochastics. Stochastic prognoses may be characterized as reflexive statements containing the structural components focus, evaluation and justification. Examples are given to illustrate these components. The paper outlines some a priori determined conceptional requirements for stochastic prognoses to give a first orientation of what can be expected from primary school children. It is assumed that the topics, questions and problems stochastics is concerned with, are part of a culture that a child is just entering. To learn more about the ways in which primary school students understand and express stochastic prognoses, a series of half-structured interviews with 3rd graders (age 8-9) were videotaped and transcribed before and after a series of lessons. This contribution concentrates on the foci that children might adopt when dealing with prognosis tasks in interviews for the first time. An overview of the reconstructed types of foci is given and illustrated by examples. The stochastic foci reconstructed so far may be classified as simple foci that could be further described as sequential or aggregate foci. A case study of one child in a pre-interview shows what and how foci might be articulated when being confronted with the new semiotic means of a list.

There seems to be consensus that stochastics should be implemented already in primary school mathematics (see also Greer & Mukhopadhyay, 2005, p. 315) and that it should be oriented at fundamental ideas. The explicit contents, however, still seem to be a matter of discussion. Research on stochastic thinking of primary school children mainly follows a normative perspective by using elaborated concepts from

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Stanja, J. (2013). The first foci of elementary school students dealing with prognosis tasks in interviews. *Nordic Studies in Mathematics Education*, 18 (4), 29–50.

stochastics (such as sample space, random variables, law of large numbers, compound events) as a lens on primary school children's thinking. The present contribution will address both the contents as well as the perspective on children's thinking and is accordingly organized in two parts. The first part proposes stochastic prognoses as a key concept for elementary school stochastics related to the fundamental idea of the specific nature of stochastics (see Batanero, Henry & Parzysz, 2005; Heitele, 1975; Steinbring, 1980, and the next section). Making predictions is a common task in stochastics education. In this contribution, the structure of stochastic prognosis is considered and some of its characteristic components will be outlined. To illustrate them, elementary examples from every day life, as well as random experiments are presented. These examples also show the richness of the concept of stochastic prognosis. To get a first orientation for what we may expect from primary school children when confronted with the task of giving a stochastic prognosis, some requirements and constraints will be outlined in a brief a priori analysis. The analysis regards knowledge from other domains of elementary school mathematics such as arithmetic, the awareness of time dimensions and the availability of appropriate tools to study and communicate stochastic prognoses.

The second part of the contribution will give some insight in a qualitative research project taking an interpretative stance that is dealing with elementary school students' understanding of stochastic prognoses. This contribution concentrates on the question what stochastic foci might be adopted by primary school children when dealing with prognosis tasks in interviews for the first time. It will be argued, that to know the typical questions and topics stochastics is dealing with, is an important requirement for entering the domain of stochastics. Examples from interviews with 3rd graders illustrate the variety of stochastic foci taken and again show the richness of the concept. The presented stochastic foci may be classified according to the distinction in the first part as sequential or aggregate foci. At the end of the paper, a summary is given and questions as well as directions for further analysis of the data are formulated.

Stochastic prognoses – a key concept

Stochastics in primary school is regarded as propaedeutic and should be oriented at fundamental ideas of stochastics. In order to accomplish that, concepts have to be identified and defined that may be the basis for more sophisticated considerations and that are connected to the fundamental ideas of stochastics. To understand this, one may compare the situation with elementary arithmetic. Prior to primary school, children can make concrete experiences in their everyday life with situations involving counting, ordering, measuring and so forth. These particular aspects of the number concept may be brought to school and form the basis for a more general number concept, operations of numbers and so forth. From a more elaborated view, one may consider number aspects as describing the purposes and contexts where numbers are used or applied. Using this analogy, what comparable key concepts might there be for primary school stochastics? This part of the paper proposes *stochastic prognoses* as a key concept for elementary school stochastics. A structural definition of stochastic prognoses will be given and illustrated by means of examples from everyday life, as well as random experiments. Here some references to more elaborated stochastic concepts will be made to show that a more differentiated treatment is possible. The last section refers to some requirements for dealing with stochastic prognoses. They concern knowledge from other domains of elementary school mathematics such as arithmetic, the awareness of time dimensions and the availability of appropriate tools to study and communicate stochastic prognoses.

Motivation and structural definition

In 1975, Heitele published a list of fundamental ideas for stochastics from an epistemological point of view. He arrived at this list by using the following perspectives: the definition of fundamental ideas according to Bruner, the results of developmental psychology at that time, the difficulties of adults with stochastics, and the historical development of stochastics. Heitele understands fundamental ideas as

> [...] those ideas which provide the individual on each level of his development with explanatory models which are as efficient as possible and which differ on the various cognitive levels, not in a structural way, but only by their linguistic form and their levels of elaboration. [...] What matters here is the constancy of the structure of the explanatory model. The more intuitive model is a coarser – and thus refinable – version of the more elaborate one. (Heitele, 1975, p. 188)

According to Bruner, such explanatory models ”pre-establish the later analytic knowledge” and may contribute to the child's understanding of its environment ”by its own means, long before it can understand the linguistic complexity and sophistication of the underlying mathematical models in their analytic form” (Heitele, 1975, p. 189).

The fundamental idea that is most important here, concerns the nature of stochastics as applied science. Heitele writes that the difficulties in stochastics lie in its applications and he considers the relation of model and reality as a basic idea for stochastics (see Heitele, 1975, p. 191). The importance of the relationship between model and reality or application may be emphasized in the light of the variety of interpretations of stochastic theories or the role and functions that are attributed to the applications (Batanero et al., 2005, p. 19; Steinbring, 1980, pp. 34). Heitele (1975) gives an illustration (p. 192) of the relationship between reality and stochastic model which may be summarized as follows: Reality (in the form of random experiments, subjective beliefs, symmetry arguments or relative frequency) functions as the basis for stochastic models (a priori probabilities, sample space, axioms, propositions). The models have consequences that are used for predictions in reality. In reality, tests are performed that may lead to modifications in reality or of the model.

For probability theory and its applications, Steinbring (1980) reconstructed from historical case studies that the relationship of reality and model itself became a subject to be considered by probability theory as well[1]. This appeared for the first time with what is now known as Bernoulli's law of large numbers. The starting point for the development of this theorem was the question whether relative frequencies could be used to estimate probabilities when they are not a priori known. It is interesting to note that a relation of frequencies and probabilities did not seem to be obvious in the beginning of stochastics. If this rather elementary relation is not obvious for learners either, it becomes clear why we cannot focus only on one side of the relationship of model and reality (see Steinbring, 1991) if we want students to understand stochastics. As has been shown in stochastics education research, purely axiomatic and formal instructions do not lead to a deep understanding (Steinbring, 1991). The same is true for a pure experimental approach as has been argued by Batanero et al. (2005, p. 33).

Though arguing for the importance of this fundamental idea, we shall refer to the doubts Heitele expressed concerning the implementation at all school levels:

> I do not dare answer the question whether the postulate of separation of reality and model[57], or even of consciousness about it – the rationalisation[58] in aloofness can be possible and effective on every cognitive developmental level, but I think it would be worthwhile leading the individual to very early empirical experiences of this phenomenon of "individual liberty under collective constraint".
> (Heitele, 1975, p. 201)[2]

The relation between classical and frequentist perspectives (of probability) is the concern of the first item that Jones, Langrall and Mooney have put on their agenda for future research directions (2007, p. 946). Examples for research studies addressing this topic are the work of Stohl and Tarr (2002), of Nilsson (2009) or more recently of Schnell (in press). All three studies focus on middle school students. Until today, Heitele's question has still not been answered for primary school students.

Stanja and Steinbring (in press) adapted the epistemological characterization of stochastic knowledge to early stochastics. It was argued that from the beginning, the particular nature of stochastics should be taken into account. Therefore, the idea of stochastic prognoses was proposed as a possibility for a first approach at the primary school level "to grasp the particularity of stochastic knowledge and to learn that there could be different qualities of what scientific knowledge can express." (Stanja & Steinbring, in press) They also referred to the complexity of the notion of stochastic prognoses. To deal with this complexity one needs semiotic means that allow to study stochastic prognoses and to articulate one's ideas. They proposed that one could introduce stochastic prognoses in the context of random experiments. Generally speaking, a working definition of a stochastic prognosis may be that it is a *justified* reflexive statement about some future event that contains a *focus* and an *evaluation*. These components can serve as one basis for studying primary school students' understanding of stochastic prognoses. To understand what is meant by the structural components of focus, evaluation and justification, table 1 gives some examples from everyday life (situations of personal interest [3]) and random experiments. The examples show the variety

Table 1. *Examples of prognoses from everyday life and random experiments.*

<table>
<tr><th>Domain</th><th>Focus</th><th>Evaluation</th><th>Justification</th></tr>
<tr><td>Weather</td><td>Occurrence of rain
Amount of rainfall
Max./Min.Temperature
range of temperature
Time for thunderstorm</td><td rowspan="5">Degree of beliefs: Probability; expressions like probably, impossible, ...

Comparisons: more probable than, most likely,...

Deviations: Standard deviations; expressions like approximately, around, ...</td><td rowspan="5">Reference to: data, context information, distribution models, propositions from the theory of time series analysis, stochastic processes, ...</td></tr>
<tr><td>Bus arrivals</td><td>Time of arrival
Distribution of delays</td></tr>
<tr><td>Traffic lights</td><td>Time until turning red/green</td></tr>
<tr><td>Sport Competition</td><td>Winning/Losing party
Scores</td></tr>
<tr><td>Random experiments (spinners)</td><td>First occurrence of yellow;
Number of successive outcomes of blue (length of runs);(Absolute or relative) Frequency of blue;
Regularity of outcomes;...</td></tr>
</table>

of possible foci and thus illustrate the potential and the richness of the concept of stochastic prognoses.

Let us look at the random experiment example. The *stochastic focus* expresses what we want to give a prognosis for. These foci may be further characterized as *sequential* or *aggregate foci*. Sequential foci encompass sequences of single turns, runs or block length, waiting times and patterns. Aggregate foci comprise the consideration of absolute or relative frequencies, ranges for frequencies of an outcome, relations (more/less, maximum/minimum) between frequencies for outcomes.

The evaluations express rationalized probabilities or degrees of belief and by means of expressing possible deviations taking variability into account. The evaluations should be "grounded in reason and analysis" (Langrall & Mooney, 2005, p. 95) which brings us to the *justification* component. This encompasses for example references to the structure of the spinner (comparison of areas, part-whole considerations), data from previous experiments with this spinner, or when interested in the convergence of relative frequencies reference to the mathematical law of large numbers according to Bernoulli.

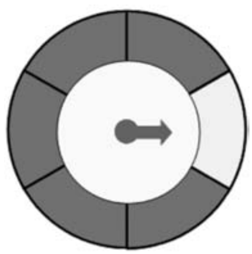


Figure 1. *Spinner*

Note. Blue fields are dark in the figure; the yellow field is bright.

Let us consider the focus of the relation of the frequencies of yellow and blue when a spinner (as shown in figure 1) is turned 20 times. A primary school child might say, that there will probably be more blue than yellow because the spinner shows more blue fields than yellow ones. The evaluation is made by the use of the word "probably" and the child refers to the relation between blue and yellow fields for justification. The focus can be expressed more precisely by $X > 10$, where X is a random variable that may take the values $\{0, 1, \ldots, 19, 20\}$. For justification one might e.g. assume that all turns are independent and carried out under the same conditions, and thus X has a binomial distribution $B(20, p)$. To determine the parameter p one may refer to the proportion of blue fields on the spinner (5/6) or estimate it by the relative frequency of the occurrence of blue from previously conducted experiments. The reference to this binomial model may become ”visible” in the calculation of the probability $P(X > 10)$. This probability may be used for the evaluation of ”more blue than yellow”.

So, the prognosis formulated by the child may be related to more elaborated stochastic considerations. The formulation of a prognosis containing an evaluation by means of probabilities referring to available relative frequency information could then be questioned. The question ”Is it a good idea to use frequencies to estimate probabilities and under what conditions can this be done?” again leads to stochastic considerations. These may then again be used to formulate new prognoses.

Requirements and constrains

The last section of the first part refers to some requirements and constraints that were identified by analyzing the concept of stochastic prognoses and some additional literature. The outlined requirements and constraints provide an orientation for what may be expected from primary school children.

In developmental psychology the development of the time concept in children was studied by Piaget (1955) who found that the time dimensions and differentiations develop at the age when children are in primary school. The awareness of the time dimensions past, present and in particular future, is a basic requirement for the concept of stochastic prognosis. For a child that is not aware of the future dimension, it would make no sense to give a statement about some future event. Moreover, Wissing (2004) investigated time concepts of primary school children (3rd grade) from a pedagogical point of view. Part of her study was concerned with ideas about the time dimensions. For stochastic prognoses the ideas about the future dimension are of interest here. The study informs about some general ideas of primary school children about the future. Almost all children in her study expressed an interest in this dimension and most of the children expressed that they do not know what will happen in the future.

We may wonder whether children may adopt stochastic foci at all, since the topics, questions and problems stochastics is concerned with, are part of a culture that the child is just entering. Moreover, Fischbein (1975) detected a cultural bias towards determinism. Keeping this in mind and taking into account the type of statements children are used to in elementary mathematics prior to an introduction of stochastics, stochastic prognoses represent a new form of statement in elementary school mathematics. Moreover: "Researchers do not agree on the age at which children understand the construct of uncertainty" (Langrall & Mooney, 2005, p. 97). But this is partly due to the tasks used in the studies. According to Langrall and Mooney, elementary school students might grasp aspects of the construct of uncertainty (see Langrall & Mooney, 2005, p. 97).

In the section before, the importance of the relationship between frequencies and probabilities has already been addressed. If we think of the laws of large numbers, the notion of infinity or several types of convergence are of great interest. These notions are not part of elementary mathematics curricula. Even middle school students seem to have great difficulties understanding the concept of infinity (Schimmöller, 2011). Thus, it is reasonable not to expect that children are able to articulate an understanding of the law of large numbers. However, children may use frequency information available to them and they may consider relations of expected frequencies and actual frequencies. But this is still not clear. Preforms of the convergence ideas may be the distinction between single outcomes and a series of outcomes (that is seen as a whole and thus requires a change of perspective).

Further constraints are provided by the knowledge from other domains of elementary mathematics, such as arithmetic and geometry, that restrict the availability of semiotic means and the usage of available ones. So there are limitations in means to study and to understand stochastic situations, but also to formulate prognoses. These limitations concern conventional means usually used in stochastics, as for example, symbolic notations (fractions, percentages) and language, but also diagrams. The development of stochastic understanding is regarded here as being determined by both the development of techniques and the development of concepts/interpretations. It is reasonable to think that the limitations concerning the means may influence the construction of new concepts, and in consequence constrain what stochastic thinking at the primary school level could mean. Comparing the situation to arithmetic and the development of the number concept, it is necessary to introduce students to social artefacts and means to enable them to express the complexity of stochastic prognoses. In order to include stochastics in the elementary school classroom, some conventional means may be adjusted (see the templates for lists and diagrams in the second part of the paper), others have to be excluded. It is assumed that children need some appropriate semiotic means that allow them to articulate their ideas and to get from a phenomenological to a more sophisticated or conceptualized understanding. For research on stochastic understanding, this means two things. On the one hand, a setting should be created such that primary school children may articulate their ideas despite all limitations. But on the other hand one should keep in mind possible influences of the provided means.

Dealing with prognosis tasks for the first time

In the second part of the contribution some insight in a research project concerned with children's understanding of stochastic prognoses will be given. But first, some remarks concerning normative and descriptive perspectives on students' thinking shall be made. It will be argued for a descriptive perspective that motivates looking at the first foci that primary school children might adopt when being confronted with the task to give a prognosis for future outcomes of random experiments. For the purpose of this contribution additional information will be given about the design of the interviews that have been carried out with 3rd graders (age 8–9). Then, some information will be provided about the data analysis that has been done to create the overview for the initial foci. Afterwards, the overview will be presented, illustrating the various types by examples from the interviews. In order to give first insights in how the foci might change when children are confronted with the list as new means of communication, the example of Johann is presented and the foci will be reconstructed from his verbal expressions and gestures.

Normative versus descriptive perspectives

Analysis of students' understanding in the stochastics education literature has usually been more content oriented. Research started from concepts or aspects of particular concepts (e.g. sample space, and probability), and it has been described what aspects of these concepts students possess. This holds for older works on misconceptions (for an overview see Shaughnessy, 1992) but also for newer level models such as the framework for assessing probabilistic thinking (Jones, Langrall, Thornton & Mogill, 1997). For an overview of newer studies on stochastic thinking with an emphasis on probability see Jones, Langrall and Mooney (2007). In most of the existing studies emphasizing probability, stochastic knowledge has been considered as something that is fixed or "ready", "objective" in the sense of being shared by a stochastic community. From this perspective, knowledge is represented by rather unambiguous semiotic means. The conventional character of those means is seldom discussed.

In research on statistical thinking, there has been a change from a concept-orientation to a process-orientation. An interesting notion that emerged from research on statistical thinking is that of informal inferential reasoning (IIR). Referring to different studies dealing with informal inferential reasoning, Zieffler, Garfield, delMas and Reading (2008) proposed a working definition of IIR that includes

> 1. Making judgments, claims, or predictions about populations based on samples, but not using formal statistical procedures and methods [...]; 2. Drawing on, utilizing, and integrating prior knowledge [...], to the extent that this knowledge is available; and 3. Articulating evidence-based arguments for judgments, claims, or predictions about populations based on samples. (Zieffler et al., 2008, p. 45)

Makar and Rubin (2009) discussed, reorganized and broadened this working definition. They identified three components (or principles) that are central to IIR which are 1. *generalization beyond the data* that include predictions, parameter estimates, and conclusions, 2. the use of *probabilistic language* and 3. the use of *data as evidence* for the generalizations. The first is seen as particular to the process of inference whereas the other two are considered as specific to statistics. Though there has been a change to look at the processes, these studies still emphasize a normative perspective.

Since instructions should not only build on the aims we wish children to accomplish, but also on the ideas, that children have (see also Hawkins & Kapadia, 1984, p. 355) other perspectives should be considered as well. The last years, however, a need has been identified for non-normative perspectives on students' thinking and understanding (see for example Pratt, 2000). Examples for studies going into this direction are those of Metz (1998) or Nilsson (2009).

Non-normative or descriptive perspectives put more emphasis on students' interpretations of the situations, tasks and means provided by the researcher. It might be useful to consider the child's knowledge that is not fixed, knowledge that varies, and that changes due to the multiple interpretations they construct of the supplied means. Whether the resulting (pre-) concepts are appropriate or useful for a further learning in stochastics can be evaluated after the analyses are done. So, methodologically this would mean to distinguish between the *meaning making* and the *evaluation* of that meaning making in respect of a shared understanding of the stochastic community (see also Sierpinska, 1994).

To get hints and find traces of children's understanding and to be able to reconstruct it, it is necessary to interact with them in some way (such as interviews or paper-pencil tests). At the same time, one cannot ensure that, while working on the test items or being engaged in the interview, students don't change their understanding. Moreover, the following phenomena are described in the literature: it is observed that (probabilistic) thinking seems to be influenced by the nature and structure of tasks or problem situations (see for example Langrall & Mooney, 2005, or Metz, 1998). So, understanding cannot be seen as something that is fixed but it can be seen as something that emerges in the engagement with a particular situation (interview, test). After all, to say it with Gal:

> Probability is not a tangible characteristic of events, but rather a perception, whether expressed via a formal mathematical notation or informal means, of the chance or likelihood of occurrence of events. Such perceptions depend on the interaction between factors operating in external situations and within persons who face these situations. (Gal, 2005, p. 40)

So, if probability is understood as a way one could perceive real phenomena, it becomes particularly interesting to look at the perceptions of children in random situations. To learn stochastics also means to know what questions might be asked and dealt with in stochastics. Therefore, the contribution is concerned with the foci that primary school children might initially take when asked to produce a prognosis for the first time. The next section refers to a research project undertaken in the frame of epistemological interaction research (see Steinbring, 2009) that encompassed pre-interviews, a series of lessons and post-interviews.

Design of the study

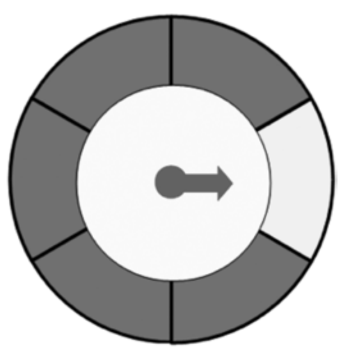
A qualitative empirical study was conducted with children in grade 3 (age 8–9) who had no or only little experience in stochastics. The children were interviewed in a 1-1 situation in half-structured interviews. A series of 12 lessons (each of 45 minutes) followed with the focus on stochastic prognoses. The content of the lessons included: introduction to random experiments, elementary tools to formulate and study stochastic prognoses for random experiments with spinners (lists, diagrams, technical terms for evaluations for frequencies); finding and formulating statements about future outcomes of random experiments, evaluate and justify them; comparison of the outcomes of various spinners; the reconstruction of spinners from available data; general discussions about prognoses (such as, What are they?, Where do you know them from? Is it the same as guessing or looking into the future?) and the evaluation of statements about future events. Afterwards, the children that were interviewed prior to the series of lessons were interviewed again. For the purpose of this contribution the following section provides the necessary information about the interviews.

Interviews

A short version of the whole interview guideline is shown in figure 2. In the following, only those questions and materials are described that are relevant for this contribution. The guideline encompassed questions, background information from the pilot study with possible further questions and actions of the interviewer. The interviewer asked further questions to clarify what a child meant by idiosyncratic expressions, unfinished sentences and so forth. If a child asked the interviewer about the questions or a task or given materials, the interviewer had to be careful not to influence the child with certain interpretations.

The data used for this contribution are transcribed episodes from the videotaped pre-interviews and the episodes of interest stem from the beginning of the interviews (see 1–3 in the guideline). Consider the following questions from the detailed guideline: "I like you to do an experiment with the spinner, where you turn 20 times – just as you did before. But before you do that, I like to know, what you think, what the outcome of an experiment with 20 turns could be?" and "How did you come to this idea?" To that point, the children have been asked whether they have had experiences with spinners. The interviewer has shown the spinner to the child and how it may be turned. The child has tried to turn the spinner. The interviewer has also explained to the child how one speaks about the outcomes of a turn (If the pointer points at yellow at the end of a turn, we say that the outcome is yellow). No material was available to the child besides the spinner itself. The questions above intended to get a

1. Experiences with the spinner
2. Statement about the outcomes without notation
3. Justification without notation
4. Explanation of the list, record of possible outcomes in the list
5. Explanation of the statement and justification with the list
6. Evaluate other given lists
7. Experiment I – record in a list
8. Description and explanations for the outcomes of the experiment
9. Comparison with initial statement
10. New statement of the outcomes with given focus of absolute frequencies
11. Explanation of elementary diagram, record of the articulated frequencies
12. Justification and additional explanations
13. Evaluations of cards with given justifications
14. Evaluation of the own statement
15. Evaluation of other given filled diagrams
16. Second Experiment
17. Description and explanations for the obtained frequencies
18. Comparison with initial statement
19. Possibility to make statements for future outcomes of experiments
20. Evaluation of cards with different statements regarding the possibility to say something about the outcomes of experiments
21. (Game context)



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Figure 2. *Short version of the guideline for the interview, template for lists and spinner*

first statement from the child about future outcomes of the experiment with the spinner. After the first statement(s), the children were introduced to the list and they recorded possible hypothetical outcomes. The template for a list represents one of two semiotic means introduced in the interview. The second one was a template for an elementary diagram with which one could record absolute frequencies or ranges of frequencies. The list was intended as an additional semiotic mean to be used to articulate and communicate ones ideas. It offers the possibility to record possible outcomes that one might refer to by verbal or gestural means for further explanations.

Data analysis

To create the overview of initial foci, the episodes prior to the confrontation with the list and before an experiment was carried out were chosen and analyzed with the aim of reconstructing the very first foci that children might adopt. It was determined whether the children interpreted the task in the intended way. Then, the verbal expressions and gestures were interpreted to find evidence that allows to categorize the statements. The general distinction between aggregate and sequential foci (see the first part of the paper) was used. Additionally, the categories ”None” and ”Other” were included. The None-Category refers to cases, where children either denied the possibility to give a prognosis, could not make sense of the demand, or thought the question was about something else. The Other-Category includes cases that concern extreme cases (in a mathematical sense) that would fit as well in the aggregate and the sequential focus category or cases that could not be well interpreted.

For getting insight into the way a child might express foci when using the new communicative means of a list, I chose the episode that starts after the production of a list. The episode begins with the question from the interviewer who asks Johann to explain what he was thinking about when producing the list. The analysis of this episode concentrates on the foci that might be reconstructed from Johann's verbal utterances and gestures using the list and how these are related. Here I want to emphasize that the produced list alone will not tell so much, but rather the verbal expressions related to it. I understand language not only as a means of communication but also as a representational system that seems to be particularly interesting for early stochastic learning (see for example Tatsis & Kafoussi, 2008, or Makar & Rubin, 2009). Taking the verbal utterances and gestures into account in the interpretative analysis is supported by an understanding of spoken language and gestures as an integrative system for the person speaking and listening (see Huth, 2011 who refers to McNeill, 2005). Johann's case also shows how the gestures may give additional information not verbally expressed.

Table 2. *Overview of the initial foci*

Focus	Total number of children	Examples (Translations)
None (No Prognosis:3, Task Interpretation:5)	8	**Thomas**: Hm (...), eh, (...), mm, I do not know at all, ehm (*hunches his shoulders*), don't know.
Sequential Foci (Single turns:2)	2	**Maira**: I believe, first this (*points at yellow*), then this again (*points at a blue field of the spinner*) and then maybe this (*points to a different blue field*), this (*points at the same blue field*) and this (*points to the first blue field*), then comes this (*points at a third blue field*) and then maybe a few times blue again and then one time yellow again and then maybe yellow again and then a few times blue again and then yellow and then a few times blue again.
Aggregate Foci (Relation(B,Y):8), Range of Frequency of Yellow: 1, Absolute Frequency of Yellow:1)	10	**Johann**: Mh (*strokes his chin, looks at the interviewer, then in the air, then at the spinner*) eh, (*looks in the air, 15 sec*), well, allright (*looks at the interviewer*), ehm, yes (*looks in the air, 6 sec., looks at the spinner, 5 sec.*), eh, I, but I belief (*strokes his chin*), then it comes rather more to blue (*points to the spinner*) |Because blue is more. (*points at the blue fields of the spinner and looks at the interviewer*)| Because, it is then always here around (*moves his finger over the blue area*) and yellow is ony once (*points at the yellow field*) and blue is five times. Then, one get almost only rarly at yellow and very often at blue.
Other (Extreme cases: 3,could not be categorized:1)	4	**Imre**: One turns always the same color?| And, and one does not have the other one then.

Overview of the stochastic foci initially taken in pre-interviews

Overall, the statements of 24 children were analyzed and categorized. Table 2 summarizes the foci that could be reconstructed from the first statements of the children.

There were eight children where no focus could be reconstructed. These children either expressed that it is not possible to give a prognosis, that they do not know what will happen (three children), they could not make sense of the demand, or understood it in a completely different way (five children). For instance it appeared that children mixed up "interview" and

”experiment”. One child thought that he was being asked to say something about his own abilities that he would show in the interview.

A sequential focus was only adopted initially by two children that were both focusing on single turns. Maira’s statement illustrates this very well.

Interestingly, 10 children adopted an aggregate focus. Here, the main focus to be adopted was the relation between the frequencies of yellow and blue (8). This was mostly expressed in an informal way (more blue than yellow). One child focused on the absolute frequency of yellow and another one considered a range of frequencies of yellow (”two or three times”). All children that adopted a focus only considered one. Children expressed their ideas about future outcomes by verbal means or additional gestures referring to the spinner. The next section presents the case of Johann, an average student in mathematics, and shows how he articulates his statements about the future outcomes when using a list as a new mean of communication.

Case of Johann

Johann’s initial focus fell in the category of aggregate foci (see table 2). More precisely, he focused on the relation between frequencies of yellow and blue outcomes. After his first statement about the future outcomes of the random experiment, the interviewer confronted Johann with a template for a list. She explained that it consists of 20 boxes, one box for each turn and that one could record in the list what could come out when turning the spinner. She put a yellow and a blue pen on the table. Then she asked Johann to fill in the template what he imagined the outcomes could look like. He silently produced the list shown in figure 2 and put it in front of the interviewer. The following transcribed episode started at that moment.

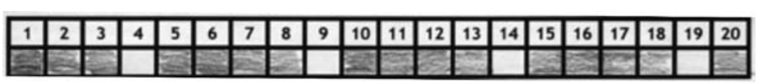


Figure 2. *List produced by Johann*

1 I Ehm, explain to me, what you were thinking [puts the list in front of Johann].

2 J Well, first of all, blue will come for sure [*german: *bestimmt*] [moves a finger from F1 (the first field) to F3], then yellow has to come some time [points at F4], that's why I have put it here. Then blue comes first for sure* [moves a finger from F5 to F9], then yellow again not until then [points at F9],

and here blue will come first again and then maybe yellow again, because yellow [points at F14] is always only one colour [points in the direction of the spinner and then points at F4, F9 and F14].

3 I Mhm.

4 J And that's why blue comes more [moves a finger from F15 to F20 and points at F20] and that's why almost everywhere blue comes, but sometimes yellow comes as well [looks at me and then at the list] in between [points successively at F19, F14, F9 and F4], I have thought [looks at me, then in the air and then at the table and wipes the table with a hand].

5 I Mhm. Ehm, and what gave you the idea to put yellow at these places [points successively at F4, F9, F14 and F19]?

6 J Ah, I have always made a bit [points at F1, then moves the finger from F1 to F3], then I have thought a bit, if there, now [points at F4, moves the finger left and right over F4] and then I have, if I, if there again [wipes the table with his hand], if there bl-, blue [points at F4] or yellow, there, I have put yellow there [points at F4], because [...] well [moves the finger from F1 to F4] [...] because [points at F4] so often blue [moves the finger from F1 to F4, stops, then to F5, stops, to F6 and F7] does not come successively [moves the finger over F7], always so often blue successively [moves the finger from F4 to F5], that's why I have put yellow here [points at F4] and here also put yellow [points at F9], because first comes blue again then [moves the finger from F5 to F8] and then I have put yellow here again [points at F9] and here, I have done that again [moves the finger from F10 to F13], that first comes a bit blue [moves the finger from F10 to F13] and then I have put a bit yellow again [points at F14] and then, here I have put blue again [moves the finger from F15 to F18] and then I have put yellow here [points at F19], because then it comes yellow maybe after, after four [moves the finger from F15 to F18] for sure* again [points at F19] and then here comes blue again at the end [points at F20].

7 I Mhm. And now you have said, ehm, blue could also not come so often successively. How did you mean that?

8 J Yes [wipes the table with his hand], that it can not always, always come [points at F4 and F9]. It comes, it comes for sure* yellow in between once in a while [points at F4], that is why I have also put yellow here in between [points successively at F9, F14 and F19], that's why I have not put blue always, that's why I have also put yellow in between [moves the finger right and left over the list].

9 I Mhm, ok. Ehm, I like to know from you, how sure you are that the experiment will come out like this now?

10 J [looks at me] Yes, well, I am also not sure about that [looks at the floor, takes a deep breath in and exhales]. Oh, hm [looks in the air], but I am a bit sure, but I am also not so sure [looks at me]. [...] Mm [...] [looks at the list, touches his chin with his hand] I kn- [10 sec.] [looks at me].

11 I What are you thinking about now?

12 J Well [looks in the air], I am, I am a bit sure [looks at me, then aside], but I am also not quite sure.

13 I Mhm.

14 J But I am sure [**german: *schon sicher*] [looks at me, then aside], that it could come out somehow like that [moves the hand left and right over the list].

For space reasons, a condensed version of the analysis is given and it only concentrates on the foci that Johann adopted and how he related them. In this episode one can find different foci from his initial one. First, in #2, Johann focused on the waiting times until yellow occurs for the first time ("yellow has to come some time") and then on the waiting times for further occurrences of yellow ("then yellow again not until then" and "blue will come first again and then maybe yellow again"). #4 provides evidence that Johann related the occurrences of yellow to his initial aggregate focus on the relationship of the frequencies of yellow and blue ("blue comes more and that's why almost everywhere blue comes, but sometimes yellow comes as well in between"). In #5, the interviewer took up his focus on the waiting times and asked him why he has put yellow at F4, F9, F14 and F19. The statements in #6 support the interpretation that he related the occurrences of yellow to the length of runs of blue ("so often blue does not come successively"). Here, further hints can be found in his gestures referring to the length of the runs (*moves the finger over the first blue fields in the list, stops at the first yellow field and then goes on*). With the repetition in the following verbal statement Johann reinforced his focus ("always so often blue successively"). Then he came back to the waiting times which becomes clear in his verbal statements ("I have put yellow here, and here also put yellow, because first comes blue again then and then I have put yellow here again and here, I have done that again [...] that first comes a bit blue and then I have put a bit yellow again and then, here I have put blue again and then I have put yellow here, because then it comes yellow maybe after, after four for sure again "). The pointing gestures make clear what turns (for example: *points at F4*) and length of runs (for example: *moves the finger from F5 to F8*) Johann is referring to. At the end of his statement, he expressed the length of the waiting time verbally for the first time ("after, after four"). In #7, the interviewer took up his focus on the length of runs of blue and asked for clarification. Johann expressed that blue cannot come always ("that it can not always, always come") and thus makes clear that he thinks about the length of blue runs as finite.

In summary, when confronted with the list, Johann expressed further foci as the length of runs of blue (number of successive turns with blue outcome) and waiting times for yellow that belong to the sequential foci. This is particularly interesting since the mathematical treatment of runs is considered as rather difficult, see Moore (1990, p. 121), who mentions that runs are usually difficult to deal with in probability computations and longer runs may be surprising. Expressing his ideas regarding this focus verbally was quite difficult for Johann. He used the list to show what he was referring to by pointing or moving the finger over several fields. Similarly, his ideas regarding the focus on waiting times for the occurrences of yellow were expressed by verbal and gestural means. It is interesting, that his expressions changed from a rather vague ("some time") form to a quantitative form ("after four").

Summary and outlook

In the first part of the paper it has been argued that prognoses may serve as a key concept in elementary stochastics that allows to introduce students to the fundamental idea of stochastics as applied science. A structural definition of stochastic prognoses was given and illustrated by means of examples from everyday situations and random experiments. Those examples illustrated the variety of possible foci and the richness of the concept. It was shown how stochastic prognoses may be related to the fundamental idea of stochastics as applied science. Some requirements and constraints for the understanding and the articulation of stochastic prognoses were discussed. These considerations provided an orientation for what one may expect from primary school children.

The second part gave insight into a qualitative research project in the frame of epistemological interaction research studying primary school students' understanding of stochastics prognoses. It was argued that the awareness of the questions and problems stochastics is dealing with is a basic requirement for entering the field. Therefore this contribution dealt with the first foci that primary school children may adopt when asked to give a prognosis for future outcomes of random experiments. An overview was provided showing that some children adopted stochastic foci. Those children were able to articulate ideas about the future outcomes of a random experiment with the spinner. In the first statements, those children regarded only one focus. However, the overview has also shown that 8 out of 24 children where not able do give a prognosis, either because they thought that this was not possible, or because the demand of giving a prognosis made no sense to them in the first place. The last point could be resolved by the help of the list for almost all those children.

The case of Johann gave some insight into how the foci initially adopted might change and be connected when being confronted with the new communicative means of a list template. The produced list of a possible outcome itself was not of particular interest here but rather the ideas that a child could express with the use of the list.

Further analysis of interviews will address the interplay of foci, evaluations and justifications and how these relate to the situation and the semiotic means used. This will be a first step to get closer to an answer to Heitele's (1975) problem. It will be interesting to see how the way of using the provided means (in a concrete or rather theoretical/ hypothetical manner) relates to the ideas expressed by the child. Comparisons of looking forward and backward (statements before and after experiments have been carried out) will probably show the time dependence of children's ideas about the outcomes of experiments. Analysis of post-interviews will give insights in possible developments and obstacles that may inform instruction and motivates further research.

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Notes

1 For statistics, the relation of model and reality was addressed by M. Borovcnik (1984) in his dissertation on the meaning of statistical statements.

2 [57] in the citation refers to (Freudenthal, 1972, p. 583) and [58] to (Müller, 1974, p. 167).

3 When asked what a prognosis is, primary school children may refer to weather forecasts, soccer games or events of personal interest that lie in the future.

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