Feeling of innovation in expert problem posing

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This paper is one of the reports on a multiple-case study concerned with the intertwining between affect and cognition in the mechanisms governing experts when posing new mathematical problems. Based on inductive analysis of a single case of an expert poser for mathematics competitions, we suggest that the desire to experience the feeling of innovation may be one of such mechanisms. In the case of interest, the feeling was realized through expert's reflections on the problems he created in the past, by systematically emphasizing how a new problem was innovative in comparison with other familiar problems based on the same nesting idea. The findings are discussed in light of past research on expert problem posers and expert problem solvers.

Mathematics education research has accumulated an extended body of knowledge on problem posing by school children and mathematics teachers who, as a rule, are novices in problem posing (Kontorovich, Koichu, Leikin & Berman, 2012). On the other hand, empirical evidence on problem posing conducted by experts, i.e. mathematicians and mathematics educators, who create new problems for various mathematical and educational needs as an integrative part of their professional practice, barely exists. This is in spite of the established practice of using research on experts as a source of ideas for fostering mathematical competences in novices. For instance, the mathematics education community has benefited from studies on how experts in mathematics *solve* problems (e.g. Carlson & Bloom, 2005), *learn* mathematics (e.g. Wilkerson-Jerde & Wilensky, 2011) and *discover* new mathematical facts (e.g. Liljedahl, 2009). By analogy, research on how experts *pose* problems may also be profitable and lead to new ideas about how to improve problem-posing competences in school children and mathematics teachers.

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In the framework of a larger on-going research (Kontorovich, in prep.) we study a community of expert problem posers for mathematics competitions for secondary school children. This community is particularly interesting because mathematics competitions are widely recognized as a valuable source of elegant and surprising problems for the use not only in out-of-school educational settings, but also in a regular mathematics classroom (e.g. Grugnetti & Jaquet, 2005). Many competition problems have served as powerful means of engaging school children in challenging mathematics, fostering their mathematical thinking and creativity (e.g. Koichu & Andžāns, 2009; Thraser, 2008). In addition, it is just intriguing to understand how the experts succeed to come up with new and surprising problems after that so many mathematical gems have been created for competitions during the last century.

We decided to look at expert problem posers' ways of thinking and practice through the lenses that intertwine cognitive and affective domains. The decision is set in-line with the recent stream of research in different fields of mathematics education. Generally speaking, the research agenda that stimulated us is that each domain has its own limitations, but considering both of them together may help in exposing a "bigger picture" (see Furinghetti & Morselli, 2009 for an elaborated substantiation of this claim).

In the current paper we continue analyzing a case study of one expert poser, Leo (pseudonym). The case of Leo was chosen because it was particularly informative with respect to mathematical, cognitive and affective aspects of creating problems. Our previous work based on the case of Leo (Kontorovich & Koichu, under review) was focused on a cognitive aspect of his problem posing, and, specifically, on the emergence of the notion "nesting idea" as an organizational unit of expert knowledge base for posing problems. In the current work, we use the case of Leo for another goal: to formulate evidence-based suggestions on how affect may be involved when the expert operates upon his knowledge base when posing new mathematical problems[1].

Two open issues in expert problem posing

Previous research has systematically pointed out the intertwining nature of problem posing and problem solving (e.g. Kontorovich et al., 2012; Pelczer & Gamboa, 2009; Silver et al., 1996). In light of this, theoretical background of the paper is a combination of selected insights from both fields.

Experts' problem posing and affect

Publications, in which expert problem posers open the doors of their kitchens, are rare. We found only three self-reflective publications of this kind: by Konstantinov (1997), Sharigin (1991) and Walter (1987). Overall, the papers inform the readers about sources of new problems, problem-posing techniques and the authors' quality criteria for the posed problems. On one hand, Walter (1978) illustrates a claim that a problem can be created "almost from anything", i.e. almost from any situation including drawings or numerical information. On the other hand, Sharigin (1991) points out that new (competition) problems usually come from other problems the poser is familiar with.

The aforementioned self-reflective writings create an impression that problem posing is a very affectively loaded experience for the experts. This impression gets even stronger when we look at the ways Sharigin (1991) and Konstantinov (1997) describe good competition problems, which can be considered as high-quality products of problem posing. They use such descriptors as "graceful", "attractive", "surprising", "sophisticated", "natural", "beautiful", "impressive", "rich", "mathematically valuable", "interesting" and "original" (translated from Russian). However, the self-reflective writings of the masters do not fully enable the readers to understand the meaning beyond the descriptors. In Konstantinov's (1997) words: "It is impossible to formulate what is a "good problem". But when the problem is posed it claims for itself (or against itself)" (p. 168, translated from Russian). Indeed, Konstantinov's (1997) view is emotionally loaded and high-quality product oriented. In turn, it leaves room for inquiring how affect is involved in the processes the experts are going through when posing high-quality problems.

Experts' problem posing and organization of their knowledge base

Expert mathematical knowledge base is more than just storage of pieces of information, like definitions, facts and routine procedures. It also includes the ways this information is represented, stored, organized and accessed (e.g. Schoenfeld, 1992). Another important part of mathematical knowledge base for problem solving, and, apparently, for problem posing, is a set of rules and norms that exist in the particular domain about legitimate and prototypical connections between different pieces of mathematical information (Schoenfeld, 1992). This kind of knowledge is constructed through continuous exposure to various mathematical problems, elaboration on a part of them and storage of problems in the knowledge base. To this particular type of mathematical knowledge we refer to as *personal pool of familiar problems*.

A personal pool of familiar problems of an expert problem poser is immense. According to Miller (1956), when experts operate with a big amount of information, their first thing to do is to break it down into meaningful *chunks*, which make the information more accessible. Then experts imply their extended arsenal of *schemas* to the chunked information. Schemas are referred to as organized structures of mental actions for associating new information with already existing one (e.g. Schoenfeld, 1992). They are used for making a personal sense of information, coding and storing it in the long-term memory as well as for recalling and decoding it back (e.g. Chi, Feltovich & Glaser, 1981).

How can *chunking* and *schemes* be used to characterize an expert's pool of familiar problems for mathematical problem solving and problem posing? Namely, what kinds of familiar problems are grouped in the same chunk? Empirical studies on problem solving showed that experts group problems together in a good agreement with a *deep* versus *surface structure* theory (e.g. Chi et al., 1981). Generally speaking, experts tend to identify problems as being similar because of the fundamental principles and strategies that lead to their solutions (deep structure), and not according to their surface structure, such as similar scientific fields or topics, usage of the same mathematical terms etc.

Let us note that in the study mentioned in the previous paragraph, the participants were experts in problem solving. In this case, associating an unfamiliar problem with familiar ones using the schema of "looking for deep-structure similarities between the problems" has been summoned. In our case, Leo's expertise is in posing challenging problems for the solution by others (i.e. students attending mathematics competitions). Therefore, the question of which schemas he is using and how he takes advantage of his immense pool of familiar problems is worth asking.

Method

Our approach has been to interview people who are professionally engaged in problem posing activities. In this report, we are analyzing how one of our interviewees, Leo, perceives his activities in problem posing.

Leo is a coach of the Israeli team for International Mathematical Olympiad (IMO) for high school students and a practicing problem poser. His problems have appeared in high-level competitions such as the Tournament of the Towns, IMO for university students and national-level Olympiads in Israel. The data on Leo's problem posing was collected in the framework of two interviews, a master class for a group of prospective mathematics teachers, a meeting, in which Leo and his colleagues constructed a questionnaire for one of the preparatory stages for the Israeli national-level Olympiad and a meeting during which Leo gave feedback on our analysis of his problem-posing practices. All the meetings with Leo were video- or audiotaped, so, overall, the case of Leo is based on more than 10 hours of recorded data. Prior to the interview, Leo also sent us a list of seventeen of his problems.

We present below several fragments of data gathered in the framework of the reflective interview (we plan to present more data elsewhere, Kontorovich & Koichu, under review). The interview was organized as a conversation around selected problems created by Leo in the past and took about 125 minutes. The problems to be discussed at the interview were sent to us by Leo in advance, which enabled us to prepare well-focused questions about each problem.

The data were analyzed using an inductive approach in order "[...] to allow research findings to emerge from the frequent, dominant or significant themes inherent in raw data, without the restraints imposed by structured methodologies" (Thomas, 2006, p. 238). To make the inductive analysis more transparent we chose to present the findings based on the way in which the categories emerged from the data.

Findings

We opened the interview with a general question about Leo's typical problem-posing behaviors. His response was:

> When I pose problem, I try not to repeat myself [i.e. a problem that he created in the past], even when it is hard. [...] I could get on the internet, chose some problems, vary their formulations a little bit and an Olympiad [questionnaire] is done in less than half an hour; but it would make no one any good.
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> [...] When I'm supposed to create a challenging problem I should come up with some surprising and unfamiliar idea. Where can I take it? I could start recalling beautiful ideas that I saw recently. However, it's very difficult to describe how it [problem posing] happens.

Two phenomena, that seem conflicting at first glance, can be observed in Leo's statement: First, Leo's problem posing is driven by a *desire to innovate*, i.e. he is incentive to pose a problem which would be different from the ones existing in his pool of familiar problems. A desire to innovate is acknowledged as being common, natural and primary motivating factor among humans (e.g. Doboli et al., 2010; Knight, 1967). Second, Leo is capable of using his pool of familiar problems and previously seen ideas for innovating.

In order to clarify the apparent conflict between the phenomena we turned to the list of problems that he sent us prior to the interview. The problems belonged to the fields of Euclidean, analytical and spatial geometries, algebra, graph theory, logic and combinatorics. Two problems, which have appeared in Israeli national-level competition for 8^{th} and 9^{th} graders, drew our particular attention because of their apparent similarity: they shared the same question and could be solved by using the idea of (algebraic) conjugate numbers.

Problem 1: Simplify

$$\frac{\sqrt{\sqrt{2}-1}}{\sqrt{\sqrt{2}+1}}+\frac{\sqrt{\sqrt{3}-\sqrt{2}}}{\sqrt{\sqrt{3}+\sqrt{2}}}+\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}$$

Problem 2: Simplify

$$\frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}+\frac{1}{\sqrt[3]{9}+\sqrt[3]{12}+\sqrt[3]{16}}+\frac{1}{\sqrt[3]{16}+\sqrt[3]{20}+\sqrt[3]{25}}$$

When Leo was asked to reflect on these problems, he chose to reflect on the second one and said:

> I needed an algebraic problem for a competition. What can be done in algebra so it would be elementary, but still unexpected? I like [algebraic] conjugate numbers since they are unexpected enough. [...] Especially when one number is a predecessor of the other, since then the numerator of 1 is masked [i.e. $\sqrt{n+1}-\sqrt{n}=\frac{1}{\sqrt{n+1}+\sqrt{n}}$].
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> [...] Ok, [I wanted to use] conjugate numbers! But quadratic conjugate numbers is hackneyed, boring and everybody knows them. So let's take a step forward: cubic conjugate numbers. This thought gave birth to the second problem. [...] There are not so many tricks like this in algebra.

Leo's reflection shows that for the creation of the second problem he turned to the idea of "conjugate numbers". It seems like Leo used the idea of "conjugate numbers" as a code name referring to a whole class of problems. The class contains problems that involve expressions with roots, which can be simplified using the properties of algebraic conjugate numbers. The existence of Problem 1 implied that Leo had already successfully turned to this class of problems. Thus, it is reasonable to assume that this prior positive experience awarded special status to the idea of "conjugate numbers" for Leo. Then Leo scanned the whole class, noticed that it embraces problems which only involve quadratic roots and introduced cubic conjugate numbers in Problem 2. Overall, it can be said that Leo *manipulated* the special idea for making an innovation.

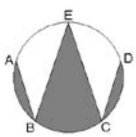
The exposed characteristics of an idea of "conjugate numbers" stimulated us to resort to a metaphor of a nest that encloses familiar problems (i.e. "egges") and serves as a useful framework for "laying" new ones. Thus, we refer to this kind of ideas as *nesting ideas*. Note that, as any metaphor, this one has its limitations. For instance, although it cannot be seen from the presented data, Leo's nests of ideas embrace problems created by Leo as well as problems created by others [2].

Leo has stopped the particular problem posing session after introducing cubic conjugate numbers. Thus it can be suggested that the manipulation ended up with a problem which was innovative enough in his eyes. Moreover, Leo remembered so well the story of creation of Problem 2, which had appeared at the competition two years ago. In this way, the creation of Problem 2 can be recognized as a significant experience for him, which left traces in Leo's memory for a long time after it actually occurred. This kind of experiences are accompanied by a highly-emotional impact and, in particular, by strong feelings (e.g. Hochschild, 1983).

From the literature on innovations (and from our everyday experiences) it is known that the fulfillment of the desire to innovate creates a pleasant feeling related to the positive self-perceptional "package" including pride, success, self-efficacy, development, improvement and significance (e.g. Doboli et al., 2010; Knight, 1967). In the problem-posing context we refer to this feeling as *a feeling of innovation*; a feeling which appears after a poser created a problem which is different enough from the problems s/he is familiar with.

Leo's last sentence in the reflection on the creation of Problem 2 ("There are not so many tricks like this in algebra") led us to two interrelated hypotheses: (1) the entire pool of Leo's familiar problems, consisting of problems from different mathematical areas, may be organized in classes of problems structured by nesting ideas; (2) the feeling of innovation is likely to appear as the result of manipulating or modifying nesting ideas. Having these conjectures in mind, we explored the data set and identified more than forty nesting ideas in Leo's arsenal belonging to various mathematical branches and topics. Leo also told us about more than twenty problems that he created by manipulating or modifying nesting ideas from his arsenal. Three additional examples of problems created by Leo are presented in the left column in table 1. The problems' formulations are presented in the left column in table 1; the right column includes the code names of the classes used by Leo, the description of commonalities between the problems belonging to the class and the essence of Leo's innovation inherent in the posed problem.

Table 1. *Additional examples of problems created by Leo*

Problem	Class the problem belongs to
Problem 3:	Clock problems:
At what time the clock hands are perpendicular? (Appeared at Israeli national-level competition for secondary students in 2010).	The class consists of problems about analogical clocks and special positions of their hands. According to Leo: "[...T]he most known problem of the class is: 'When do the hour and minute hands coincide after 12 o'clock?'" Leo told us that "the most known problem" was suggested for a competition and then: "I changed it a little bit, so it won't look like we lack ideas". Leo's innovation in Problem 3 was a question about perpendicular position of the clock's hands.
Problem 4:	Cutting problems:
The points A, B, C, D, E are located on the circle so that the distance between two neighbouring points is constant.  The broken line divides the area of the circle into two areas: below the line (the grey area) and above the line (the white area). Which area is bigger: the grey or the white one? (Appeared at Israeli national-level competition for 8^{th} and 9^{th} graders in 2007).	The problems of this class are based on a figure divided into two areas by a curve. According to Leo "The typical question of the class is 'Which area is larger and why?'" and the typical answer is that the areas are equal, whereas they do not look equal. Leo explained why he likes this class of problems: "In Israel there is a huge gap between typical scholastic geometry problem and Olympiad one. The average participant of a competition is not even familiar with classical geometrical topics as constructions with ruler and compass, Cheva's theorem etc. Because of that, many thinking directions [of a problem poser] are automatically disqualified. This [situation] creates a serious lack of appropriate ideas for the 8th and 9th graders " Leo continued: "They [the cutting problems] are based on quite basic knowledge of Euclidian geometry and do not require knowledge of rarefied facts." Leo's innovation in Problem 4 was that the white area is larger than the grey one, although it is not obvious from the picture.
Problem 5:	Ellipse:
Two ellipses share a focus. Prove that the ellipses intersect in two points at the most. (Appeared in IMO for college students in 2008).	The problems with ellipse belong to this class. Leo told us that ellipses are one of his favourite topics in plain geometry and that he frequently uses them in his problem posing. He explained that it is "because ellipses have many interesting properties, and not many people know them". Therefore, the innovation is realized through creating problems using a rarefied property of an ellipse. Indeed, Problem 5 can be solved using an uncommon definition of ellipse involving a point and a directix.

Summary and discussion

In the paper we presented fragments of data from a case study of an expert who professionally poses problems for mathematics competitions. Our goal was to substantiate a claim that when creating problems, the expert desires to get a *feeling of innovation*, when his pool of familiar problems serves as a baseline. We illustrated that the feeling may be achieved at the result of manipulating with *nesting ideas* – special organizational units which are ubiquitous in different mathematical topics and fields in expert's pool of familiar problems. In this way, the paper provides an evidence-based example of a symbiotic relationship between cognitive and affective domains. Namely, we illustrated how a cognitive structure (i.e. nesting idea) is intertwining with the achievement of a desire of affective nature (i.e. the feeling of innovation). Taken together, these two may partially explain how high-quality mathematical products (i.e. problems for high-level mathematics competition) appear in the expert's practice. In the following subsections we discuss the introduced notion of nesting ideas in light of the well-known cognitive structures from past research and point out possible explanations for expert's motivation to achieve the feeling of innovation.

Nesting ideas versus chunking and schemas

The notion of nesting idea bears a resemblance to a notion of chunk, since they both are operational ways of dealing with a large amount of data (see the section on experts' problem posing and organization of their knowledge base). Thus, nesting ideas can be considered as special chunks of expert's pool of familiar problems. Leo's practice of manipulation with or modification of various nesting ideas can be considered as a problem-posing scheme: a structured mental action with familiar piece of knowledge aiming at the creation of a new one (see the section on experts' problem posing and organization of their knowledge base once again).

The notion of nesting ideas can turn to be instrumental for pointing out the differences between expertise in problem solving and problem posing. In the context of problem solving experts tend to focus on the similarities in the problems' deep structures. The fragments of presented data exemplify two additional types of nesting ideas, i.e. two additional types of reasons for Leo to include familiar and newly constructed problems in the same class: surface structure nesting ideas (see "cutting problems") and nesting ideas based on particularly rich situations (see "ellipses"). The former type reflects *deep versus surface structure theory* mentioned in the second section on experts' problem posing and organization of their knowledge base; the latter refers to situations with a considerable number of mathematical properties, when each property is represented by a problem in the class. In this type of nesting ideas the deep-level connection between problems' solutions are possible but non-obligatory. In this way, considering alternatives to nesting ideas of a deep structure can be useful at least in some cases in the context of problem posing.

Expert's motivation to achieve the feeling of innovation

We suggest that expert's desire to achieve the feeling of innovation steams from three sources. *The first source is pedagogical*: From the perception analysis of 22 adult participants of the competition movement (Kontorovich, 2012), we know that competition problems are aimed at achieving four (interrelated) pedagogical goals: to supply opportunities for learning meaningful mathematics, to strengthen a positive attitude towards a particular problem and mathematics in general, to create a cognitive difficulty and to surprise. These goals cannot be achieved without a permanent innovation of the pool of competition problems.

The second source is intellectual: Fulfilling a desire to innovate can end up with a creation of a new problem which integrates in the pool of familiar problems of an expert and enriches it. Creating such problems can be seen as an act of acquiring significant knowledge by the expert. This perspective is in line with Ericsson's (2006) one, who wrote that experts tend to engage themselves in deliberate practices in order to extend their already well-developed knowledge base and to sharp their professional skills. In the problem-posing context the journey from the desire to innovate to the feeling of innovation may be accompanied by positive "research" feelings such as excitement of scientific exploration, the thrill of discovery and the sense of ownership for the result (e.g. Liljedahl, 2009).

The third source is social: Mathematics competition movement is a special case of a professional community of practice. One of the characteristics of such communities is an aspiration to gain knowledge. Thus the participants of the community appreciate collaboration, innovation and enrichment of an existing community knowledge base. Their appreciation can fulfill expert's social needs in belonging, esteem and respect (Maslow, 1943).

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Notes

1 Note that this paper and Kontorovich & Koichu's (under review) paper inevitably intersect, though the research goals, the ways of the data analysis and the conclusions are different. This is because of the need to make each of the papers as self-contained as possible.

2 The concept of nesting ideas will be gradually developed in the continuation of the paper.

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