Preparing future teachers for interdisciplinarity

Designing and implementing a course for pre-service upper secondary teachers

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Educational researchers and policy-makers have for some time touted the need for interdisciplinary teaching. But while there are many educational, democratic, and economic arguments for bringing an increased attention to interdisciplinary teaching, there has been a striking lack of exposure of the question of how future teachers, who are largely educated in a mono-disciplinary fashion, can best become equipped to introduce genuinely interdisciplinary teaching activities to their future students. This article presents some preliminary reflections upon a graduate course at the University of Southern Denmark, which aims to prepare future science and mathematics teachers for interdisciplinary teaching, and which has been designed on the basis of influential theoretical expositions of the concept of interdisciplinarity.

In 2005 the Danish upper secondary school system underwent a structural reform. The reform implies that more lessons are allocated for optional subjects organized as so-called packages. An important feature of a package is that the participating subjects form a *coherent* program, which is ensured by a closer interaction between the subjects. The reform thus calls for interdisciplinary teaching across the traditional boundaries between the subjects, both at the level of subject matter and at the level of pedagogy. Indeed, the very aim of the regular upper secondary school system (studentereksamen) is "to prepare the students for further education, hereunder to acquire [...] knowledge and competencies through the education's combination of disciplinary breadth and depth and through the interplay of the disciplines" (Ministry of Children and Education, 2010a; our translation). This focus on interdisciplinary teaching permeates the goal descriptions of the individual disciplines. For example, one of the disciplinary goals of Physics (level A and B) is that the student –

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"through examples and in collaboration with other disciplines" – must become "able to bring a perspective to the contribution of physics on the understanding of natural phenomena as well as technological and societal development" (Ministry of Children and Education, 2010b; our translation). Such aims and objectives of the upper secondary school system necessarily influence the responsibilities of secondary school teachers. And it may even pose a challenge to science teachers who, in the Danish system, traditionally have had their academic training within one or two mono-disciplinary programs. Further, there are many conceptual pitfalls pertaining to the very idea of interdisciplinary teaching and it seems that there is a genuine need for a scholarly discussion about exactly how teachers could be equipped to implement fruitful interdisciplinary activities.

In an attempt to offer future science and mathematics teachers the possibility to prepare themselves for the practical challenges of interdisciplinary teaching, the University of Southern Denmark developed a graduate course (Nat802) in "modeling and interdisciplinarity". The focus of this paper is to thematise how the preparation of aspiring teachers can be operationalized in a university-level course. In particular, we shall focus on the question of how to design a course for pre-service upper secondary school teachers preparing them for the challenges of interdisciplinary teaching. We offer first a background (literature-based) discussion on the relationship between interdisciplinarity and modeling in science education leading to a presentation of a didactic framework for interdisciplinary teaching, which was part of the course Nat802. Next, we present the course itself and its curriculum. The main part of the article consists of a series of illustrative examples of students' work taken from the implementation of the course. Finally, we discuss some of the problems, as we see them, with preparing future teachers for interdisciplinarity in teaching.

Interdisciplinarity and modeling

To implement the objectives of the reform, interdisciplinary teaching across the traditional boundaries of disciplines and subjects is required. This is far from a trivial task, partly due to the lack of frameworks for integrating productive ideas from a variety of theoretical and practical perspectives on the relations between the disciplines of mathematics and science. To grasp the challenge from the reform there is a need for a didactical framework for interdisciplinary teaching. In *International handbook of science education*, Berlin and White (1998) argue that science and mathematics are naturally and logically related in the real world, and that educators therefore must try to capture this relationship in the classroom in an effort to improve students' achievement and attitude in both disciplines. The idea of integrated science and mathematics is not new. For example, a historical analysis of documents related to integrated science and mathematics reported by Berlin and Lee (2005) spans from 1901 to 2001. This analysis documents a strong philosophical support for the integration of science and mathematics education as a way to improve student understanding of the two disciplines. It is emphasized that although each of the human enterprises of mathematics and science has a character and history of its own, each of the disciplines depends on and reinforces the other (Berlin & Lee, 2005). However, there is still no emerging theory supporting an integrated science and mathematics education. But the extensive literature recognizing the importance of models and modeling, both in mathematics education and in science education (Blum, Galbraith, Henn & Niss, 2007; Gilbert & Boulter, 2000; Halloun & Hesteness, 1987; Kaiser & Sriraman, 2006), indicates that modeling could serve as a starting point for developing a didactical framework to advance integrated science and mathematics instruction.

The importance of modeling in science and mathematics education can be justified from a variety of positions. From the *historic-practice* position one could justify this importance by referring to the ubiquity of models and modeling in the history and practice of mathematics and natural science. Matthews (2007) points out that the central role of models in scientific practice entails that learning about the Nature of Science (NOS) will involve learning something about the functioning of models in the history of science and their epistemological import. This leads to the *learning* position referring to modeling instruction as engaging the students actively in the learning process. The act of modeling allows students to engage in a design process, which begins with a set of tentatively accepted theories that evolve into coherent understandings as represented in their models. Halloun (2006) notes that modeling is student-centered in the sense that it engages students without leaving them on their own free will, and teacher-mediated in the sense that the teachers provide guidance and cultivate the students' scientific thinking. According to Mason (2001), a major contribution towards effective teaching of modeling lies in enculturating students into how it feels to perceive the world as a modeler. Teaching modeling is more than simply rehearsing established models from mathematics and science. It is also displaying what it means to interrogate the world and to construct models to explain phenomena which we identify (Niss & Højgaard, 2011). The interrogation of the world supports the *authentic* position. Modeling is authentic in the sense that construction and use of models are used for cognition and learning in everyday life as well as in scientific practice (Hesteness, 2008).

Now, one could say that despite the overwhelming amount of literature on modeling in science and mathematics education, the *interdisciplinary* position is seldom addressed explicitly. However, modeling is a specific problem solving strategy with scientific, mathematical and pragmatic purposes (Hesteness, 2008). As a rule of scientific problems, phenomena of science and everyday life problems do not accept traditional and historical determined boundaries between subjects. This fact is beautifully exemplified by the pendulum story which deals with the interrelatedness of timekeeping, pendulum science, philosophy and social forces. The International Pendulum Project demonstrates how the pendulum can support an extended and integrated pedagogical journey, where the interplay between mathematics, technology, physics, philosophy and experiment can be explored and appreciated (Matthews, Gauld & Stinner, 2005). Furthermore, it illustrates the dependence of science upon mathematics as well as the importance of mathematics in doing science. And one could add, along with Osborne (2002), that in science education it is often accentuated that many phenomena and their patterns of interaction are best described in the language of mathematics, which then becomes a bridge between the students' verbal language and the scientific meaning we seek to express.

An example of a didactic framework for interdisciplinary teaching

The Realistic Mathematics Education (RME) approach, derived from the works of Treffers (1987) and Freudenthal (e.g. 1991), is based on the conception that mathematizing is the key process of mathematical activity. There are two types of mathematizing in an educational context: horizontal and vertical mathematizing. In *horizontal mathematizing*, the students come up with mathematical tools, which can help to organize and solve a problem located in a real-life situation. V*ertical mathematizating* is the process of reorganization within the mathematical system itself, e.g. finding shortcuts and discovering connections between concepts and strategies, and the application of these discoveries. Doorman and Gravemeijer (2009) elaborate on this and introduce the notion of *emergent modeling*. The notion has as the departing point situation specific problems, which are subsequently modeled. The problems first offer the students the opportunity to develop situation-specific methods and symbolizations. Then the methods and symbols are modeled from a mathematical perspective and in this sense the models emerge from the students' activity. The models first come into being as a model of the situation, and then the model gradually becomes an entity in its own right and begins to serve as a model for mathematical reasoning. The shift presented from a model *of* to a model *for* should concur with a shift in the way students perceive and think about the models; from models that derive their meaning from the context situation modeled to thinking about the mathematical content.

Inspired by the RME approach, Michelsen (2005) suggests a didactical framework for coordination and mutual interaction between mathematics and science based on the idea of modeling as a didactical tool for interdisciplinary learning activities in mathematics and science education. The framework consists of two phases: *horizontal linkage* and *vertical structuring* (see figure 1). In the horizontal phase thematic integration is used to connect concept and process skills of mathematics and science by modeling activities in an interdisciplinary context. The vertical phase is characterized by a conceptual anchoring of the concepts and process skills from the horizontal phase by creating languages and symbol systems that allow the students to move about logically and analytically within mathematics and science without reference back into the contextual phase. The shift from the horizontal to the vertical phase might thus concur with a shift from integrated instruction to subject-oriented instruction. It should be stressed that the framework is iterative. Once the concepts and skills are conceptually anchored in the respective subjects, they can evolve in a new interdisciplinary context, as part of a horizontal linkage.

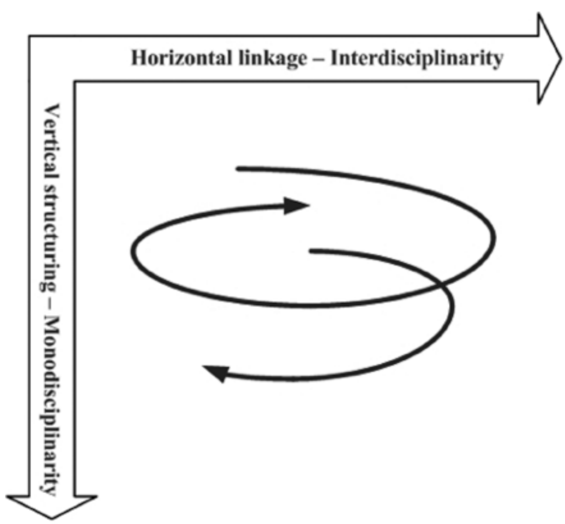

Figure 1. *The spiral shape illustrates the repetitive movements between the horizontal and vertical phases (based on Michelsen & Iversen, 2009, p. 27)*

The Nat802 course: history and present course curriculum

The development of the course was initiated in 2005, and the first curriculum aimed at equipping the students with didactical tools to replace the classical mono-disciplinary approach in mathematics and science with an interdisciplinary one, where mathematics, biology, chemistry and physics are woven continuously together. In its initial conceptualization, the course title *Modeling and interdisciplinarity* explicitly highlights the main idea in the course: modeling as a didactical tool for interdisciplinarity in mathematics and science education. The initial curriculum was centered round the above mentioned didactical framework of Michelsen (2005).

The course was offered for the first time in the academic year 2008/2009 for students enrolled at master programs in mathematics, biology, chemistry and physics. The students were introduced to the didactical framework, and as the core activity of the course they were expected to develop prototypes of interdisciplinary instructional units centered on modeling activities and the inclusion of at least two subjects.

In the academic year 2009/2010 the course was designed upon three pillars: (i) different conceptual expositions of interdisciplinarity, (ii) different conceptualizations of the notion of modeling within several disciplines, and (iii) a focus on the theoretical underpinning of requirements of interdisciplinary teaching in Danish upper secondary school. A range of scholarly articles concerning each pillar was collected and those that were deemed appropriate in terms of the level of the students were chosen to be on the syllabus.

The core of the course literature concerning interdisciplinarity consisted of the proposed taxonomies of different forms of interdisciplinarity from Jantsch (1972), Ulrichsen (2001) and Beckman (2008). For example, Jantsch's (1972) taxonomy involves a spectrum ranging from (1) multidisciplinarity to (5) transdisciplinarity. The former resembles the complete separation of disciplines that we already find in the educational system; the latter resembles interdisciplinarity to such a degree that the individual disciplines become invisible, e.g. when biology and chemistry become biochemistry, and so on. The intermediate levels of interdisciplinarity, which are the ones most often occurring in teaching, are (2) pluridisciplinarity, (3) crossdisciplinarity, and (4) interdisciplinarity proper (Jantsch, 1972), as illustrated in figure 2.

Similar taxonomies have been proposed by Klein (2010), Beckmann (2009), and Ulrichsen (2001). Although these taxonomies present very different lenses on the landscape of interdisciplinarity, the shared feature of these taxonomies is the idea of a spectrum of forms of interdisciplinarity between collaborations, where disciplines "complement" each other and

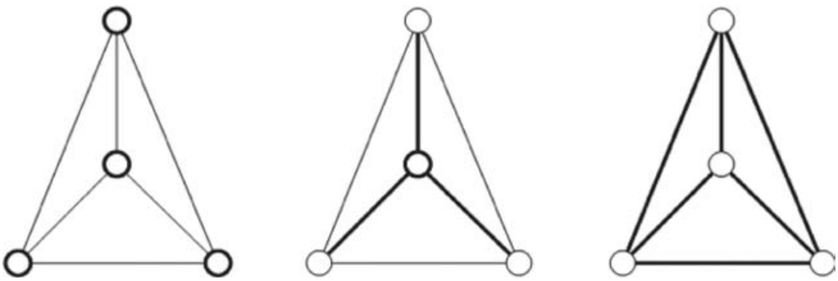

Figure 2. *From left to right: Jantsch' pluridisciplinarity, crossdisciplinarity, and interdisciplinarity proper illustrated as the emphasis (by dark coloring) on subjects (circles) and connections between subjects (lines) (Jensen, 2010)*

collaborations where disciplines are "hybridized" (e.g. Klein, 2010). Such taxonomies are important for assessing the interdisciplinary work done in education.

Introducing the students to multifarious conceptualizations of interdisciplinarity is an invitation to *negotiate* – rather than being told – the meaning of the concept of interdisciplinarity. The key aim was that students were able to identify and navigate different forms of interdisciplinary teaching from different theoretical vantage points. The core of the course literature on the concept of modeling at that time had to be somewhat altered from the initial focus on the distinction between horizontal and vertical processes. This was due to the fact that more new students had a background also in the humanities and the social sciences. A number of different conceptualizations of how disciplines other than mathematics model their object of study had to be included in the syllabus. This broad presentation of the different competing expositions of what it means to represent or model reality also led to a learning situation in which students were invited to negotiate the meaning of "modeling".

In the academic year 2010/2011, a fourth pillar was added: (iv) a conceptual background of the notion of competency. The argument for this, from a design perspective, was the fact that competency based teaching permeates the Danish educational system. Further there are fundamental potentials in terms of an overlap between interdisciplinarity and the notion of competency as "a well-informed readiness to act appropriately in situations" (Niss & Højgaard, 2011, p. 49) in a way that is guided by one's knowledge from a discipline. Dahland (1998) uses the term "coincident didactic conceptions" to express that among the didactics of various subjects, one can trace a number of analogous notions that make up a didactic intersection. For example, the didactics of mathematics, biology and history all include subject-specific elements, but in addition didactic notions can belong to more than just one subject, i.e. one may talk about intersections of didactic notions. The actual content of such intersections ultimately depends on the perspective adopted. If one adopts the perspective of competencies, it can be argued, for example, that some competencies – such as the competency to represent a phenomenon or process in different ways, or the competency to reason – are *interdisciplinary competencies* in the sense that they appear as key constituents for multiple disciplines (Iversen, 2006; Michelsen, 2005). The educational potential of this insight is that interdisciplinary teaching activities can be designed on the basis of students having to execute interdisciplinary competencies. In other words, that the interdisciplinary competencies are the putty that holds together the interacting disciplines. In this regard, the Nat802 students relied on a selection of Danish literature on competency development in relation to mathematics (Niss & Højgaard, 2011) and science, in particular physics (Dolin, Krogh & Troelsen, 2003). The fundamental aim of the course was that the students in groups were able (a point that was emphasized more fully in 2010/2011) to design interdisciplinary teaching activities themselves, activities readymade for potential implementations once they become in-service teachers.

Student projects involving interdisciplinary teaching activities

For the implementation of the course in the academic year of 2010/2011 the students were to do two group projects: one pilot project and one final exam project.

In the pilot project the students were, among other things, given the task to design an interdisciplinary teaching activity and describe (a) how the upper secondary pupils should be introduced to the interdisciplinary *problématique* of the activity; (b) how the students themselves envisaged that they would work with it during an implementation of the activity; and (c) discuss how the pupils' meta-perspective discussion in relation to aspects of general education could be *anchored* in their monodisciplinary content knowledge (Jankvist, 2011b)[1].

For the final exam project the students were to work with modeling as a didactical tool to achieve interdisciplinarity. More precisely they were asked to clearly (a) describe the form and extent of the interdisciplinarity and the place and role of modeling and (b) *argue* for their design choices on the basis of the course literature on interdisciplinarity, modeling, and competencies. Also, in this project, the students were asked to structure their reports around an actual problem formulation or research question (to be illustrated later).

At the final exam, the students' projects were read and marked by both the course teacher and an external examiner, who was either a practicing upper secondary school teacher or a mathematics education researcher with strong knowledge of teaching practice in upper secondary school. The students then had to "defend" and discuss their project in an oral examination with both course teacher and examiner present, upon which a final grade was given. Besides being based on the students' presentations of their projects and the following discussion, the final grade also took into consideration the students' abilities, as displayed in the project and during the exam, to: provide pedagogical and didactical arguments for mono-disciplinary and multi-disciplinary teaching, respectively; describe and evaluate the role of models and modeling processes in mathematics and science teaching; identify and describe different types of models, and discuss their applicability as didactical tools for interdisciplinarity in mathematics and science teaching; describe different modeling tools (e.g. software, etc.) and evaluate their applicability as resources for learning in terms of pupils' modeling activities.

In the following, we give first an example of a student pilot project. Next we display a selection of the students' problem formulations/ research questions for the exam project. And finally, we display and discuss in depth a sample student exam project.

A student project between mathematics, philosophy and history

The students state that the overall purpose of this teaching activity is to provide the upper secondary level pupils with an historical insight into the interplay between a series of philosophical considerations about the concept of infinity and the more concrete mathematical handling of the concept. The students give the following description of their overall idea for doing this (translated from Danish):

> The activity begins with Aristotle's discussion of infinity and Zeno's paradoxes as well as their forestalling of the concept of infinitesimals. This is carried on to the historical development of the calculus of infinitesimals focusing on the works of Archimedes, Newton, and Leibniz. Leibniz' use of infinity from a philosophical perspective is also brought in here. The modern conception of infinity is illuminated within mathematics through Cantor's transfinite-ism and within philosophy through "Super task theory", which again takes us back to the paradoxes of Zeno.
>
> (Thomsen, Hansen, Christensen & Svendsen, 2010, p. 8)[2]

The idea of the students is to post somewhat open problems or questions (Jensen, 2009) in the activity, thus focusing more on matematizing than on proficiency (Freudenthal, 1991) and more on the (historical) process than on the (scientific) results (Lesh & Doerr, 2003). Aristotle's discussion of infinity provides the pupils with the philosophical introduction to the problem, whereas Archimedes' work on the approximation of areas introduces them to the concept of infinity, and eventually the notion of limits, from an historical mathematical point of view. The paradoxes of Zeno, however, are thought to act as the interdisciplinary "glue" binding the philosophical and mathematical discourses together. The mathematical walk-through from Archimedes through Newton and Leibniz to Cantor as well as the simultaneous philosophical discussions of Aristotle, Leibniz' *Monadology*, and finally the modern notion of super-tasks[3] provide the pupils with the historical insights regarding development and legitimacy. As for the possibility of anchoring the interdisciplinary meta-perspective discussions in the mono-disciplinary content knowledge, the students provide the following reflections (translated from Danish):

> In relation to philosophy the activity puts focus on the significance of mathematics for history of philosophy, not only by showing that several great philosophers were practicing mathematicians who had mathematics as an ideal of realization and source of inspiration, but also by applying philosophical texts where mathematical thinking is used in the actual argumentation. [...] Besides the fact that philosophical formation evidently becomes both wider and sharper by including knowledge from mathematics and science, the activity also shows that mathematical overview and understanding actually is a precondition for making sense of a series of philosophical insights. [...] The pupils should get the idea that you can use the subject knowledge of philosophy to unveil the general education aspects of mathematics, but also that you can use subject knowledge of mathematics to strengthen and expand the general education aspects of philosophy. [...] In the actual encounter – the concept of infinity – pupils should realize that not only is this concept something to consider within philosophy, it is actually something that you operate with in mathematics, in a formal and well-defined manner. And that the development of this construct is a result of an historical interplay between philosophy and mathematics, where it has been practically impossible to separate the two.
>
> (Thomsen et al., 2010, p. 10)

Examples of problem formulations and research questions

As mentioned, for the final exam project the students were asked to structure their projects around an actual problem formulation/research question. Of course, their projects were not to be regarded as actual research papers, but still the hand-in written reports for their final exams were to be structured in such a way – in reality not all of them included actual research questions, some resembled more problem statements. In this section, we provide a selection of quotations from the students' written exam reports, in order to, on the one hand, give an idea of the kind of projects and prototypes for teaching activities that students devised, and, on the other hand, to illustrate the type of problems regarding interdisciplinary teaching that the students themselves came to realize as being imperative for the success of designing and implementing interdisciplinary teaching activities at upper secondary level. We provide five examples, all translated from Danish into English:

> The present exam project concerns an example of an interdisciplinary modeling activity in biology, chemistry, and mathematics, which may contribute to the breaking down of the sharp division between subjects and besides developing pupils' general subject knowledge also contribute to pupils' competencies being more in focus as well as fulfilling the more meta-perspective goals of general education, so that pupils come to view the particular nature of the different subjects, their particular methods, and the relationships between the subjects. The interdisciplinary modeling activity is built around the open question [Jensen (2009)]: "What constitutes a dangerous disease?" The challenge faced here is to design an activity that both reaches the above goal and at the same time bring in the three subjects on an equal footing in an interdisciplinary interplay [in Jantsch' (1972) and Ulrichsen's (2001) taxonomy].
>
> (Grube & Rasmussen, 2010, p. 2)

> How may an interdisciplinary [in Jantsch'(1972) and Ulrichsen's (2001) taxonomy] modeling activity between the disciplines of mathematics and business economics be structured for upper secondary school so that it will strengthen the mathematical competencies of modeling, communication, and representation [in Niss & Højgaard's (2011) classification[4]]? We intend to investigate this by designing an activity centered on the open question [Jensen (2009)]: "You have some funds to invest, how do you choose the best business to invest in?"
>
> (Wendelboe & Thomsen, 2010, p. i)

> How is it possible to design a well-functioning modeling activity in the subjects of mathematics and history that reaches an interdisciplinary level [in Jantsch' (1972) and Ulrichsen's (2001) taxonomy]? How may we structure such a teaching activity so that both history and mathematics play a part in the modeling activity? [...] We have chosen to design an activity on the development of calculus during the Age of Enlightenment, where focus will be on the scientific development, in particular that of Newton. [...] The history part will concern form of government and social groupings in France and England and the development of natural science in that period, the mathematics part will focus on the mathematics of Newton.
>
> (Imanilasaki & Jørgensen, 2010, p. 1)

> With this project we wish to study the degree to which one may construct a *dynamic* interdisciplinary teaching activity between the subjects of physics, philosophy, and mathematics. By dynamic we generally think of an open activity, where the relations between the subjects involved may change in character during the implementation. The question is if it is possible to identify a problem and a structure, where it is the actual relations and reciprocal actions between the subjects that drive the activity forward, so that focus is on the actual process, where techniques and subject knowledge may be taken from the different subjects according to need, instead of being isolated methods used to shed light on already delimited subject areas. The activity we have come up with is called "The growth of mankind", and the basic idea is to investigate the single question: "Is there a limit to the number of people that may exist?"
>
> (Christensen & Bjerre, 2010, p. 2)

> How may we structure a three-subject interdisciplinary teaching activity [between mathematics, physics, and sports] that make sense according to the development of subject-boundary crossing competencies [in the terminology of Michelsen (2005)], and at the same time is motivating for the pupils?
>
> (Svendsen, Pedersen & Overgaard, 2010, p. 4)

Of course, not all student projects equally well achieved what they set out to. But compared to the development of the students' acquired knowledge about and understanding of the challenges of designing – and themselves later having to implement and assess – interdisciplinary teaching activities for upper secondary school pupils, this becomes somewhat secondary. In the following section, we illustrate what a student exam project may look like. More precisely, we use the one based on the question just above, between the three subjects of mathematics, physics, and sports (in this case, swimming).

A student project between mathematics, physics and sports

The three students of this exam report state that within sports they have chosen the discipline of swimming, more precisely to investigate which style of swimming is the "best" (crawl, breaststroke, or butterfly) and continue "this involves a fair amount of physics in the form of friction etc., which in return provides the opportunity to build mathematical models" (Svendsen et al., 2010, p. 5). The students then move on to give an account of what they will consider to count as mathematical modeling in their teaching activity. This mainly include: Lesh & Doerr's (2003) model eliciting activities as well as making other relevant references to the course literature, e.g. Freudenthal's (1991) and Treffers' (1987) description of horizontal and vertical mathematizing; Michelsen's (2005) horizontal linking and vertical structuring, and of course the literature on competencies, i.e. Niss and Højgaard's (2011) mathematical competencies and the science and physics competencies described in Dolin et al. (2003). In regard to the latter, the students further specify the competencies which they plan to focus on in the design of their teaching activity. For mathematics they choose the *representation competency*, the *tools and aids competency*, and the *modeling competency* (Niss & Højgaard, 2011), since they consider these to be highly relevant in modeling between mathematics and physics, and also since the data treatment in their case would involve computer calculations. For physics they choose the *experimental competency* (Dolin et al., 2003), not least due to the swimming experiments. For sports they are not able to find literature defining a set of competencies to draw from, so they choose to define themselves and operate with a *bodily competency*, referring to pupils having to develop knowledge about the body as well as bodily skills in sports.

The entire design of the teaching module is built around Beckmann's taxonomy of interdisciplinary activities (Beckmann, 2008) – referring also to elements of Jantsch (1972) and Ulrichsen (2001) along the way. More precisely, the students design their teaching unit so that it is to run over 6 double lessons (2 x 45 minutes each), and they classify the activities in these lessons according to Beckmann's (2009, pp. 6–12) four forms of cooperation in interdisciplinary and cross-curricular teaching: (1) topic- and major subject-related form; (2) parallel topic-related form; (3) parallel planning form; and (4) joint planning form. To obtain an idea of the actual activities in the designed module, the subject topics involved, and the lessons form of cooperation in Beckmann's taxonomy. Please, refer to the students' own illustration of this in figure 3.

	Sports	Physics	Mathematics
1	Intro to the module Crawl	Upward force	Modeling and modeling competency
2	Breaststroke	Friction	Modeling
3	Pull through water Beginning of project Video taping of swimming styles Data collection	Modeling experiment	Differential equations
		Project	
4		Modeling and data treatment	
5		Analysis of results Preparation of presentation	
6		Presentation Handing in report	

Figure 3. *Students' illustration of their interdisciplinary teaching module between sports, physics, and mathematics (Svendsen et al., 2010, p. 13, translated from Danish)*

Note. The numbers in the left column refer to the double lesson in question. The three shadings in the figure indicate the form of interdisciplinarity in Beckmann's taxonomy: form 1 and 2 white; form 3 light grey; and form 4 dark grey.

Some of the insights which the students gained from having to design a future interdisciplinary teaching module may be seen from the final sections in their report, where they reflect on possible implementations of their teaching activity. One of the things which the students discuss is the cooperation between teachers of the activity's three subjects. They state (translated from Danish):

> As the level of interdisciplinarity is being raised [during the duration of the activity], we anticipate that so will the demand for organization between the [three] subjects. Neither Ulrichsen (2001) nor Beckmann (2009) discuss this point, but it seems logical to us that the more the subjects are to cooperate the more the teachers will have to coordinate.
>
> (Svendsen et al., 2010, p. 20)

The students also discuss problems with assessing the pupils' work in such interdisciplinary activities, also in relation to the intended level of interdisciplinarity[5], and they continue in relation to competency based teaching (translated from Danish):

> [...] teachers often have difficulties seeing what each others' subjects are good for. Competency based teaching may make this easier, but requires that we as teachers are aware what competencies the other subjects contain and how they are defined. This is difficult since we often come to see our own subject as being the most important and since it requires substantial overview to relate to other subjects. Furthermore, it isn't even sure that there are well-defined competency descriptions available for all subjects at upper secondary level, as for example was the case with sports. But it must be emphasized that we as upper secondary teachers must try to change our conceptions and ideas about how the teaching must be conducted. Offhand this must be considered a difficult barrier to overcome for teachers, who have taught in upper secondary school for a long time, if it was not imposed on them to conduct competency based teaching before the reform of 2005.
>
> (Svendsen et al., 2010, pp. 22–23)

Discussion and concluding remarks

With this paper we have tried to illustrate how future upper secondary school teachers can be better prepared for the challenges of having to deal with an increasing element of, and demand for, interdisciplinarity in their teaching. The problem of their preparation is twofold: On the one hand, future teachers have to abide to the upper secondary school programs and their demands for interdisciplinary teaching activities; on the other hand, they have to do this in cooperation with established teachers who may not be so willing to make drastic changes in teaching styles and activities, etc. The solution to both these problems, as we have envisioned it, is to a large degree the same. Namely, to familiarize future teachers with a solid basis of literature, including research literature, on interdisciplinary teaching and related topics (cf. the aforementioned four pillars), which not only enable them to design their own interdisciplinary teaching activities and to do so on a theoretical foothold, but also to equip them to discuss the benefits of taking such a theoretical approach with future (possibly more experienced and established) colleagues.

The sample exam project on mathematics, physics, and sports shows how the students are able to design such activities applying the theoretical constructs from the course literature, e.g. the notion of model eliciting activities from Lesh and Doerr (2003), horizontal and vertical mathematizing from Freudenthal (1991) and Treffers (1987), competency development from Niss & Højgaard (2011), and also to structure the activities based on the taxonomies of interdisciplinary teaching, e.g. Beckmann (2008) and Ulrichsen (2001). Also, it illustrates the previously argued (central) role that modeling can play in the design of interdisciplinary teaching activities.

Besides illustrating how to possibly design an interdisciplinary activity between subjects from different faculties, the pilot project on mathematics, philosophy, and history also illustrate a different aspect – namely, that of trying to anchor the treatment of the different subjects in each other into the design of the activity (Jankvist, 2011b). Not only does this allow for the possibility of the students anchoring the interdisciplinary meta-perspective discussions in the mono-disciplinary content knowledge of mathematics and philosophy (Jankvist, 2011a), it also illustrates the initiation of a setting for the students to invoke mathematics and science content knowledge when discussing historical, cultural, and societal issues beyond the mono-disciplinary boundaries, which is otherwise a general problem in science education, as pointed out by Nielsen (2011, in press).

The excerpts of problem formulations from the students' exam reports add further support to the claim of the students' developing an awareness of the different levels of interdisciplinarity and the different ways of bringing these in play, for example by focusing on a few selected competencies within the chosen disciplines and structuring the design of the teaching activities around the development of these. One competency often in play in the problem formulations is the mathematical modeling competency, which may be considered more or less natural due to the course literature's focus on modeling as a didactical tool for interdisciplinarity between mathematics and the natural sciences. Another aspect from the course curriculum, which some students tried to bring in, was the notion of Michelsen (2005) that the educational research discussion about competency based teaching (as introduced to the students through the texts of Niss and Højgaard, 2011 and Dolin et al., 2003) can be synthesized in the sense that there are overlapping or even straightforwardly *interdisciplinary competencies*. One such competency, as envisaged by Michelsen (2005), is the representational competency which is stipulated as aim of both mathematics and science education. Further, as Iversen (2006) has argued, the reasoning competency seems to cross the boundaries between almost all disciplines in the (Danish) school system. From both a teacher preparation course design perspective as well as one of purely educational research, such observations lend credence to the idea that competency development could be seen as a lynchpin for interdisciplinary activities, not just across mathematics and natural sciences, but also across disciplines from all faculties.

Furthermore, the students experience some of the problems related to designing interdisciplinary activities, for example when it comes to trying to achieve a high degree of interdisciplinarity. And, as illustrated by the final quotations from the sample exam report, they may become aware of some of the potential problems with actually having to implement such interdisciplinary modules or activities – problems some of which are quite general for the realization of interdisciplinary teaching, e.g. the cooperation between teachers, the assessment of such activities, etc. This level of reflection, along with the students' willingness to pick up the course's invitation to discuss and negotiate the concept of interdisciplinarity and its realization, seem promising factors in the preparation of future teachers who are to conduct interdisciplinary teaching. Also, the fact that both the course teacher and second readers (external examiners) read, discussed and marked the final exam projects acted as a practice-oriented control mechanism for ensuring a high degree of feasibility of the students' work.

As mentioned in the abstract, the results of the Nat802 course as presented in this paper are of course somewhat preliminary. By this we are referring both to the fact that the course has a very short history and to the fact that if we are to truly see the outcome of the course, then we will have to seek out the course participants once they have begun teaching at upper secondary level and see to what extent they are applying their knowledge from Nat802, when realizing the interdisciplinary requirements of the curricula. Still, from the looks of it, the preliminary results of the implementation of Nat802 for future upper secondary school teachers are promising.

Of course, in order for future interdisciplinary teaching to be sustainably integrated in teachers' practice, more systemic changes need to occur on the level of the in-service teaching communities and in terms of the continued professional development that occurs within the culture of teachers at a school. It is our hope that adequate pre-service courses such as Nat802 can be the basis for more productive teaching cultures vis à vis interdisciplinary teaching.

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Notes

1 Regarding anchoring, see also Jankvist (2011a) and Jankvist & Kjeldsen (2011).

2 Due to the complexity of some of the mathematical and philosophical constructs in play here, the students state that their activity is directed towards the upper secondary advanced mathematics level, which in Denmark

means that the pupils are taught mathematics throughout all three years of upper secondary school.

3 The references the students give here are James F. Thomson and Paul Benacerraf.

4 The students read the Danish 2002 version of Niss and Højgaard (2011).

5 For a discussion of how to possibly assess the level of interdisciplinarity, see (Jankvist, 2011b)

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