How mathematics students perceive the transition from secondary to tertiary level with particular reference to proof

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This paper reports on a research project concerning the difficulties met by undergraduate students in mathematics during the first year of university. Our aim is to provide elements for studying the transition from secondary to tertiary level as perceived by the students who live it. The combination of different analytical tools (questionnaires, interviews, problem solving and proving activities) allows shedding light on aspects which are not purely cognitive, but also pertain to the affective domain.

There is wide concern about the gap observed in an increasing number of countries between the supply and demand for mathematics students and personnel in educational institutions, the workplace, and the future of the subject itself, see the "Pipeline" project coordinated by ICMI (http://www.mathunion.org/pipeline/pipeline-project/). This gap does not only depend on the fact that students are moving away from enrolling in undergraduate mathematics courses, but also on the sometimes traumatic effect many students experience in the transition from school to university mathematics.

Gueudet (2008) identifies two main lines of research on the topic of transition: detecting students' difficulties and planning interventions for making the transition easier. The study reported in the present paper belongs to the first line of research, with reference to undergraduate students in mathematics. These students, when entering the university program in mathematics, are aware that their curriculum will be mainly based on mathematics. Hence, we may assume that they are motivated in regard to this discipline or, at least, they have not negative feelings about it. We add that, in general, at secondary school they have been good students. Nevertheless, many of these students experience difficulties in the

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transition from secondary school to university. The research presented in this paper is aimed at detecting the factors affecting this transition. As we'll detail in the subsequent sections, our study is characterized by two theoretical and methodological choices: 1) we collected different kinds of qualitative data (questionnaires, interviews, and also written proving processes); 2) we analyzed data adopting a perspective that considers both cognitive and affective factors, in line with our previous studies (Furinghetti & Morselli, 2007, 2009).

Theoretical framework

In order to answer our research question about the factors affecting the transition we investigated the way students perceive their experiences in the first university year, and their process of developing the identity as "apprentice mathematicians". A specific focus is on the way they deal with proof, that we consider a typical mathematical activity. Answers to questionnaires and interviews on the transition and on the first examinations together with the analysis of proving processes carried out by students provided the data for our investigation. These data are analyzed with the theoretical lens of affective factors, in interaction with cognition. Hence, we organize our theoretical framework in three parts: studies on transition, proof at university level, affective factors (and their intertwining with cognition).

Transition

As observed by Clark and Lovric (2008), any transition from a school level to another presents problems for students. As regards mathematics, these problems are particularly marked in the secondary–tertiary transition because it is asked to work at an advanced level, which implies a formal approach with mastering of symbols and language (Tall, 2008). Stadler (2011) points out that the change in dealing with topics often makes students' experience of the transition from school to university mathematics as a bewildering re-visiting of contents and ways of working that seems simultaneously familiar and novel.

The studies on the secondary–tertiary transition or undergraduates' difficulties stress different aspects. Ferrari (2004) focuses on the problem of language in advanced mathematical thinking. Moore (1994) examines the cognitive difficulties that university students experience in learning to do formal mathematical proofs. Castela (1995) and Praslon (1999) analyze the ways the same subject (tangent for Castela and derivative for Praslon) is presented at school and university and the consequent cognitive ruptures. Robert (1998) considers the different organization of knowledge and the complexity of the new contents. Some studies, such as (Di Martino & Maracci, 2009), pay a particular attention to affective factors. Other authors focus on aspects more linked to the context: the didactic contract (Bosch, Fonseca & Gascón, 2004), the influence of social phenomena (De Guzmán, Hodgson, Robert & Villani, 1998), the institutional problems linked to the organization of the courses (Hoyles, Newman & Noss, 2001), or the way university examinations in mathematics are carried out (Griffiths & McLone, 1984).

Proof

Among the factors affecting the transition from secondary to tertiary level the Survey Team 4 presented at ICME-12 in Seoul (Korea) lists changes in competencies "with regard to essential technical facility, analytic powers, and perception of the place of precision and proof in mathematics" (Thomas et al., 2012, p. 90). In the chapter dedicated to transitions of the ICMI Study 19 on proof Selden (2012) assumes as a starting point that "The nature of proofs and proving at tertiary level, with its increased demand for rigour, constitutes a major hurdle for many beginning university students." (p. 392). As a matter of fact, in university courses a big amount of time is dedicated to proof. As an example we cite the case of the lectures of three mathematics professors reported in (Mills, 2011): roughly half the lecture time was dedicated to present proofs. In his investigation about mathematicians' perspectives on their pedagogical practice with respect to proof, Weber (2012) reports that: on one hand, there is a large consistency between mathematicians' and mathematics educators' goals for presenting the different roles of proof; on the other hand, the mathematicians participating to the study "appeared to lack an arsenal of pedagogical strategies to achieve their goals" (p. 478). In light of the concerns outlined above, proof appears as a pivotal element for dealing with the issue of transition, though, as observed by Thomas et al. (2012), there are few studies directly addressing proof as an issue of transition. In our research we have considered proof as an element central for enlightening students' perception of the transition. As Hemmi (2008, 2010) does, we espouse the view of proof as an artifact in mathematical practice, since, following (Säljö, 2005), artifacts are defined "as tools that mediate between the individual and the social practice. They stabilise human practice, facilitate continuities across generations and co-ordinate and discipline human reasoning by suggesting how to do things" (quoted in Hemmi, 2010, p. 273). For its characteristics the approach to the artifact proof marks the development of mathematics students' identity in the community of mathematicians they are entering in the first year of university.

In the present research, we consider the way students perceive the teaching of proof at university level and the way they prove. We see proof in it double status: a process to be performed (say, a problem to be solved) and a final product that must conform to the standards of the reference community.

Affect and cognition

Our study may be framed into the general trend of research in affective factors in the learning of mathematics, see (Evans, Hannula, Zan & Brown, 2006). Since we see affect and cognition as two intertwined representational systems (Furinghetti & Morselli, 2007, 2009), we assume that affective factors play a key role in the transition. More specifically, our focus will be on beliefs about self, about mathematics and about proof. Such beliefs may be the main origin of the interpretation students give for success and failure, i.e. causal attributions. According to Weiner (1980, 1985), attributions and perceptions of success and failure are categorized along three main dimensions: locus (internal vs. external), stability (stable vs. unstable), controllability (controllable vs. uncontrollable) of the causal agent.

Beliefs may reveal themselves useful to investigate how the students entering the university course in mathematics develop their identity as mathematicians. As found by Ward-Penny, Johnston-Wilder and Lee (2011) the failure in this development is one main cause of undergraduates' disaffection for mathematics. On the other hand, learning itself is seen by Heyd-Metzuyanim and Sfard (2012) as an interplay between the activities of mathematizing (talking about mathematical objects) and of identifying (talking about participants of the discourse).

Methodology

Data collection

We set up and carried out a qualitative research, (see Patton, 1990) that developed according to the following steps:

- Administration of a questionnaire to the students enrolled in the first year of the university program in mathematics (50 students); see below for a presentation of the questionnaire.

- Identification of a sample of nine students on which to focus our investigation.
- A first audio-recorded interview to a sample of students, in order to deepen the themes of the questionnaire.
- Written individual problem solving activity performed by the sample students.
- Second audio-recorded interview concerning the written individual problem solving activity.
- Third audio-recorded interview to the sample, at the end of the year, for a sort of concluding outlook.

At the University of Genoa the academic year starts in October. We collected data from November 2010 to May 2011, in order to have a global panorama of the facts experienced by students along the first year of university. As shown the previous list, a peculiarity of our work is the combination of different research instruments (questionnaires, interviews, etc.). Table 1 summarizes the steps of the data collection and the schedule of the university examinations so as to link the steps of our research with the academic life of the students.

Table 1. *The schedule of the academic examinations and the steps of the data collection and population*

Dates	Academic examinations	Data collections
November, the 2nd		Questionnaire (50 students)
November, 15–19	Intermediate examinations (algebra, infinitesimal analysis, geometry)	
November, 22–30		Interview no. 1 (sample)
December, the 6th		Problem solving (sample)
December		Interview no. 2 (sample)
January–February	Intermediate examinations (algebra, analysis) and mid-term examinations (algebra, analysis, mathematical laboratory)	
April, 28–29	Intermediate examinations (physics, probability)	
May, 2–18		Interview no. 3 (sample)

The questionnaire was administered at the beginning of the academic year (around one month after the beginning of the courses). The aim was twofold: gathering data on the whole population and selecting a sample for the in-depth interviews and the problem solving activity. The first part of the questionnaire, designed by a commission appointed by the Mathematics Department, is administered every year to the freshmen. The second part of the questionnaire was explicitly designed and administered by one of the authors (C. M.) for the purpose of the research. The students were informed about the project they were involved in.

The first part of the questionnaire contained multiple-answer questions on the attended secondary school, the motivations for the choice of the university program, the possible role of orientation activities proposed by university in such a choice, expectations and satisfaction about the university program, attendance to tutorial activities, suggestions to improve activities for secondary students aimed at orienting in the choice of the university career.

The second part of the questionnaire contained open-answer questions and Likert scale questions concerning students' previous university studies (if any) in different disciplines, expectations and projects about work after university, the most and the least favorite topics in mathematics at secondary school level, relationship to proof in secondary school, evaluation (in term of Likert scale) of some factors for success in mathematics. Furthermore, students were asked to narrate their own story with mathematics. The idea of studying students' personal stories is in line with the recent trend of narrative research (Lieblich, Tuval-Mashiach & Zilber, 1998), see (Di Martino & Zan, 2010) for a telling example concerning mathematics. Our aim was to collect data in a very open way, so as to allow students to report what they felt as relevant in their past experiences with mathematics (stories of success, difficulties, overcoming difficulties, relevant figures such as teachers etc.). The individual stories may be very different. Nevertheless, our aim was not comparing stories, rather using information coming from each story as a complementary data.

After the administration of the questionnaire, we chose a sample of nine students among those who had declared their willingness to take part in the research project. Criteria for the choice were the kind of secondary school they attended (e.g., secondary school with scientific orientation, with humanistic orientation and so on), their declared intentions about the future job and the answer to the question concerning factors for success in mathematics. We tried to have students coming from different secondary institutes in order to see the effects of different mathematical preparation. Non-commuter students were preferred, so as to work with students who should have more time at disposal for the additional activities we proposed.

The first semi-structured interview to the sample took place in November. Themes of the interview were: the preparation offered in secondary school, first course examinations, factors for success in mathematics, expectations and satisfaction about the university program, (possible) difficulties and strategies to overcome them, proofs (with particular reference to the difference between proofs in secondary schools and proofs at university level). Often there have been questions concerning future job, and influence of parents, friends, teachers or events on the choice of the university program. Some answers to the questionnaire were also discussed during the first interview.

The third step (developed in December) concerned an individual problem solving activity proposed by us, which consisted of five problems, three of which were proving activities. The five problems were given at the same time, and each student could choose in which order to attack the problems. There was not time limit. The students solved the problems individually. They were asked to write down, as much as possible, also their emotions, ideas, etc. This request, unusual for the students, was aimed at leading them be reflective about their way of working.

In the subsequent days, one of the authors (C. M.) carried out individual retrospective interviews on the solving processes. The interviewed students were invited to reconstruct verbally their solving process; in case of mistakes or incomplete solutions, they could amend or complete the solution, autonomously or with the help of the interviewer. During the interview the students were encouraged to discuss the emotions experienced during the solving process.

The last individual semi-structured interview took place in May. The themes of the interview were: examinations already taken during the academic year (expectation and difficulties, strategies to pass them, results), feelings about the choice of the university faculty (regret, satisfaction, surprise, etc.), possible changes in the relationship to mathematics, possible changes in the intentions about the future job, general comments about the participation to the research project.

Data analysis

The different data (questionnaires, interviews, written individual solutions of problem solving tasks) were analyzed separately.

The questionnaires were analyzed according to the categories that were established a priori, following the theoretical references. More specifically, data provided information in terms of beliefs about mathematics and proof, causal attribution, development of identity. The interviews were analyzed according to Grounded Theory (Strauss & Corbin, 1998). We coded data independently; different codes were compared and so on, thus leading to a cycle of analysis.

The solving processes were analyzed with the method already used in other studies, see (Furinghetti & Morselli, 2007, 2009): the whole solving process is seen as made up of two intertwining paths (the cognitive and the affective one). The written data provided by the students encompass mathematical sentences (belonging to the cognitive path), descriptions of the process and comments on the process. Descriptions and comment are interpreted, so as to give an account of the proving path, and after analyzed in terms of the intertwining between affect and cognition, using the interpretive tools at disposal. After the analysis of each kind of data, we combined the findings.

Findings

The analysis of the first interview, combined with the answers to the questionnaire, gives direct insight into students' perception of the passage from secondary school to university. Specific questions on the first examinations provide additional information on causal attribution, perception of controllability, and the way students live the transition. The analysis of the problem solving processes, which included proofs, gives information on the way students deal with a key activity in their university experience and provides indirect insights into their perception of the transition.

Students' views about transition (from the first interview)

As mentioned in the methodology section, the students of the sample come from secondary schools with different orientations. For the purpose of the present paper the main distinction is between schools with scientific or technological orientation aimed at giving strong scientific, in particular mathematical, preparation and schools aimed at giving a humanistic or artistic preparation, where mathematics is rather neglected. The students who studied in a secondary school with scientific orientation, generally deem that secondary school gave them a good preparation in terms of knowledge. Nevertheless, they point out that the transition to university is somehow problematic for them, since mathematics is presented in a different way and the organization of the university activity is different from that of school.

The student Gogear, for instance, accepts as an ineluctable fact the differences between school mathematics and university mathematics. He explains his difficulties in terms of internal causal attribution ("it's my fault"):

> If I had difficulties, it wouldn't be my secondary school's fault, since at secondary school you do so much different things, the approach is totally different. If I have difficulties, it's my fault.

Louis's explanation of the sources of his difficulties is focused on a specific element, e.g. his need of visualizing concepts, as he was used to do in secondary school:

> The work [in secondary school] was more geometrical. For instance, they said: that's a derivative. [...] We just studied limits. I was used to seeing limits in a geometric way. I understand them because I see them in that way. I see the incremental ratio and so on. While here [at university] we have only formulas [...]. It is very hard to see, to imagine what is a derivative. [...] I don't like matrices, because I see tables of numbers to be multiplied in a strange way, I don't see what they are in reality.

Following the model of the three worlds used in (Tall, 2008) to illustrate the cognitive development in mathematics, we may say that Louis's words express the difficulties of the transition secondary–university in term of difficulties in progressing from the conceptual–embodied to the formal world.

Significantly different is the situation of those students who studied in a secondary school with a non-scientific orientation. All those students feel their former preparation to be inadequate for university studies in mathematics. For instance, the student Meg points out that while other students, coming from a secondary school with scientific orientation, seem quite "at ease", because they already studied infinitesimal analysis and so on, she never met some concepts.

Focus on first examinations

The analysis of students' comments (gathered during the initial interview) on the first three mid-term examinations gives additional information on their perception of the transition. The students of the sample experienced difficulties in the examinations: only one student out of 9 (Sara) passed all the three examinations, two students (Deste and Go Gear) passed one examination; all the marks were under 25/30. Table 2 summarizes the way students commented their failure in first three examinations; such comments are organized in terms of causal attribution according to our interpretation of locus. The controllability is dependent on students' interpretation. We may see that some causal attributions are clearly linked to beliefs about self and about mathematics.

Table 2. *Students' comments to failures in terms of causal attribution*

External causal attribution	Lack of time to solve all the exercises (Louis, Deste). Difference of method (in comparison with secondary school) (Meg).
Internal causal attribution	Examinations were difficult for me (Cucky). I was not used in doing exercises by my own (Cucky). I am not yet "into the subject" (Meg, Deste). I am emotional (Meg). I did not study enough (Louis, Ro, Cucky, Lulù). I studied too much (Meg). I wrote down my ideas in a confused way (Paola, Lulù, Deste).

Students' views of proof

From the questionnaire we know that only a part of the students worked on proofs at secondary school. All the students point out that proof is central in university mathematics.

In order to deepen our understanding of students' relationship to proof, three of the five tasks proposed to the students of the sample concerned proof. In the present section we present some findings from the following task:

> Prove that, if x and y are two odd natural numbers, and they are not both equal to 1, then x^2+y^2 is not a prime number.

Four students carried out an algebraic proof by "pushing symbols". Two students proved by means of natural language. One student, Sara, alternated symbols and natural language. The student Cucky started using symbols, but after a while gave up and showed a numeric example. Another student, Paola, used symbols without succeeding. Two students planned to prove by contradiction, but did not succeed, one student claimed to use the proof by contradiction, but, indeed, carried out a direct proof.

The students were asked to write down, as much as possible, also the emotions and thoughts that accompanied their solving process. Although not all were able or were willing to do that, the written documents provide information that goes beyond the pure cognitive and it will be better analyzed through the tools presented in (McLeod, 1992). We may distinguish between "preliminary comments", generated before starting the solving process, and "ongoing comments" generated during the process when the students met a moment of difficulty or impasse and wrote down their emotions associated to this moment. Students' beliefs emerge in terms of insights on their self-confidence and self-efficacy provided by preliminary comments, and from the traces of causal attribution detected in the ongoing comments.

With regard to the "preliminary comments" Sara wrote down at the very beginning: "Ok, that's easy". Afterwards, she carried out the proof without problems. Analogously, Ro started by writing: "My mood in front of the problem is quiet and I decide to start with an [numeric] example". Both comments reveal a good self-efficacy, which makes the approach to the proof smooth.

Totally different is the first comment by Meg, who, after having written down hypothesis and thesis, wrote down: "Panic! I hate proving!". Afterwards, she started reflecting on the text, but, being her approach driven by a low self-efficacy, she seems to be looking for possible causes of difficulties rather than for a real comprehension of the text. This intertwining between affective and cognitive factors resounds the findings in (Furinghetti & Morselli, 2009) about the proving process carried out by mathematics students of the final year. The present study shows that such an intertwining may be present already at the beginning of university studies.

With regard to the "ongoing comments", we consider the performance by Ro. As reported before, she started with a numeric example: she chose two odd numbers, 3 and 5, and computed $x^2+y^2=32+52=34$. Afterwards, at first she wrote down "This is a counterexample". This last sentence was erased and Ro wrote: "I was absent minded and made a great mistake! 34 is not a prime number …". We highlight the causal attribution performed by Ro: she made a mistake because she was "absent minded". This is an internal and controllable cause. We found other occasions in which Ro resorted to internal controllable causal attribution, e.g. she claimed that she did not pass the examinations because "she did not study enough".

We reported on the previous episodes to provide insights on how also in the case of proof the different mathematical preparation may affect that way proof is faced at university. From the interviews we know that Sara and Ro already met proofs in secondary school. They show a good self-efficacy and their causal attributions appear controllable, even when the cognitive factors crash with the affective factors. The lack of continuity in the path towards proof is a real problem for those who face formal proofs for the first time in university. The case of Meg confirms that, as claimed by Moore (1994), the abrupt encounter with proof generates difficulties.

Discussion

We presented and analyzed three kinds of data: preliminary questionnaire, first interview with comments on examinations, proving activities. All data were analyzed with a specific focus on causal attribution and on its relation to beliefs.

The findings show that students, in front of difficulties in coping with university mathematics, perceive a discontinuity between their previous school experience and the current academic experience. This discontinuity generates different causal attributions in relation to students' mathematics preparation in secondary school (scientific vs. non-scientific orientation).

As for students with scientific background the discontinuity concerns the approach to mathematics. They point out that, in comparison with secondary school mathematics, university mathematics is less procedural. Furthermore, the concepts already studied in secondary school are presented in a different, more theoretical and rigorous way:

> In infinitesimal analysis, I thought: I already studied this at secondary school, so I don't need to study it a lot, because I already know it ... but this is not true. Thus, I had to change my mind. (Deste)

The successful transition of these students depends on their capacity of realizing that there is a continuity between school and university mathematics at content level, but that the approach to the same topics may be different so that there is the need for a different formatting of their personal approach. The key point for overcoming the problem of this "false continuity" generated by dealing with the same topics, is to perceive that the cause of difficulty is controllable, that is to say modifiable.

On the contrary, the prevailing causal attribution that some mathematical topics (that were not present or were rather neglected in their secondary school curriculum) are completely unknown (as it is the case of the students coming from secondary school with non-scientific orientation), may generate a feeling that there is a discontinuity which will not possible to overcome. For this reason some students may decide to give up.

Referring to the theory of causal attribution, we may say that the students from secondary school with scientific orientations interpret failures and difficulties in terms of internal causal attribution (*I must change my approach*), while students from non-scientific oriented schools attribute their difficulties to the kind of the secondary school they attended, the past teachers or other external causes. An internal causal attribution is often perceived as controllable, that is to say students feel it is their own responsibility to change, while an external causal attribution is often felt as "without possibility of remedial".

Finally, we note that the presence of internal causal attributions, both in interview and proving activities, may be interpreted as a symptom of an acceptance of the way of teaching in mathematics department as appropriate, even when it is clearly problematic for many students. In the mathematical discourse there is not devolution of authority from the teacher to students, see (Lampert, 1990). This lack of devolution suggests that such students are far from having developed the identity that allows them to enter the mathematical community.

Conclusions

The integration of different analytical tools allowed investigating on how students live the transition by exploiting the different potentialities of questionnaires, interviews, and problem solving or proving activities. Beliefs about self, mathematics, locus of control shape the way the students enter into the university mathematical discourse. There is the need of being aware of the different approaches in secondary school and university and of a clear cut with the past in terms of challenging certain beliefs coming from secondary school that may be a burden in the transition.

All that said, though our research was aimed at detecting students' difficulties, we glimpse some potentialities of our tools of investigation as a didactic tool for promoting awareness in the process of transition, as epitomized by Sara's sentence:

> [Participating in the research] forced me to reflect on such things; all those questions about the reason, how I feel, my method of study ... maybe usually you don't even think about it. But if you are forced, you must focus on it.

This awareness may be the stepping-stone for a successful entrance into the mathematical discourse.

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